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A new distributed Kalman filtering based on means-quare estimation upper bounds☆

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Abstract

In this paper, a novel distributed Kalman filter consisting of a bank of interlaced filters is proposed for a signal model whose dynamic equation and measurement equation are coupled. Each of the interlaced filters estimates a part of state rather than the global state using its and its neighbor information, which is different from other distributed filters already existed (e.g., distributed Kalman filter based on diffusion strategy or consensus strategy, distributed fuzzy filter and distributed particle filter with Gaussian mixer approximation, etc). This relieves the calculation and communication burden in networks. In addition, the proposed distributed Kalman filtering contains no consensus strategies, which is useful in some cases since consensus usually requires an infinite number of iterations.

1. Introduction

Rapid advances in the areas of sensor design, information technologies, and wireless networks have paved the way for the proliferation of wireless sensor networks. More and more network applications require hundreds or thousands of sensor nodes, often deployed in remote

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and inaccessible areas. With the increasing of demands of people, a wireless sensor has not only a sensing component, but also on-board processing, communication, and storage capabilities. With these enhancements, a sensor node is often responsible for data collection, as well as for in-network analysis, correlation, and fusion of its own sensor data and data from other sensor nodes [1].

In response to rapid development and vast deployment of low-cost sensors and sensor networks, distributed filter design has been an extremely active research topic for over a decade. Up to now, there are so many distributed filters for both the linear systems and non-linear systems. For the case of linear systems, such as: A cluster-based distributed Kalman filtering technique is proposed for target tracking in [2]. Nodes which can detect the target at a given time instant forms a cluster. However, the cluster-based processing is still not fully distributed because a node is chosen as a cluster head which collects measurements from other nodes and updates the state estimate collectively in this cluster.

Instead of doing full average consensus, the diffusion technique (e.g., [3,4]) approximates a global average with an average among the neighboring nodes. In addition, in [5], a distributed filed estimation algorithm is given based on the so-called Kriging interpolation technique. The algorithm employs the well-known Jacobi over-relaxation method and a dynamic average consensus method, both requiring an infinite number of iterations in theory. It is not clear how to choose the right number of iterations in practice, especially for large networks. Lately, one common approach to DKF is to use an average consensus strategy as introduced in [6], see [7–9] for examples of this approach. Different from the method employing a consensus strategy to design distributed filter, in [10], author designed a kind of distributed filter, which consists of a bank of interlaced Kalman filters. Each of interlaced Kalman filters only estimates a part of the state and consider the remaining parts as known time-varying parameters whose values are evaluated by the other filters at the previous step. Moreover, in [11], authors concerned with a problem of distributed fuzzy filter design for a class of sensor networks described by discrete-time T-S fuzzy systems with time-varying delays and multiple probabilistic packet losses. For the case of non-linear systems, in [10], authors gave an interlaced extended Kalman filter based on the idea of adaptive estimation, reference [12,13], etc. In [14], authors addressed a consensus-based networked estimation of the state of a nonlinear dynamical system. A family of distributed state estimation algorithms which relies on the extended Kalman filter linearization paradigm is given in. In [15], authors investigated a target tracking problem using a distributed particle filter (DPF) over sensor networks. To approximate the posterior distribution of weighted particles, the parameters exchange of Gaussian mixture model is implemented by the consensus strategy there. In addition, in [16], authors gave two novel distributed particle filters with Gaussian Mixer approximation based on signalling among neighbors and information fusion respectively. More relevant works see references therein.

Before state our writing motivation, we first give an example.

$$x(t+1) = F(t)x(t) + \omega(t), \tag{1a}$$

$$z(t) = \bar{A}(t)x(t) + v(t), \quad t \in \mathbb{N}, \tag{1b}$$

where $x(t) \in \mathbb{R}^n$, $F(t) \in \mathbb{R}^{n\times n}$, $\bar{A}(t) \in \mathbb{R}^{m\times n}$. $\{\omega(t)\}$, $\{v(t)\}$ are individually zero mean, Gaussian processes with known covariances, the initial state $x(0)$ is an independent Gaussian variable with mean x_0 and covariance Σ_0. The signal model Eqs. (1) are the compact form of the following sub-systems:

$$x_i(t+1) = F_i(t)x(t) + \omega_i(t),\ \ i = 1, \dots, N, \tag{2a}$$

$$z_j(t) = \bar{A}_j(t)x(t) + v_j(t),\ \ j = 1, \dots, M,\ \ t \in \mathbb{N}, \tag{2b}$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $z_j(t) \in \mathbb{R}^{m_i}$, $F_i(t) \in \mathbb{R}^{n_i \times n}$ is the time-varying transition matrix, $\bar{A}_j(t) \in \mathbb{R}^{m_i \times n}$ is the time-varying measurement vector. $\omega_i(t)$ is the system noise which is a zero-mean i.i.d. Gaussian noise with covariance $Q_i(t) \geq 0$, and $v_j(t)$ is the measurement noise which is a zero-mean i.i.d. Gaussian noise with covariance $R_j(t) > 0$. i.e., $x(t) = (x_1^{'}(t), \dots, x_N^{'}(t))^{'}$, $z(t) = (z_1^{'}(t), \dots, z_M^{'}(t))^{'}$, $F(t)$ and $\bar{A}(t)$ are given similarly. In addition, the covariance of $\omega(t)$, $v(t)$ are $Q(t) = \text{diag}\{Q_i(t)\}$, $R(t) = \text{diag}\{R_j(t)\}$ respectively. Finally, $x_0 = (\bar{x}_1^{'}(0), \dots, \bar{x}_N^{'}(0))^{'}$, and the principal block of Σ_0 is $\text{diag}(\Sigma_1(0), \dots, \Sigma_N(0))$.

As is well known, (centralized) Kalman filter can be used to estimate the state of the signal model Eq. (1). However, in some cases (e.g, a smart grid network), it is sufficient for each node to estimate state itself rather than the global one in a distributed way. Motivated by this, our interest in this paper is to design a distributed estimation method, such that each of nodes estimates state itself rather than the global one using its and its neighbor information.

Secondly, to meet the needs of some cases, we change the dynamic system model Eq. (1a) to

$$x(t+1) = f(t, x(t)) + \omega(t), \tag{3}$$

where $f : \mathbb{N} \times \mathbb{R}^n \to \mathbb{R}^n$, satisfies some mild conditions which will be given below. The Eq. (3) is the compact form of

$$x_i(t+1) = f_i(t, x_{i_1}, \dots, x_{i_{p_i}}) + \omega_i(t), \quad i = 1, \dots N, \tag{4}$$

where $f(t, x(t)) = (f_1^{'}(t, x_{1_1}, \dots, x_{1_{p_1}}), \dots, f_N^{'}(t, x_{N_1}, \dots, x_{N_{p_N}}))^{'}$, others are the same with the description about Eq. (2a). For each $i = 1, \dots, N$, each element of the subscript set $\{i_1, \dots, i_{p_i}\}$ of the coupled states $x_{i_1}(t), \dots, x_{i_{p_i}}(t)$ with $x_i(t+1)$, presented in $f_i(t, x_{i_1}, \dots, x_{i_{p_i}})$, lies in $\{1, \dots, N\}$.

