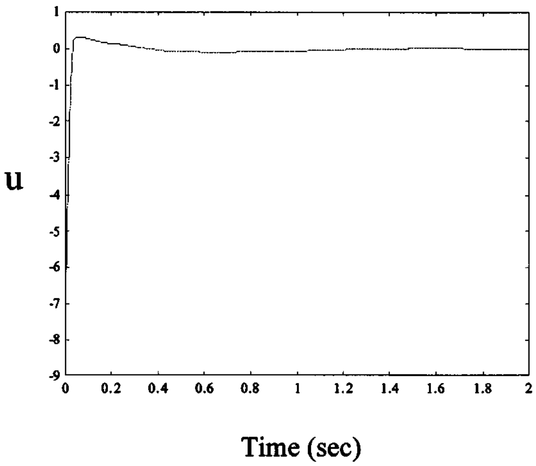

Fig. 3. Control input.

can see that the proposed controller has quite good performance and is quite effective in dealing with system uncertainties.

REFERENCES

[1] S. H. Zak and S. Hui, "On variable structure output feedback controllers for uncertain dynamic systems," *IEEE Trans. Automat. Contr.*, vol. 38, pp. 1509–1512, Oct. 1993.

[2] C. Kwan, "On variable structure output feedback controllers," *IEEE Trans. Automat. Contr.*, vol. 41, Nov. 1996.

[3] J. Slotine and W. Li, *Applied Nonlinear Control*. Upper Saddle River, NJ: Prentice-Hall, 1991.

[4] L. Hsu and F. Lizarralde, "Comments and further results regarding "On variable structure output feedback controllers," *IEEE Trans. Automat. Contr.*, vol. 43, pp. 1338–1340, Sept. 1998.

[5] B. S. Heck and A. A. Ferri, "Application of output feedback variable structure systems," *AIAA J. Guid., Control, Dyn.*, vol. 12, pp. 932–935, 1989.

[6] Y. P. Chen, *Variable Structure System*. Taipei, Taiwan: OpenTech, 1999.

[7] S. Hui and S. H. Zak, "Low-order state estimator and compensators for dynamical systems with unknown inputs," *Syst. Control Lett.*, vol. 21, pp. 493–502, Dec. 1993.

[8] B. M. Diong and J. V. Medanic, "Dynamic output feedback variable structure control for system stabilization," *Int. J. Control*, vol. 56, pp. 607–630, Sept. 1992.

[9] C. Edwards and S. K. Spurgeon, "Compensator based output feedback sliding mode controller design," *Int. J. Control*, vol. 71, pp. 601–614, Nov. 1998.

[10] ——, "Sliding mode control: theory and applications," in *Systems and Control Series*. London, U.K.: Taylor and Francis, 1998.

Comments on "A Revisit to the Gain and Phase Margins of Linear Quadratic Regulators"

U. Holmberg

Abstract—**In the above paper,[1] an example is given, showing that the LQ controller gives an arbitrary small gain margin with respect to variations of the open-loop plant. As a remedy, a dynamic-state feedback is proposed which is claimed to give an arbitrary large gain margin. This is incorrect. In fact, the proposed dynamic state feedback controller does not even stabilize the nominal system.**

Zhang and Fu give several thoughtful examples about the interpretation of the guaranteed gain and phase margins for the linear quadratic regulator (LQR). In Example 2, a particular parameterization of the performance index is chosen, showing that an arbitrarily small gain margin with respect to open-loop variations can be obtained. Then a modification of the controller is claimed to solve the problem and even achieve arbitrarily large gain margins. However, this modification is incorrect. The mistake is due to an unstable pole-zero cancellation. The unstable mode is, therefore, present in the closed-loop system. A disturbance entering between the cancellation point will excite the unstable mode and make some states unbounded.

The studied system consists of two first-order systems in series

$$x_1 = \frac{B_1}{A_1}u \qquad x_2 = \frac{B_2}{A_2}x_1$$

with A_1, A_2, B_1, B_2 being polynomials in the differential operator. The LQ regulator is of the form $u = -f_1x_1 - f_2x_2$ and the proposed dynamic modification can be written

$$u = -\frac{S_1}{R_1}x_1 - \frac{S_2}{R_2}x_2$$

with R_1, R_2, S_1, S_2 being polynomials in the differential operator. The closed-loop system has a cascade control structure with an inner and outer loop. The inner loop is driven by $v = -(S_2/R_2)x_2$ according to

