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Localizability and Distributed Localization of Sensor Networks using Relative Position Measurements

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Abstract: The paper introduces the localization problem of sensor networks using relative position measurements. It is assumed that relative positions are measured in local coordinate frames of individual sensors, for which different sensors may have different orientations of their local frames and the orientation errors with respect to the global coordinate frame are not known. A new necessary and sufficient condition is developed for localizability of such sensor networks that are modeled as directed graphs. That is, every sensor node should be 2-reachable from the anchor nodes. Moreover, for a localizable sensor network, a distributed, linear, and iterative scheme based on the graph Laplacian of the sensor network is developed to solve the coordinates of the sensor network in the global coordinate frame.

1. INTRODUCTION

Many existing localization schemes for sensor networks utilize pairwise distance measurements between sensor nodes to compute the position of each node in a global coordinate frame. With the development of micro-electromechanical systems (MEMS), more and more meanings are given to the terminology *sensor* in variable networks such as robotic networks, aircraft networks etc. Thus, it is possible for a sensor to easily and also necessarily get additional useful measurements such as bearing angles [3][5], which together with relative distance measurements lead to the availability of relative positions in local coordinate frames.

On the other hand, depending on whether a central station or a central node exists or not, localization schemes can be divided into centralized schemes [10] and distributed schemes [6]. The former asks every sensor node to transmit its information to a central station or a central node and then computes the positions of the entire senor network centrally, while the latter lets every node only exchange information with its neighbors and conduct the computation of its own position locally. The distributed one is more preferable since it is obviously energy efficient. This paper aims at solving the distributed localization problem using relative position measurements on local coordinate frames.

A fundamental and important issue in sensor network localization is called *localizability* [9]. It checks whether a sensor network is localizable based on available measurements. Typically, it counts on two aspects: the total number of measurements and where to place these measurements between sensor nodes in the sensor network. For range-based localization, a sensor network is usually characterized by a distance graph [1] and then graph rigidity theory is applied for the localizability problem. That is, for a sensor network in the 2D plane, it can be uniquely localized if and only if it contains at least three location-known anchor nodes and its distance graph is globally rigid [4].

However, there is no known result for localizability of a sensor network based on relative position measurements. This paper develops a fundamental result to answer this question. Necessary and sufficient conditions are obtained to characterize the localizability of a sensor network with relative position measurements. It is shown that a sensor network in the 2D plane with relative position measurements is localizable for almost all distribution of sensor nodes if and only if it contains at least two location-known anchor nodes and its sensing graph holds a 2-reachability property. Additionally, the algebraic necessary and sufficient conditions are characterized by the rank of graph Laplacians.

In addition to provide a complete answer for the localizability problem, this paper also provides a distributed localization scheme to calculate the position of each sensor node by itself based on its own local measurements and a few information exchanged from its neighbors. The proposed localization algorithm is in an iterative and linear form with guaranteed global convergence. This is more attractive compared with those distance-based localization algorithms that usually cannot avoid being trapped into local optima.

In the paper, we assume that each sensor node can measure the relative positions of only two neighbors (the least number required) to simplify the analysis in obtaining the localizability conditions as well as the distributed localization algorithms. However, the localizability conditions also hold for a more general case where each sensor node may access relative position information of more than two neighbors. The distributed localization algorithms work for the more general case as well.

Notations: $\mathbb{C}$ denotes the set of complex numbers. $\iota = \sqrt{-1}$ denotes the imaginary unit. I denotes the identity matrix of appropriate order.

2. PRELIMINARY AND PROBLEM FORMULATION

2.1 Graph Theory

First, we are going to introduce some notions from graph theory and algebraic graph theory. A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a non-empty node set $\mathcal{V}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. We define $\mathcal{N}_i$ as the neighbor set of node i, i.e., $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}$. As defined in [8], for a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, a node $v \in \mathcal{V}$, is said to be *2-reachable* from a non-singleton set $\mathcal{U}$ of nodes if there exists a path from a node in $\mathcal{U}$ to v after removing any one node except node v.