The reason for dealing with the signal model Eqs. (3) and (1b) contains two aspects: For one thing, in [17], authors discussed a dynamic estimation problem in the power systems. Writing their dynamic and measure equations into a compact form respectively, it is easy to see their model is a special case of ours; For another, in [18], authors investigated a sensor localization problem using minimal number of anchor nodes (to obtain the locations of each node in a network based on the knowledge of the locations of a few anchor nodes as well as the mutual distances between neighbor nodes). Due to the inaccuracy in the distance measurement, the measure can be written as Eq. (2b) with the invariable time instant, see also [19]. Our measurement Eq. (2b) is given to deal with the dynamic estimation problem with a more general dynamic system (4).

Because both the dynamic system Eq. (4) and the measure Eq. (2b) are coupled in state variable, it is hard to design a distributed filter such that each node to estimate the state coordinate itself only using its and its neighborhoods' information. To overcome this difficulty, we firstly give the filtering equations for our signal model Eqs. (4) and (2b) based on the extended Kalman filtering (EKF), then use the method of undetermined coefficients to calculate the unknown parameters presented in the filter equations given initially. A particular advantage of this method is its guaranteed stability, i.e., the error covariance is guaranteed to be bounded. This method was proposed originally in [20] to solve the state estimation problem for a kind of continuous-time nonlinear signal model, the discrete version may reference [21], in which the authors called this kind of filter the bound optimal filter. The idea of this method is: Firstly, designing the filter based on the EKF (the filter equations are the same with the EKF's, but the gain is unknown). Secondly, using the method of undetermined coefficients under some assumptions called cone bound conditions to calculate the unknown gain based on the principle of minimal upper bound for the state estimation error covariance. We remark there that different from their work, we calculate an unknown parameter appeared in our measure update equation instead of the gain to obtain the distributed filter and it makes the calculation more complex. In addition, they deal with the problem of centralized dynamic estimation, while we design a distributed estimation method to deal with the problem of distributed dynamic estimation.

In this paper, a novel distributed Kalman filter consisting of a bank of interlaced filters is proposed for a signal model whose dynamic and measurement equation are coupled. Each of the interlaced filters estimates a part of state rather than the global state using its and its neighbor information (neighbor state estimation and measurement), which is different from other distributed filters already existed (e.g., distributed Kalman filter based on diffusion strategy or consensus strategy, distributed fuzzy filter and distributed particle filter with Gaussian mixer approximation. etc). This relieves the calculation and communication burden in networks. In addition, compared with the interlaced extended Kalman filter [10], the signal model equations and the filter equations are decoupled in our filter designed which needs less amount of calculation than the former one that depends on the Taylor expansion. In fact, in the case of non-linear systems, Taylor expansion leads the linearization equations and the interlaced extended Kalman filter equations to be coupled. Compared with consensus-based distributed EKF or consensus-based DPF, the proposed DKF contains no consensus strategy and it is applied to solve some estimation problems appearing in power systems and sensor networks, see [17,22].

The paper is organized as follow. Section 2 describes the system model. Section 3 gives our main results. Section 4 designs an algorithm to implement the DKF constructed in Section 3. Section 5 illustrates its effectiveness by simulations. Section 6 summarizes our paper and appendix contains proofs of our theorems.

In this paper, we use the following notations. For vectors and matrices, the superscript$'$ denotes their transpose. I denotes the unit matrix, tr$M$ denotes the trace of matrix M and diag(.) denotes the block diagonal matrix. For a column x, $\|x\|$ denotes the Euclidean norm $\|x\| = (x^{'}x)^{\frac{1}{2}}$. For two sets A and B, $A \times B$ denotes their Cartesian product and $(A)^{\sharp}$ denotes the cardinal number of set A. Inequalities between square symmetric scalar matrices are taken in the following sense: $A \geq B$ means that $A - B$ is nonnegative definite.

2. Problem statement

Consider a network formed by N nodes. For each time $t \in \mathbb{N}$, node i has an associated parameter vector $x_i(t)$, which complies with Eq. (4). i.e.,

$$x_i(t+1) = f_i(t, x_{i_1}(t), .., x_{i_{p_i}}(t)) + \omega_i(t), \quad i = 1, .., N \tag{4}$$

and the measurement equations are given by Eq. (2b). i.e.,

$$\begin{aligned} z_j(t) &= \bar{A}_j(t)x(t) + v_j(t), \\ &= \sum_{l=1}^{N} \bar{A}_{j,l}(t)x_l(t) + v_j(t), \quad j = 1, \dots, M, \end{aligned} \tag{2b}$$

where $\bar{A}_j(t) = (\bar{A}_{j,1}(t), \dots, \bar{A}_{j,N}(t))$, and the describe about the states, measurements, etc. are the same as above. Stacking up all the states and the measurements respectively give a compact form Eqs. (3) and (1b), i.e.,

$$x(t+1) = f(t, x(t)) + \omega(t), \tag{3}$$

$$z(t) = \bar{A}(t)x(t) + v(t) \tag{1b}$$

For the coupling both in dynamic Eq. (4) and the measurement Eq. (2b), we introduce the following notations. It is convenient to express the distributed filtering constructed later. With respect to the dynamic Eq. (4), we denote $\mathcal{I}_i^{time}(t) = \{j : x_j(t+1) = f_j(t, \dots, x_i(t), \dots)\}$, i.e., the set of nodes whose states at time $t+1$ involve the state $x_i(t)$ of node i at time t. And we denote $\mathcal{O}_i^{time}(t) = \{i_1, \dots, i_{p_i}\}$, i.e., the set of nodes whose states are involved in the state $x_i(t+1)$ of node i. Furthermore, Let $\mathcal{N}_i^{time}(t) = \mathcal{I}_i^{time}(t) \bigcup \mathcal{O}_i^{time}(t)$ be the system neighborhood of node i at time t. In parallel, with respect to the measurement Eq. (2b), we denote $\mathcal{I}_i^{meas}(t) = \{j : \bar{A}_{j,i}(t) \neq 0\}$, i.e., the set of nodes whose measurements involve the state of node i at time t, and denote $\mathcal{O}_i^{meas}(t) = \{j : \bar{A}_{i,j}(t) \neq 0\}$, i.e., the set of nodes whose states are involved in the measurement of node i at time t. Moreover, Let $\mathcal{N}_i^{meas}(t) = \mathcal{I}_i^{meas}(t) \bigcup \mathcal{O}_i^{meas}(t)$ be the measurement neighborhood of node i at time t in the process of measurement.

Remark 1. In Eq. (2b), some $\bar{A}_{i,j}(t), i = 1, \dots, N, j = 1, \dots, M$, may be zero, which depend on the measure $z_i(t)$. In addition, to simplify the notation, we drop time index t appearing in $\mathcal{I}_i^{time}(t)$ and $\mathcal{O}_i^{time}(t)$, etc. in the following description.