$$x_1 = \frac{A_1R_1}{A_{c1}}v \quad A_{c1} = A_1R_1 + B_1S_1.$$

From the outer loop perspective, this inner closed loop is in series with the second subsystem. It is between these systems the cancellation occurs since R_1 is chosen to be equal to A_2 in the paper. For clarity, introduce a disturbance w between the inner closed loop and the second subsystem according to

$$x_2 = \frac{B_2}{A_2}(x_1 + w).$$

The closed-loop response from w to x_2 is

$$x_2 = \frac{A_{c1}R_2B_2}{A_c}w$$

where

$$A_c = A_2A_{c1}R_2 + B_1B_2R_1S_2$$

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The author is with the School of information science, Computer and Electrical Engineering, Halmstad University, S-301 18 Halmstad, Sweden (e-mail: Ulf.Holmberg@ide.hh.se).

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[1] C. Zhang and M. Fu, *IEEE Trans. Automat. Contr.*, vol. 41, pp. 1527–1530, Oct. 1996.

is the closed-loop characteristic polynomial, which determines the stability of the system. Since the authors choose $R_1 = A_2$ it follows that $A_c = A'_c A_2$, where $A'_c = A_{c1} R_2 + B_1 B_2 S_2$. Thus, the closed-loop response is passing through the open-loop second subsystem

$$x_2 = \frac{A_{c1} R_2}{A'_c} \frac{B_2}{A_2} w.$$

Since $A_2 = s$ (where $s = d/dt$) the closed loop is unstable. In the paper, the modified feedback from x_1 is $S_1/R_1 = f_1 + (s - 1/s)Q$ where $Q \neq 0$ modifies the original state feedback. Clearly, for all values $Q \neq 0$ it follows that $R_1 = s = A_2$. Thus, the proposed modified feedback makes the system unstable even in the nominal unperturbed case.

Authors' Reply

M. Fu and C. Zhang

U. Holmberg correctly pointed out an error in the above paper,[1]. Specifically, the dynamic state feedback controller in (23) of the paper[1] does not lead to internal stability. However, this error can be easily fixed by modifying the controller to

$$u(s) = -\left(2\sqrt{r} + r + \frac{s-1}{s+\rho} Q(s)\right) x_1(s) + \left(r + \frac{s}{s+\rho} Q(s)\right) x_2(s)$$

where $\rho > 0$ and $Q(s)$ is a stable transfer function, both to be determined. For $\rho = 0$, the modified controller reduces to (23) in the paper.[1] It is easy to see that the modified controller has the same features as (23) in the paper,[1] i.e., it reduces to optimal nominal controller when$Q(s) = 0$ and that the $Q(s)$ term does not alter the closed-loop transfer function. Similar to the paper,[1] we choose $Q(s) = -r$. It is straightforward to verify that the closed-loop characteristic polynomial is given by

$$s^3 + (\rho + 2\sqrt{r})s^2 + (r(1 - \rho\epsilon) + 2\rho\sqrt{r})s + \rho r(1 + \epsilon)$$

where $1 + \epsilon$ is the "perturbed" plant gain. Using Routh-Hurwitz criterion, the roots of the characteristic polynomial are Hurwitz stable if and only if

$$\begin{aligned}\Delta :=& (\rho + 2\sqrt{r})(r(1 - \rho\epsilon) + 2\rho\sqrt{r}) - \rho r(1 + \epsilon) \\ =& 3\rho r + r(2\sqrt{r}(1 - \rho\epsilon) - \rho\epsilon) + \rho(r(1 - \rho\epsilon) + 2\rho\sqrt{r}) > 0.\end{aligned}$$

Choose

$$\rho = \frac{2\sqrt{r}}{\epsilon^*(2\sqrt{r} + 1)}$$

where $\epsilon^* > 0$ is any upper bound for ϵ. Then, it is verified that $(1 - \rho\epsilon) > 0$ and $(2\sqrt{r}(1 - \rho\epsilon) - \rho\epsilon) > 0$. Hence, $\Delta > 0$ and an arbitairly large gain margin is achieved.

It is noted that large gain margin requires ρ to be small, leading to small stability margin for the close-loop poles. However, this seems an understandable tradeoff.

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M. Fu is with the Department of Electrical and Computer Engineering, University of Newcastle, Newcastle, N.S.W. 2308, Australia (e-mail: eemf@ee.newcastle.edu.au).

C. Zhang is with the Department of Electrical and Electronic Engineering, University of Melbourne, Parkville, VIC 3052, Australia (e-mail: cishen@unimelb.edu.au).

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