Then, we introduce two notions called complex Laplacian matrix and Dirichlet matrix. For a directed graph, associated each edge $(j, i) \in \mathcal{E}$ with a complex value w_{ij} called the weight of the edge, its complex Laplacian matrix is defined as

$$L = \begin{cases} -w_{ij}, & \text{if } i \neq j \text{ and } j \in \mathcal{N}_i \\ 0, & \text{if } i \neq j \text{ and } j \notin \mathcal{N}_i \\ \sum_{j \in \mathcal{N}_i} w_{ij}, & \text{if } i = j. \end{cases} \tag{1}$$

Similarly to the one defined in [2], the Dirichlet matrix H is the matrix obtained from the Laplacian matrix L by deleting all rows and columns that correspond to a subset of specific nodes.

2.2 Problem Formulation

We consider a sensor network consisting of *anchor nodes*, whose position are already known, and *normal sensor nodes*, which are to be localized. In the rest of this paper, we call normal sensor nodes as sensor nodes for short. Suppose each sensor node is equipped with an onboard sensor which can measure the relative distances and bearing angles of some other nodes, called its neighbors.

We model such a sensor network as a directed graph, called a *sensing graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, in which an edge $(j, i) \in \mathcal{E}$ indicates that node i can measure the relative distance and bearing angle to node j as ρ_{ij} and θ_{ij} in its local frame. To be more explicit, each sensor node i holds a *local coordinate system* Σ_i, in which the origin is set on the position of itself. Let p_i denote the absolute position of node i in a *global coordinate system* Σ_g. Then $(p_j - p_i)$ represents the relative position between node j and i in Σ_g and $(p_j - p_i)e^{-\iota\alpha_i}$ is the relative position of node j in the local coordinate system Σ_i, where α_i is the offset angle between Σ_i and Σ_g. Thus, the local measurement holds the following formula:

$$\rho_{ij} e^{\iota\theta_{ij}} = (p_j - p_i)e^{-\iota\alpha_i}.$$

An illustrative example is given in Fig. 1. In the example, the arrowed line pointing from j to i means node i can measure the relative position of j in its local coordinate system. We can understand this arrow as the direction of information flow in the network. In the following, we will treat sensor networks and sensing graphs interchangeably.

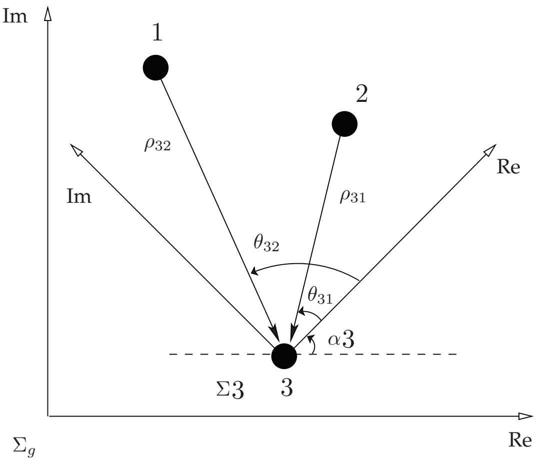


Fig. 1. An illustration of sensing graph and relative position measurements.

For a sensor network $\mathcal{G}$ containing m location-known anchors and n sensor nodes to be localized, the $(m+n) \times (m+n)$ Laplacian matrix and $n \times n$ Dirichlet matrix H corresponding to the n sensor nodes are represented as

$$L = \begin{bmatrix} 0 & 0 \\ B & H \end{bmatrix}, \tag{2}$$

where B is an appropriate dimensional matrix, indicating the links of sensor nodes to the anchor nodes in the sensing graph.

Note that, all local coordinate systems (Σ_i, $i = 1, \ldots, n$) are not required to have the same offset angle. This is in agreement with the fact that distributed sensor nodes do not have global knowledge of, or it is costly to achieve agreement on, a common orientation.

In this paper, we introduce the following fundamental problems for sensor network localization based on relative position measurements.

(P-1) What are the necessary and sufficient graphical (algebraic) conditions for the localizability of the whole sensor network by only using the measurements of relative distances and bearing angles in local coordinate systems?

(P-2) Find an efficient distributed localization algorithm for the sensor network when it is entirely localizable.

3. NECESSARY AND SUFFICIENT CONDITION FOR LOCALIZABILITY

3.1 A Necessary Localizable Condition

First, we provide a necessary condition for localizability in terms of graph connectivity.

Theorem 1. If a sensor network $\mathcal{G}$, consisting m anchor nodes and n sensor nodes, is localizable, then

(NC-1): $m \geq 2$, and
(NC-2): each sensor node is 2-reachable from the set of anchor nodes.