Let $\hat{x}(t|t) = \mathbb{E}\{x(t)|z(1), \dots, z(t)\}$ and $\Sigma(t|t)$ be the associated estimation error covariance matrix. Similarly, $\hat{x}(t|t-1) = \mathbb{E}\{x(t)|z(1), \dots, z(t-1)\}$ and its error covariance matrix is $\Sigma(t|t-1)$. For the signal model Eqs. (3) and (1b), suppose $f(\cdot)$ satisfies some well conditions, then the iteration equations for the state estimation of the EKF are given as follow, reference such as [21]

$$\hat{x}(t|t) = \hat{x}(t|t-1) + K(t)\Big(z(t) - H(t)\hat{x}(t|t-1)\Big), \tag{5a}$$

$$\hat{x}(t+1|t) = f(t, \hat{x}(t|t)), \tag{5b}$$

where $K(t) = \Sigma(t|t-1)H'(t)\Big(H(t)\Sigma(t|t-1)H'(t) + R(t)\Big)^{-1}$, and initialization is provided by $\hat{x}(0|-1) = x_0 = (\bar{x}_1'(0), \dots, \bar{x}_N'(0))'$. Via the well-known matrix inversion lemma [23], we have

$$\begin{aligned} &\Sigma(t|t-1)H'(t)\Big(H(t)\Sigma(t|t-1)H'(t) + R(t)\Big)^{-1} \\ =&\Big(\Sigma^{-1}(t|t-1) + H'(t)\Sigma(t|t-1)H(t)\Big)^{-1} H'(t)R^{-1}(t) \\ =&\Sigma(t|t)H'(t)R^{-1}(t). \end{aligned}$$

Thus, Eq. (5a) can be rewritten as:

$$\hat{x}(t|t) = \hat{x}(t|t-1) + \Sigma(t|t)H'(t)R^{-1}(t)\Big(z(t) - H(t)\hat{x}(t|t-1)\Big). \tag{6}$$

Because the covariance matrix $\Sigma(t|t)$ is not block diagonal, it is hard to obtain a distributed filtering. If we use only the principal block of $\Sigma(t|t)$ to obtain a distributed filter, obviously, there will be some data loss. Our method is to assume the covariance matrix $\Sigma(t|t)$ to be a block diagonal parameter matrix $\text{diag}\{L_i(t)\}$, we don't know what each $L_i \in \mathbb{R}^{n_i\times n_i}$, $i = 1,\dots,N$ is, but we will use the method of undetermined coefficients to pin them down based on the principle of minimal upper bound for the state estimation error covariance.

Considering the ith state variable $\hat{x}_i(t|t)$ of Eq. (6), with the aid of the notations $\mathcal{I}_i^{meas}$, $\mathcal{O}_i^{meas}$ and via a simple calculation, we have

$$\hat{x}_i(t|t) = \hat{x}_i(t|t-1) + L_i(t)\sum_{k\in\mathcal{I}_i^{meas}} \bar{A}'_{k,i}(t)R_k^{-1}(t) \times \left[z_k(t) - \sum_{j\in\mathcal{O}_k^{meas}} \bar{A}_{k,j}(t)\hat{x}_j(t|t-1)\right]. \tag{7a}$$

Similarly, considering the ith state variable $\hat{x}_i(t+1|t)$ of Eq. (5b), we have

$$\hat{x}_i(t+1|t) = f_i(t,\hat{x}_{i_1}(t|t),\dots,\hat{x}_{i_{p_i}}(t|t)). \tag{7b}$$

Based on the discuss above, for each $i = 1,\dots,N$, we consider distributed filters Eq. (7) given above. In Eq. (7a), $\{L_i(t)|t \in \mathbb{N}\}$ is to be a fixed sequence, and in the rest of this paper, our main goal is to calculate them. To start, let us first give the following assumptions.

Assumption 1. There exists a coefficient $\overline{F}_i(t)$ for the nonlinear function $f_i(.)$ to satisfy the cone bound for each $i = 1,\dots,N$. About the cone bound conditions, reference for example [20,21]

$$\|f_i(t,x_{i_1}+\delta_{i_1},\dots,x_{i_{p_i}}+\delta_{i_{p_i}}) - f_i(t,x_{i_1},\dots,x_{i_{p_i}}) - \overline{F}_i(t)\cdot\delta\| \le \phi_i(t)\|\delta\|, \tag{8}$$

where $\overline{F}_i(t) = (\bar{F}_{i,i_1}(t),\dots,\bar{F}_{i_{p_i}}(t))$, $\delta = (\delta'_{i_1},\dots,\delta'_{i_{p_i}})'$ and $\phi_i(t) \ge 0$ is a real-value function. For all x_{i_j}, δ_{i_l}, $\overline{F}_i(t)$ and $\phi_i(t)$ are independent of x_{i_j}, δ_{i_l}.

Remark 2. Regarding Assumption 1, it holds if the nonlinear function satisfies the hypotheses of the finite-increment theorem [24]. With respective to Eq. (8), evidently, if x is a scalar and $f_i(.)$ is differentiable, its slope lies between $\overline{F}_i(t) - \phi_i(t)$ and $\overline{F}_i(t) + \phi_i(t)$. Similarly, the property about the slope holds for the case of x a vector, just change $\phi_i(t)$ into a vector each of whose coordinate is a copy of $\phi_i(t)$. Moreover, it is easy to verify that the linear dynamic system considered in [17,22] satisfies this cone bound conditions.

Without this assumption, to our knowledge, the interlaced extended Kalman filter [10] still works. It is an interesting problem that how to design the distributed particle filter, distributed unscented filter, etc., under the constraint that each of filter of them only estimates a part of the global state (e.g. in power grid, each of node estimates the state itself rather than the global one).

Assumption 2. For each $i = 1,\dots,N$, the node i can send/receive information to/from all its neighbors. Also, $\bar{F}_{i,j}(t)$ for all $j \in \mathcal{O}_i^{time}$, $\bar{A}_{i,j}(t)$ for all $j \in \mathcal{O}_i^{meas}$, and $R_i(t)$ are available at node i.

Remark 3. Regarding with Assumption 2, Marelli and Fu [19] has taken the same assumption. We remark here that without this assumption, an estimate problem with packet losses should be considered. For example, see [4,25] for the relevant works.

3. Main results

In this section, we give our main theorem, and put their proofs into appendix. In Theorem 1, we calculate upper bounds for the state estimation covariances. It is well to keep in mind that the looseness of the sector bound Eq. (8) and the relevant coupling presented in Eq. (4), Eq. (2b) determine the looseness of the performance bounds to be derived. In Theorem 2, based on the upper bounds obtained from Theorem 1, we derive the optimal parameter L_i such that the upper bounds, which relies on the parameter L_i, to be minimal.

Theorem 1. *Let* $\tilde{x}_i(t|t) = x_i(t) - \hat{x}_i(t|t)$, $\Sigma_i(t|t) = \mathbb{E}\{\tilde{x}_i(t|t)\tilde{x}_i^{'}(t|t)\}$, $\tilde{x}_i(t|t-1) = x_i(t) - \hat{x}_i(t|t-1)$ *and* $\Sigma_i(t|t-1) = \mathbb{E}\{\tilde{x}_i(t|t-1)\tilde{x}_i^{'}(t|t-1)\}$. *With the signal model Eqs. (4) and (2b), the filter Eq. (7), the filter error covariances are bounded as* $\Sigma_i(t|t) \leq \bar{\Sigma}_i(t|t)$ *and* $\Sigma_i(t+1|t) \leq \bar{\Sigma}_i(t+1|t)$, *for each* $i = 1, \ldots, N$, *where* $\bar{\Sigma}_i(t|t)$ *and* $\bar{\Sigma}_i(t+1|t)$ *be given by the following iterative Eq. (9):*