Proof: First, we prove the necessity of $m \geq 2$. When $m = 0$, the whole sensor network can freely rotate and translate in the plane without any constraint and thus cannot be localizable. When $m = 1$, the whole sensor network can not translate, but can freely rotate around the unique anchor node. So again it is not localizable.

Second, we show the necessary condition (NC-2). Suppose by contrary that there is one node, say v_i, that is not 2-reachable from the anchor set, denoted here as $\mathcal{S}_a$. By the definition, it is then known that there exists another node v_* such that when node v_* is removed, node v_i is not reachable from $\mathcal{S}_a$. Denote by $\mathcal{S}_*$ the set of sensor nodes that are not reachable from the anchor set $\mathcal{S}_a$ after removing node v_*. That is, there is no edge connecting any node in $\mathcal{S}_*$ to any node not in $\mathcal{S}_* \cup \{v_*\}$. Thus, look at the sub network composed of nodes in $\mathcal{S}_*$ and v_*. Then by the necessary condition (NC-1), even in the case v_* is localizable, the sub network is still not localizable and it can freely rotate around v_*. ■

It is shown in Theorem 1 that 2-reachability of every sensor node from the set of anchor nodes is a necessary graphical condition. Certainly, 2-reachability implies that every sensor node has at least two neighbors in the graph. So we assume that every sensor node in the sensor network has at least two neighbors. In this paper, we consider the minimum sensing graph, that is, every sensor node has only two neighbors, requiring the minimum number of measurements. The assumption is stated formally below.

To simply demonstrate our ideas in deriving necessary and sufficient conditions for localizability as well as developing distributed localization algorithms, we consider the above assumption. However, our results will also hold if each sensor node in the sensor graph has relative position measurements of more than two neighbors.

Assumption 1. Every sensor node in the sensor network $\mathcal{G}$ measures relative positions of only two neighbors.

3.2 Necessary and Sufficient Algebraic Conditions

Under Assumption 1, it holds that for any sensor node i, there exists a set of complex weights w_{ij} to make

$$\sum_{j \in \mathcal{N}_i} w_{ij}(p_j - p_i)e^{-\iota\alpha_i} = 0$$

represented in the local coordinate system Σ_i and equivalently

$$\sum_{j \in \mathcal{N}_i} w_{ij}(p_j - p_i) = 0 \tag{3}$$

represented in the global coordinate system Σ_g. An easy way to choose the complex weights w_{ij} is to take the values that normalize the relative position vectors of its neighbors and project onto the positive and negative real axis of its local coordinate system Σ_i. That is, if j is a neighbor of node i then

$$w_{ij} = \frac{e^{-\iota\theta_{ij}}}{\rho_{ij}}, \tag{4}$$

and if k is the other neighbor of node i then

$$w_{ik} = -\frac{e^{-\iota\theta_{ik}}}{\rho_{ik}}. \tag{5}$$

In this way, the complex weights use only the measurements of relative distances and bearing angles of sensor node i. Taking Fig. 1 for example, we can choose the complex weights for edges $(1,3)$ and $(2,3)$ as $w_{31} = \frac{e^{-\iota\theta_{31}}}{\rho_{31}}$, $w_{32} = -\frac{e^{-\iota\theta_{32}}}{\rho_{32}}$. Then it holds that

$$w_{31}(p_1 - p_3) + w_{32}(p_2 - p_3) = 0. \tag{6}$$

Denote by

$$p = [p_1 \cdots p_m, p_{m+1}, \cdots, p_{m+n}]^T \tag{7}$$

the aggregate position vector of the sensor network. Then the aggregate equation of (3) can be written as

$$Lp = 0 \tag{8}$$

where L is the complex Laplacian matrix for the sensing graph with the nonzero complex entries being the complex weights chosen above.

Let p_a and p_s be the position vectors of anchor nodes and sensor nodes respectively. Thus, eq. (8) can be re-written as

$$Lp = \begin{bmatrix} 0 & 0 \\ B & H \end{bmatrix} \begin{bmatrix} p_a \\ p_s \end{bmatrix} = 0, \tag{9}$$

where H is the Dirichlet matrix. Equivalently, we could write as

$$Hp_s = -Bp_a. \tag{10}$$

Now, the problem of localizability of the sensor network is equivalent to the existence of a unique solution for p_s to the equation (10) for which p_a is known. Thus, a necessary and sufficient algebraic condition is presented below.