$$\begin{aligned}\bar{\Sigma}_i(t|t) =& (1+\alpha_i(t))\Big(I - L_i(t)\Psi_{i,i}(t)\Big)\bar{\Sigma}_i(t|t-1)\Big(I - L_i(t)\Psi_{i,i}(t)\Big)^{'} \\ &+ (1+1/\alpha_i(t))\Bigg(\sum_{k\in\mathcal{I}_i}(\mathcal{O}_k^{meas,o})^{\sharp}\Bigg)L_i(t) \\ &\times \Bigg[\sum_{k\in\mathcal{I}_i}\sum_{j\in\mathcal{O}_k^{meas,o}}\Psi_{i,j}^{(k)}(t)\bar{\Sigma}_j(t|t-1)\Psi_{i,j}^{(k)}(t)^{'}\Bigg]L_i^{'}(t) \\ &+ L_i(t)\Psi_{i,i}(t)L_i^{'}(t),\end{aligned} \tag{9a}$$

$$\begin{aligned}\bar{\Sigma}_i(t+1|t) =& (1+\beta_i(t))(\mathcal{O}_i^{time})^{\sharp}\sum_{j\in\mathcal{O}_i^{time}}\bar{F}_{i,j}(t)\bar{\Sigma}_j(t|t)\bar{F}_{i,j}^{'}(t) \\ &+ (1+1/\beta_i(t))\phi_i^2(t)\sum_{j\in\mathcal{O}_i^{time}}tr\Big(\bar{\Sigma}_j(t|t)\Big)I_{n_i} \\ &+ Q_i(t),\end{aligned} \tag{9b}$$

which is initialized by $\bar{\Sigma}_i(0|-1) = \Sigma_i(0)$. *Herein, we denote* $\mathcal{O}_k^{meas,o} =: \mathcal{O}_k^{meas} \setminus \{i\}$, *and denote* $\Psi_{i,j}^{(k)}(t) = \bar{A}_{k,i}^{'}(t)R_k^{-1}(t)\bar{A}_{k,j}(t)$. *Similarly,* $\Psi_{i,i}^{(k)}(t) = \bar{A}_{k,i}^{'}(t)R_k^{-1}(t)\bar{A}_{k,i}(t)$ *and* $\Psi_{i,i}(t) = \sum_{k\in\mathcal{I}_i}\Psi_{i,i}^{(k)}(t)$.

Remark 4. By searching the $\alpha_i\beta_i$ space to minimize $tr(\bar{\Sigma}_i(t|t))$ and $tr(\bar{\Sigma}_i(t+1|t))$. Setting the relevant partial derivatives to zero yields the optimal values:

$$\begin{aligned}\alpha_i^*(t) =& \left[\frac{tr\{\sum_{k\in\mathcal{I}_i}\sum_{j\in\mathcal{O}_k^{meas,o}}L_i(t)\Psi_{i,j}^{(k)}(t)\bar{\Sigma}_j(t|t-1)\Psi_{i,j}^{(k)}(t)^{'}L_i(t)^{'}\}}{tr\{(I - L_i(t)\Psi_{i,i}(t))\bar{\Sigma}_i(t|t-1)(I - L_i(t)\Psi_{i,i}(t))^{'}\}}\right]^{\frac{1}{2}} \\ &\times\left[\sum_{k\in\mathcal{I}_i^{meas}}(\mathcal{O}_k^{meas,o})^{\sharp}\right]^{\frac{1}{2}},\end{aligned}$$

$$\beta_i^*(t) = \left[\frac{n_i\phi_i(t)\sum_{j\in\mathcal{O}_i^{time}}tr(\Sigma_j(t|t))}{(\mathcal{O}_i^{time})^{\sharp}\sum_{j\in\mathcal{O}_i^{time}}tr(\bar{F}_{i,j}(t)\bar{\Sigma}_j(t|t)\bar{F}_{i,j}^{'}(t))}\right]^{\frac{1}{2}}.$$

For simplicity, we take $\alpha_i(t) \equiv \beta_i(t) \equiv 1$ for each $i = 1, \ldots, N$ and $t \in \mathbb{N}$.

Remark 5. An interesting thing is that as the cone bounds collapse, i.e., the nonlinearities becomes more linear in the sense that the cone bound parameter $\phi_i(t)$ approaches zero, and the signal model Eq. (4), Eq. (2b) have no coupling, then the N filters Eq. (7), Eq. (9) become all the standard Kalman filters.

Theorem 2. *With notation as earlier, for each $i = 1, \ldots, N$ and $t \in \mathbb{N}$, let*

$$L_i^*(t) = \arg\min_{L_i(t)} \bar{\Sigma}_i(t|t),$$

then the optimal sequence $\{L_i^*(t)|t \in \mathbb{N}\}$ is given by:

$$L_i^*(t) = (1+\alpha_i(t))\bar{\Sigma}_i^*(t|t-1)\Psi_{i,i}(t)V_i^*(t)^{-1}, \tag{10}$$

where

$$\begin{aligned} V_i^*(t) = {} & (1+\alpha_i(t))\Psi_{i,i}(t)\bar{\Sigma}_i^*(t|t-1)\Psi_{i,i}(t) + (1+1/\alpha_i(t)) \\ & \times \left(\sum_{k\in\mathcal{I}_i^{meas}} (\mathcal{O}_k^{meas,o})^\sharp \right) \sum_{k\in\mathcal{I}_i^{meas}} \sum_{j\in\mathcal{O}_k^{meas,o}} \Psi_{i,j}^{(k)}(t)\bar{\Sigma}_j^*(t|t-1)\Psi_{i,j}^{(k)}(t)' \\ & + \Psi_{i,i}(t). \end{aligned} \tag{11}$$

and the corresponding upper bounds are given by:

$$\bar{\Sigma}_i^*(t|t) = (1+\alpha_i(t))\Big[\bar{\Sigma}_i^*(t|t-1) - \bar{\Sigma}_i^*(t|t-1)\Psi_{i,i}(t)L_i^*(t)'\Big], \tag{12a}$$

$$\begin{aligned} \bar{\Sigma}_i^*(t+1|t) = {} & (1+\beta_i(t))(\mathcal{O}_i^{time})^\sharp \sum_{j\in\mathcal{O}_i^{time}} \bar{F}_{i,j}(t)\bar{\Sigma}_j^*(t|t)\bar{F}_{i,j}'(t) \\ & + (1+1/\beta_i(t))\phi_i^2(t) \sum_{j\in\mathcal{O}_i^{time}} tr\Big(\bar{\Sigma}_j^*(t|t)\Big)I_{n_i} + Q_i(t), \end{aligned} \tag{12b}$$

which is initialized by $\bar{\Sigma}_i^*(0|-1) = \Sigma_i(0)$.

Remark 6. For each $i = 1, \ldots, N$, Eqs. (7), (10)–(12) give the DKF which is initialized by $\hat{x}_i(0|-1) = \bar{x}_i(0)$ and $\bar{\Sigma}_i^*(0|-1) = \Sigma_i(0)$. Via the iterations of the upper bounds Eq. (12), the optimal parameters $L_i(t)$ can be calculated, then the filter Eq. (7) runs. Based on Eqs. (7), (10)–(12), each filter estimates its corresponding state variable x_i, only using its and its neighborhood's measurements, estimations, etc. Furthermore, compared with DKF based on the consensus strategies, this kind of DKF maybe useful in some cases, because consensus requires an infinite number of iterations in theory.

4. Distributed algorithm reality

In this section, we design a distributed algorithm to implement our filters Eqs. (7), (10)–(12). To this end, we need the following observations. Based on Assumption 2, for each time $t \in \mathbb{N}$, $\bar{A}_{k,j}(t)$, $R_k(t)$ are only available at node k. That is, in the process of measurement update, node k acts as an intermediary between node j, which transmits $\hat{x}_j(t|t-1)$, and node i, which receives $\sum_{j\in\mathcal{O}_k^{meas}} \bar{A}_{k,i}'(t)R_k^{-1}(t)\bar{A}_{k,j}(t)\hat{x}_j(t|t-1)$. This means that node j should transmit $\hat{x}_j(t|t-1)$ to all node k with $j \in \mathcal{O}_k^{meas}$, or equivalently, to all nodes in $\mathcal{I}_j^{meas}$. However, node j doesn't know which node are in $\mathcal{I}_j^{meas}$. Thus, node j simply transmits $\hat{x}_j(t|t-$

1) to all nodes in $\mathcal{N}_j^{meas}$, and it is up to the receiving node k to detect whether node $j \in \mathcal{O}_k^{meas}$ or not. Now, we give the algorithm, in which each node i, $i = 1, \ldots, N$, just use its and its neighborhoods' information to estimate the state $x_i(t)$ itself, and all nodes finish their state estimations via this kind of cooperation.

Algorithm 1. *Initialization:*

For each $i = 1, \ldots, N$, node i sets $\hat{x}_i(0|-1) = \bar{x}_i(0)$ and $\bar{\Sigma}_i^*(0|-1) = \Sigma_i(0)$.

Main Loop: At time $t \in \mathbb{N}$

Measurement update:

(1) For each $j = 1, \ldots, N$ and $k \in \mathcal{N}_j^{meas}$, node j sends $\left(\hat{x}_j(t|t-1), \bar{\Sigma}_j^*(t|t-1)\right)$ to node k.

(2) On reception, for each $k = 1, \ldots, M$ and $i \in \mathcal{O}_k^{meas}$, node k computes:

$$\Psi_{i,j}^{(k)}(t) = \bar{A}_{k,i}'(t) R_k^{-1}(t) \bar{A}_{k,j}(t),$$

$$\check{x}_{i,k}(t) = \sum_{j \in \mathcal{O}_k^{meas}} \Psi_{i,j}^{(k)}(t) \hat{x}_j(t|t-1),$$

$$\Xi_{i,k}(t) = \sum_{j \in \mathcal{O}_k^{meas,o}} \Psi_{i,j}^{(k)}(t), \bar{\Sigma}_j^*(t|t-1) \Psi_{i,j}^{(k)}(t)',$$

together with $\gamma_i^{(k)}(t) = \bar{A}_{k,i}'(t) R_k^{-1}(t) z_k(t)$, $\Psi_{i,i}^{(k)}(t) = \bar{A}_{k,i}'(t) R_k^{-1}(t) \bar{A}_{k,i}(t)$, then sends $(\gamma_i^{(k)}(t), \Psi_{i,i}^{(k)}(t), \check{x}_{i,k}(t), \Xi_{i,k}(t), (\mathcal{O}_k)^\sharp)$ to node i.

(3) On reception, for each $i = 1, \ldots, N$, node i computes:

$$\hat{x}_i(t|t) = \hat{x}_i(t|t-1) + L_i^*(t) \left(\sum_{k \in \mathcal{I}_i^{meas}} \gamma_i^{(k)}(t) - \sum_{k \in \mathcal{I}_i^{meas}} \check{x}_{i,k}(t) \right),$$

$$\bar{\Sigma}_i^*(t|t) = 2\bar{\Sigma}_i^*(t|t-1) - 2\bar{\Sigma}_i^*(t|t-1) \sum_{k \in \mathcal{I}_i^{meas}} \Psi_{i,i}^{(k)}(t) L_i^*(t)',$$

where $L_i^*(t)$ and V_i^* are given by Eqs. (10) and (11) respectively just taking $\alpha_i(t)$ one there.

(4) For each $j = 1, .., N$, and $l \in \mathcal{I}_j^{time}$, node j send $(\hat{x}_j(t|t), \bar{\Sigma}_j^*(t|t)$ to node l.

Time update: For each $i = 1, \ldots, N$, node i computes:

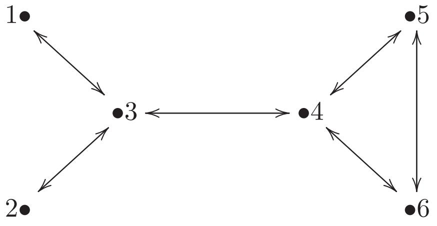
$$\hat{x}_i(t+1|t) = f_i(t, \hat{x}_{i_1}(t|t), \ldots, \hat{x}_{i_{p_i}}(t|t)),$$

$$\bar{\Sigma}_i^*(t+1|t) = 2(\mathcal{O}_i^{time})^\sharp \sum_{j \in \mathcal{O}_i^{time}} \bar{F}_{i,j}(t) \bar{\Sigma}_j^*(t|t) \bar{F}_{i,j}'(t) + 2\phi_i^2(t) \sum_{j \in \mathcal{O}_i^{time}} \mathrm{tr}\left(\Sigma_j^*(t|t)\right) I_{n_i} + Q_i(t),$$

where $\{i_1, \ldots, i_{p_i}\} = \mathcal{O}_i^{time} \subseteq \{1, \ldots, N\}$, then transmits $\left(\hat{x}_i(t+1|t), \bar{\Sigma}_i^*(t+1|t)\right)$ to node k, with $k \in \mathcal{N}_i^{meas}$.

5. Simulation results and discussions

As is well known, the centralized Kalman filter (CKF) is optimal for the linear signal system with additively white noise, in the sense of minimum variance estimate. In this section, a sparse network and a linear signal system will be studied and the performance of the DKF designed with the CKF will be compared. We neglect the influence of the time-delay and dropout, etc., in the process of information transmission. The following picture describes the measurement neighborhood of six nodes, where each node corresponds with one state variable.



We take the system noise covariance $\sigma_w = 1^2$, and measurement noise covariance $\sigma_v = 10^2$. The system matrix $\bar{F} = \text{diag}\{0.65, 0.65, 0.49, 0.72, 0.61, 0.61\}$, and the measurement matrix

$$\bar{A} = \begin{pmatrix} 1 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 10 & 0.2 & 0 & 0 & 0 \\ 5 & 6 & 6 & 5 & 0 & 0 \\ 0 & 0 & 5 & 6 & 5 & 6 \\ 0 & 0 & 0 & 0.1 & 15 & 0.1 \\ 0 & 0 & 0 & 0.1 & 0.1 & 1 \end{pmatrix}.$$

For each $k = 1, 2, 3, 4, 5, 6$, in Fig. $2k$-1, the red line denotes the real state x_k, and the green line and the blue line denote the estimate for x_k using the CKF and DKF respectively. Each of the real state x_k is initialized by 1, and all initial values of both filters are zeros. In Fig. $2k$, the green line, started from 0.5, denotes the covariance of estimate error x_k using the CKF, the blue line, started from 0.5 as well, denotes the upper bound of the estimate error covariance of x_k using the DKF.