Theorem 2. Suppose Assumption 1 holds. Then a sensor network consisting of m anchor nodes and n sensor nodes is localizable if and only if $m \geq 2$ and $\text{rank}(L) = \text{rank}(H) = n$ where L and H are the Laplacian and Dirichlet matrix with their entries defined in (4)-(5).

Proof: **(Necessity)** From Theorem 1, it is certain that $m \geq 2$ is necessary. On the other hand, we suppose by contradiction that $\text{rank}(L) \neq n$ or $\text{rank}(H) \neq n$. Note that $\text{rank}(H) \leq \text{rank}(L) \leq n$. So for both cases, it means $\text{rank}(H) < n$. Then eq. (10) has multiple solutions of p_s, which contradicts to the condition that the sensor network $\mathcal{G}$ is localizable. Therefore, it is necessary that $\text{rank}(L) = \text{rank}(H) = n$.

(Sufficiency) For a sensor network $\mathcal{G}$, if the Laplacian and Dirichlet matrix with their entries defined in (4)-(5) satisfy $\text{rank}(L) = \text{rank}(H) = n$, then there exists a unique solution p_s to eq. (10). Thus, the sensor network is localizable. ■

3.3 Necessary and Sufficient Graphical Conditions

In this subsection, we develop necessary and sufficient graphical conditions for localizability. It is shown below that the necessary condition given in Theorem 1 is also sufficient for localizability of almost all sensor networks. That is, the 2-reachability of every sensor node from the anchor set is a key to the localization problem with relative position measurements.

Theorem 3. Suppose Assumption 1 holds. Then a sensor network consisting of m anchor nodes and n sensor nodes is localizable for almost all distribution p if and only if $m \geq 2$ and every sensor node in $\mathcal{G}$ is 2-reachable from the set of anchor nodes.

Proof: **(Necessity)** Following the same argument as in the proof of Theorem 1, if either $m < 2$ or there exists one sensor node that is not 2-reachable from the set of anchor nodes, then the sensor network is not localizable for whatever distribution p. Therefore, it is necessary that $m \geq 2$ and every sensor node in $\mathcal{G}$ is 2-reachable from the set of anchor nodes.

(Sufficiency) For a sensing graph $\mathcal{G}$ consisting of m anchor nodes and n sensor nodes, let L be the Laplacian matrix and H be the Dirichlet matrix corresponding to the n sensor nodes with their entries defined in (4)-(5). We denote by H_i $(i = 1, \ldots, n)$ the right-bottom i-by-i dimensional sub-matrix of H. By Assumption 1 that each sensor node has only two neighbors in the graph and by the condition that every sensor node is 2-reachable from the set of anchor nodes, we then know that for any $i = 1, \ldots, n$, there must exist a row in H_i having at most two nonzero entries as otherwise it indicates that the subset of sensor nodes corresponding to the labels of H_i is not reachable from the set of anchor nodes, a contradiction. With the above observation, we label the anchor nodes from 1 to m and re-label the node corresponding to the row in H_n that has at most two nonzero entries as $m + 1$. Moreover, we re-label the node corresponding to the row in H_{n-1} that has at most two nonzero entries as $m + 2$, and so on. By this relabeling scheme, for any H_i it is modified so that the first row of H_i has at most two nonzero entries and one of them is at the diagonal entry. That is, the following form holds for H_i.

$$H_i = \left[\begin{array}{c|c} h_{(n-i)(n-i)} & \triangle \\ \hline * & H_{i-1} \end{array}\right]$$

where $h_{(n-i)(n-i)}$ represents the $(n - i)$-th diagonal entry of H, and there is at most one nonzero entry in $\triangle$, denoted as $h_{(n-i)j}$ if exists. Thus,

$$\det(H_i) = h_{(n-i)(n-i)}\det(H_{i-1}) + (-1)^k h_{(n-i)j}\det(M^*) \tag{11}$$

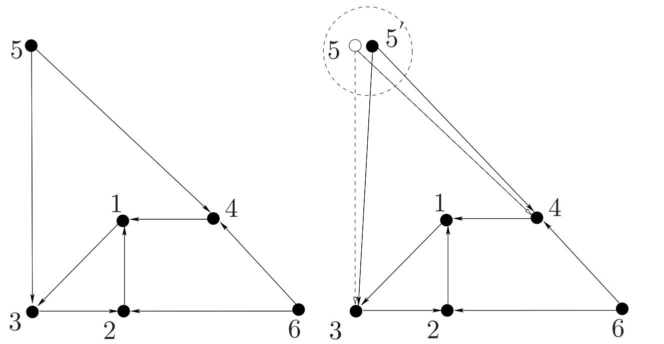
where k is a proper integer and M^* is the minor by removing the first row and the column of H_i which the nonzero entry $h_{(n-i)j}$ lies in.