Fig. 1 shows that the filtering trajectory using DKF is very close to the one by using CKF and Fig. 2 shows that the upper bound of the covariance trajectory using DKF converges more slowly than the covariance trajectory using CKF, this is reasonable since the data transformation process is needed in a distributed way. Fig. 3 and Fig. 4 show that the estimate performance of x_2. From Fig. 4, it is easy to see that the estimate performance of

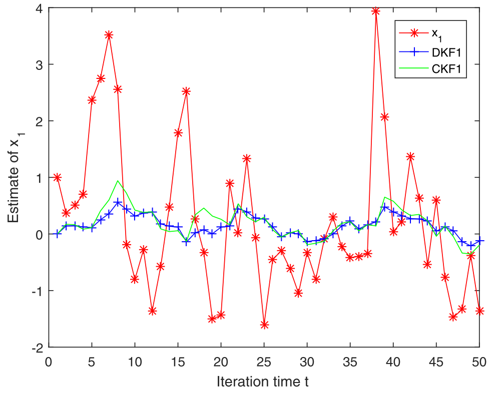


Fig. 1. State estimate of x_1.

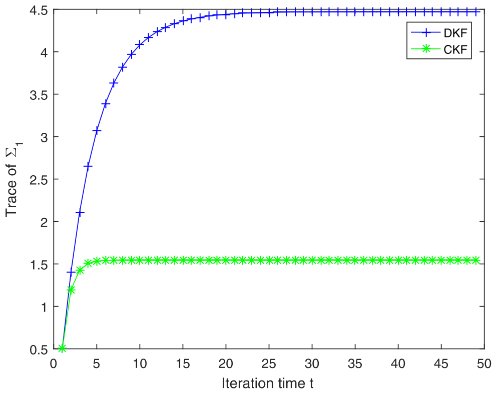


Fig. 2. Estimate error of x_1.

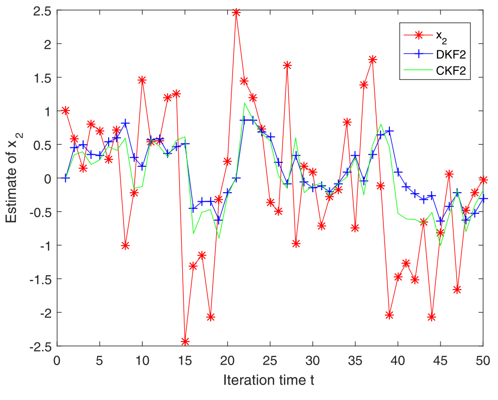


Fig. 3. State estimate of x_2.

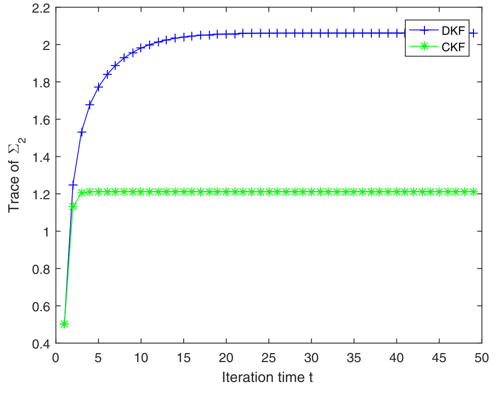


Fig. 4. Estimate error of x_2.

x_2 is better than x_1's. To be specific, the maximal upper bound of the estimate deviation of x_2 using DKF reaches about 2.1, which is less than the value 4.5 of x_1's. We explain the reason as follow. Because Agent 1 and Agent 2 have the same dynamic system parameters (0.65), the main difference are the weights of the measurement parameters of both

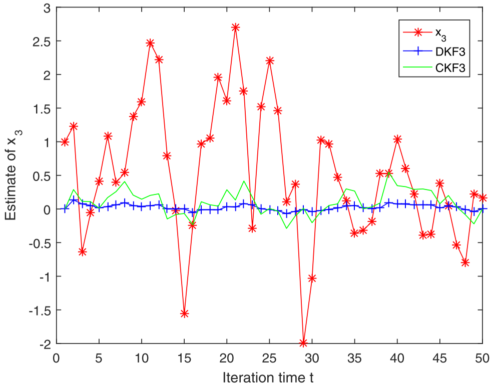


Fig. 5. State estimate of x_3.

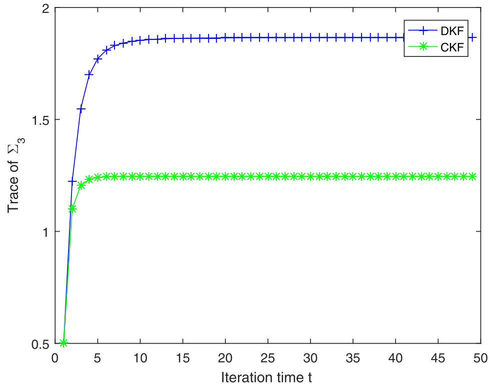


Fig. 6. Estimate error of x_3.

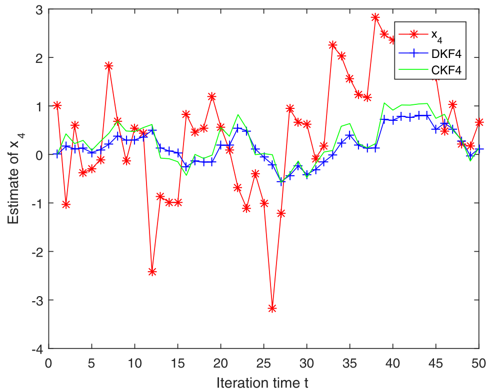


Fig. 7. State estimate of x_4.

agents. In measurement equation of Agent 1, the proportion of the state x_1 is 1/0.2, while in measurement equation of Agent 2, the proportion of the state x_2 is 10/0.2. From intuition, in the latter case, the measurement data of all coupled state variables (in this case, there is only x_3) are likely to be negligible (for simplicity, we call this situation " measure weak coupled" here). Based on this fact and Remark 5, the estimate performance of x_2 is better than x_1's.

From Figs. 5–8, it seems that the state estimate performance of x_3 is better than x_4's. Notice that both their measurement equations have the similar measure parameters, the main

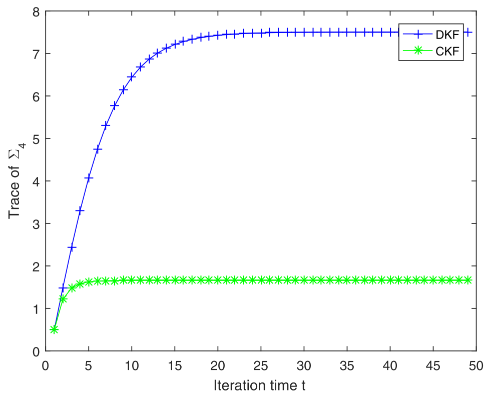


Fig. 8. Estimate error of x_4.

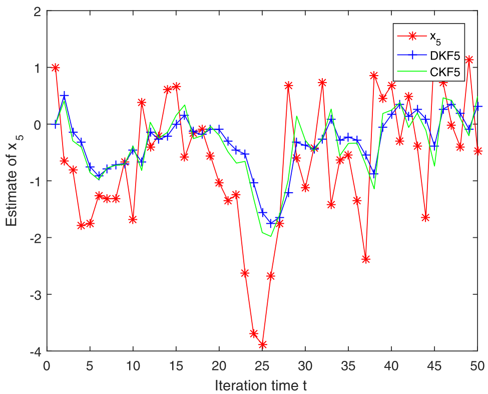


Fig. 9. State estimate of x_5.

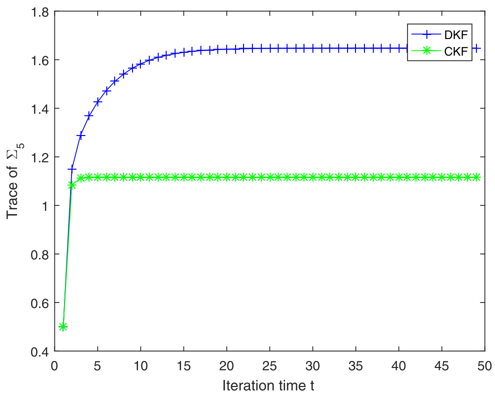


Fig. 10. Estimate error of x_5.

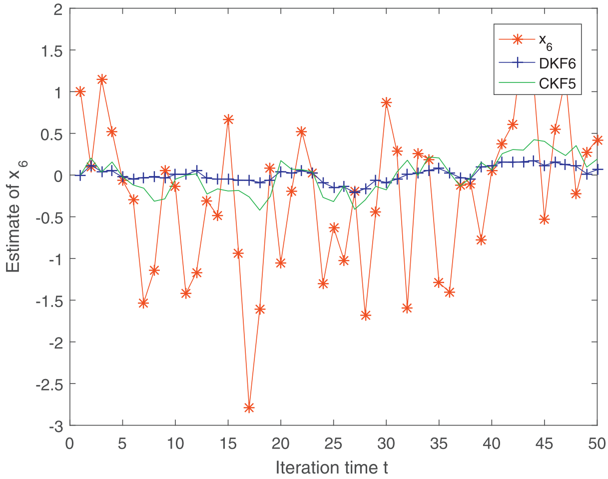


Fig. 11. State estimate of x_6.

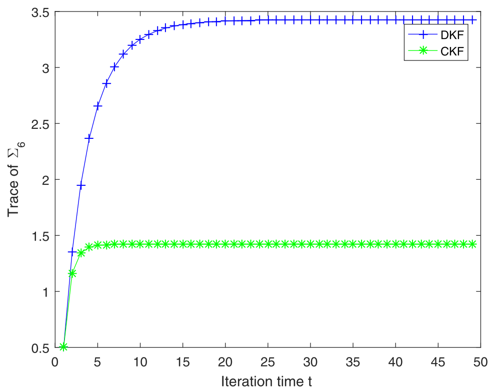


Fig. 12. Estimate error of x_6.

reason is the system parameter (0.49) of Agent 3's is less than the one (0.72) of Agent 4's, where we have neglected the effect caused by their respective neighbors. Finally, the same reason for the comparison between Agent 1 and Agent 2 yields the estimate performance of x_5 should be better than the case of x_6. The simulation results, i.e., Figs. 9–12, are also consistent with this.

From the simulation analysis above, we know that the state estimate performance of each agent is affected by both its dynamic system parameter and the weight of the coupled state variables in its measurement equation. Generally, if the dynamic system parameter is small and the measure is weakly coupled (remember Remark 5), the state estimate performance will be better, see Figs. 3, 4, 9,10.

6. Conclusion

In this paper, a novel distributed Kalman filter consisting of a bank of interlaced filters is proposed for a signal model whose dynamic system equation and measurement equation are coupled. Different from some distributed filters already existed, each of the interlaced filters estimates a part of state rather than the global state in the distributed way. This relieves the calculation and communication burden in networks. This kind of DKF maybe have wide application in power systems and sensor networks, for example, it can deal with the models appeared in [17,22].

Appendix A

A1. Proof of Theorem 1

Let

$$\tilde{f}_i(t) = f_i(t, x_{i_1}(t), \ldots, x_{i_{p_i}}(t)) - f_i(t, \hat{x}_{i_1}(t|t), \ldots, \hat{x}_{i_{p_i}}(t|t)), \tag{13}$$

$$q_i(t) = \tilde{f}_i(t) - \sum_{j\in\mathcal{O}_i^{time}} \bar{F}_{i,j}(t)\tilde{x}_j(t|t), \tag{14}$$

where we denote $q_i(t, x_{i_1}(t), \ldots, x_{i_{p_i}}(t), \hat{x}_{i_1}(t|t), \ldots, \hat{x}_{i_{p_i}}(t|t))$ as $p_i(t)$ for short, so is the case of $\tilde{f}_i(t)$'s.

Eqs. (4), (7b) and (13) yield

$$\tilde{x}_i(t+1|t) = \tilde{f}_i(t) + \omega_i(t).$$

Via Eq. (14), we have

$$\begin{aligned}
\Sigma_i(t+1|t) &= \mathbb{E}\{\tilde{f}_i(t)\tilde{f}_i'(t)\} + Q_i(t)\\
&= \mathbb{E}\left|q_i(t) + \sum_{j\in\mathcal{O}_i^{time}} \bar{F}_{i,j}(t)\tilde{x}_j(t|t)\right|^2 + Q_i(t),\\
&= \mathbb{E}|q_i(t)|^2 + \mathbb{E}\left[q_i(t)\sum_{j\in\mathcal{O}_i^{time}} \bar{F}_{i,j}(t)\tilde{x}_j(t|t)\right]\\
&\quad + \mathbb{E}\left[\sum_{j\in\mathcal{O}_i^{time}} \tilde{x}_j'(t|t)\bar{F}_{i,j}'(t)q_i'(t)\right] + \mathbb{E}\left|\sum_{j\in\mathcal{O}_i^{time}} \bar{F}_{i,j}(t)\tilde{x}_j(t|t)\right|^2\\
&\quad + Q_i(t),
\end{aligned} \tag{15}$$

where for a column vector ξ, we denote $|\xi|^2 = \xi\xi'$ and notice that it is a matrix. Since

$$\begin{aligned}
\mathbb{E}|q_i(t)|^2 &= \mathbb{E}[q_i(t)q_i'(t)] \le \mathrm{tr}\Big(\mathbb{E}[q_i(t)q_i(t)']\Big)I_{n_i}\\
&= \mathbb{E}\|q_i(t)\|^2 I_{n_i} \le \phi_i^2(t)\mathbb{E}\|(\tilde{x}_{i_1}'(t|t), \ldots, \tilde{x}_{i_1}'(t|t))'\|^2 I_{n_i}\\
&= \phi_i^2(t)\mathrm{tr}\mathbb{E}\Big(\Big|(\tilde{x}_{i_1}'(t|t), \ldots, \tilde{x}_{i_1}'(t|t))'\Big|^2\Big)I_{n_i}\\
&= \phi_i^2(t)\sum_{j\in\mathcal{O}_i^{time}} \mathrm{tr}\Big(\Sigma_j(t|t)\Big)I_{n_i},
\end{aligned} \tag{16}$$

where the sense of $|\cdot|^2$ is the same with above, and the second inequality holds because of the Assumption 1.

Now we estimate $\mathbb{E}\left|\sum_{j\in\mathcal{O}_i^{time}} \bar{F}_{i,j}(t)\tilde{x}_j(t|t)\right|^2$, for a vector $y\in\mathbb{R}^{n_i}$, we have

$$\mathbb{E}\{y'\bar{F}_{i,j}(t)\tilde{x}_j(t|t)\}\{\tilde{x}_l'(t|t)\bar{F}_{i,l}'(t)y\}$$

$$\leq \frac{1}{2}\Big[y'\bar{F}_{i,j}(t)\Sigma_j(t|t)\bar{F}_{i,j}'(t)y + y'\bar{F}_{i,l}(t)\Sigma_l(t|t)\bar{F}_{i,l}'(t)y\Big].$$

Then, considering $y'(\sum_{j\in\mathcal{O}_i^{time}}\bar{F}_{i,j}(t)\tilde{x}_j(t|t))(\sum_{j\in\mathcal{O}_i^{time}}\bar{F}_{i,j}(t)\tilde{x}_j(t|t))'y$, and via the inequality $(a_1+..+a_n)^2\leq n(a_1^2+\cdots+a_n^2)$, with a_i being all real numbers, we can obtain the following inequality:

$$\mathbb{E}\left|\sum_{j\in\mathcal{O}_i^{time}}\bar{F}_{i,j}(t)\tilde{x}_j(t|t)\right|^2 = \mathbb{E}\Big[\sum_{j\in\mathcal{O}_i^{time}}\bar{F}_{i,j}(t)\tilde{x}_j(t|t)\Big]\Big[\sum_{j\in\mathcal{O}_i^{time}}\bar{F}_{i,j}(t)\tilde{x}_j(t|t)\Big]'$$
$$\leq (\mathcal{O}_i^{time})^{\sharp}\sum_{j\in\mathcal{O}_i^{time}}\bar{F}_{i,j}(t)\Sigma_j(t|t)\bar{F}_{i,j}'(t). \tag{17}$$

At last, we estimate the two cross-terms appeared in the right hand of Eq. (15), since

$$y'q_i(t)\Big[\sum_{j\in\mathcal{O}_i^{time}}\bar{F}_{i,j}(t)\tilde{x}_j(t|t)\Big]'y$$

$$\leq \beta_i(t)|y'q_i(t)|\left|(1/\beta_i(t))\sum_{j\in\mathcal{O}_i^{time}}\tilde{x}_j'(t|t)\bar{F}_{i,j}'(t)y\right|$$

$$\leq \frac{1}{2}\beta_i(t)\Big[y'q_i(t)q_i'(t)y + (1/\beta_i^2(t))y'\Big(\sum_{j\in\mathcal{O}_i^{time}}\bar{F}_{i,j}(t)\tilde{x}_j(t|t)\Big)$$

$$\times\Big(\sum_{j\in\mathcal{O}_i^{time}}\bar{F}_{i,j}(t)\tilde{x}_j(t|t)\Big)'y\Big]. \tag{18}$$

Thus, Eqs. (15)–(18) yield

$$\Sigma_i(t+1|t)\leq(1+\beta_i(t))(\mathcal{O}_i^{time})^{\sharp}\sum_{j\in\mathcal{O}_i^{time}}\bar{F}_{i,j}(t)\Sigma_j(t|t)\bar{F}_{i,j}'(t)$$
$$+\ (1+1/\beta_i(t))\phi_i^2(t)\sum_{j\in\mathcal{O}_i^{time}}\mathrm{tr}\Big(\Sigma_j(t|t)\Big)I_{n_i}$$
$$+\ Q_i(t). \tag{19}$$

Similarly, via Eq. (7a) we may show

$$\Sigma_i(t|t)\leq(1+\alpha_i(t))\Big(I-L_i(t)\Psi_{i,i}(t)\Big)\Sigma_i(t|t-1)\Big(I-L_i(t)\Psi_{i,i}(t)\Big)'$$
$$+\ (1+1/\alpha_i(t))\Big(\sum_{k\in\mathcal{I}_i}(\mathcal{O}_k^{meas,o})^{\sharp}\Big)L_i(t)$$

$$\times\left[\sum_{k\in\mathcal{I}_i^{meas}}\sum_{j\in\mathcal{O}_k^{meas,o}}\Psi_{i,j}^{(k)}(t)\Sigma_j(t|t-1)\Psi_{i,j}^{(k)}(t)^{'}\right]L_i^{'}(t)$$
$$+\ L_i(t)\Psi_{i,i}(t)L_i^{'}(t), \tag{20}$$

just noticing that

$$\tilde{x}_i(t|t)$$
$$=\Big[I-L_i(t)\Psi_{i,i}(t)\Big]\tilde{x}_i(t|t-1)-L_i(t)\sum_{k\in\mathcal{I}_i^{meas}}\sum_{j\in\mathcal{O}_k^{meas,o}}\Psi_{i,j}^{(k)}(t)\tilde{x}_j(t|t-1)$$
$$-L_i(t)\sum_{k\in\mathcal{I}_i^{meas}}\bar{A}_{k,i}^{'}(t)R_k^{-1}(t)v_k(t).$$

Subtracting Eq. (19) from Eq. (9b) and subtracting Eq. (20) from Eq. (9a), the inequality relation $\Sigma_i(t|t)\le\bar{\Sigma}_i(t|t)$ and $\Sigma_i(t+1|t)\le\bar{\Sigma}_i(t+1|t)$ be valid by mathematical induction and the nonnegative definite initial values.

A2. Proof of Theorem 2

Taking the partial derivative of $\bar{\Sigma}_i(t|t)$ in Eq. (9a), with respect to $L_i(t)$, and letting the partial derivative zero yield

$$L_i(t)=(1+\alpha_i(t))\bar{\Sigma}_i(t|t-1)\Psi_{i,i}(t)V_i^{-1}(t), \tag{21}$$

where

$$V_i(t)=(1+\alpha_i(t))\Psi_{i,i}(t)\bar{\Sigma}_i(t|t-1)\Psi_{i,i}(t)+(1+1/\alpha_i(t))$$
$$\times\left(\sum_{k\in\mathcal{I}_i^{meas}}(\mathcal{O}_k^{meas,o})^{\sharp}\right)\sum_{k\in\mathcal{I}_i^{meas}}\sum_{j\in\mathcal{O}_k^{meas,o}}\Psi_{i,j}^{(k)}(t)\bar{\Sigma}_j(t|t-1)\Psi_{i,j}^{(k)}(t)^{'}$$
$$+\ \Psi_{i,i}(t). \tag{22}$$

Expand the first term of the right hand part of Eq. (9a), we have

$$\bar{\Sigma}_i(t|t)=(1+\alpha_i(t))\bar{\Sigma}_i(t|t-1)-(1+\alpha_i(t))\bar{\Sigma}_i(t|t-1)\Psi_{i,i}(t)L_i^{'}(t)+\varsigma(t),$$

where

$$\varsigma(t)=-(1+\alpha_i(t))L_i(t)\Psi_{i,i}(t)\bar{\Sigma}_i(t|t-1)$$
$$+\ (1+\alpha_i(t))L_i(t)\Psi_{i,i}(t)\bar{\Sigma}_i(t|t-1)\Psi_{i,i}(t)L_i^{'}(t)$$
$$+\ (1+1/\alpha_i(t))\left(\sum_{k\in\mathcal{I}_i^{meas}}(\mathcal{O}_k^{meas,o})^{\sharp}\right)L_i(t)$$
$$\times\left[\sum_{k\in\mathcal{I}_i^{meas}}\sum_{j\in\mathcal{O}_k^{meas,o}}\Psi_{i,j}^{(k)}(t)\bar{\Sigma}_j(t|t-1)\Psi_{i,j}^{(k)}(t)^{'}\right]L_i^{'}(t)$$
$$+\ L_i(t)\Psi_{i,i}(t)L_i^{'}(t),$$

Via Eqs. (21–22), it is easy to verify $\varsigma(t)=0$. Thus we obtain Eqs. (12a), and (12b) is obtained from Eq. (9b).

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