Next we prove $\det(H) = \det(H_n) \neq 0$ for almost all sensor distribution p by induction. First, it can be seen that for a generic sensor distribution p, no two sensor nodes are overlapped. Thus, the diagonal entries of H are nonzero. So $\det(H_1) \neq 0$. Second, suppose that $\det(H_{i-1}) \neq 0$. Then according to (11), it is known that for a generic sensor distribution p, $\det(H_i) \neq 0$ as both $h_{(n-i)(n-i)}$ and possible nonzero entry $h_{(n-i)j}$ are dependent solely on p. This means, for almost all sensor distribution p, $\det(H) \neq 0$. Thus, applying Theorem 2 leads to the conclusion that for almost all sensor distribution p, the sensor network is localizable. ■

Theorem 3 shows that if the number of anchor nodes is greater than 2 and every sensor node in the sensing graph is 2-reachable from the set of anchor nodes, then a sensor network with a distribution p, which is not localizable, is of zero measure. Take the sensor network in Fig. 2(a) as an example, for which, nodes 5 and 6 are the anchor nodes, and every sensor node $i = 1, \ldots, 4$ is 2-reachable from the set of anchor nodes. Suppose the sensor distribution is the one given in Fig. 2(a), i.e.,

$$p = (1 + \iota, 1, 0, 2 + \iota, \iota 3, 3).$$

Then it can be checked that $\text{rank}(L) = \text{rank}(H)$ is less than 4, meaning that (10) must have infinite number of solutions and the sensor network is not localizable. Actually, for whatever choice of w_{ij} satisfying (3), the rank of H is still less than 4, meaning that there is no other way to solve the absolute positions of the sensor network. However, it can be checked that such situations are of zero measure as shown in Theorem 3. For this example, when we shift a little bit the position of node 5 in the neighborhood of the original location (Fig. 2(b)), then the new Dirichlet matrix H is of full rank and thus the sensor network is localizable.



(a) Not localizable. (b) Localizable.

Fig. 2. A sensor network satisfying the conditions of Theorem 3 is localizable for a generic sensor distribution.

Note that if a sensor node has more than two neighbors, the corresponding row of the resultant Laplacian has more nonzero entries. Thus, the analysis will be very similar in obtaining the same necessary and sufficient localizability conditions.

4. DISTRIBUTED LOCALIZATION ALGORITHM AND CONVERGENCE ANALYSIS

For a sensor network, once we find out that it is localizable according to the conditions developed in last section, we need to find an efficient way to solve the absolute positions p_s in eq. (10). A centralized computation is trivial, but for practical applications, we are looking for a distributed scheme. To this end, we multiply a complex diagonal matrix D on both sides of (10) and obtain a new equation as

$$DHp_s = -DBp_a. \tag{12}$$

Adding p_s on both sides of equation (12) and re-arranging the terms in the equation, the following is obtained.

$$p_s = (I - DH)p_s - DBp_a. \tag{13}$$

According to (13), we then can construct an iterative algorithm for localization as follows.

$$z(k+1) = (I - DH)z(k) - DBp_a, \tag{14}$$

where $z(k) = [z_{m+1}, \cdots, z_{m+n}]^T$ is the estimate of p_s at time k. The above iterative form can also be written as an iterative process at each sensor node, $i = 1, \ldots, n$, that is,

$$z_i(k+1) = z_i(k) + \sum_{j \in \mathcal{N}_i} d_i w_{ij}(z_j(k) - z_i(k)), \tag{15}$$

where $z_i(k)$, $i = m+1, \ldots, m+n$, is the estimate of sensor i's own position and $z_i(k) = p_i$ for $i = 1, \ldots, m$ (the anchor nodes are stationary). Notice that w_{ij} is available to node i based on its local measurements about its two neighbors. So when its neighbors $j \in \mathcal{N}_i$ communicate with sensor node i their estimates of themselves positions, then the iterative localization scheme (15) can be implemented locally in a distributed manner.

Remark 1. The convergence rate of the iterative localization algorithm depends on the spectral radius of $I - DH$. To increase the convergence rate, we can choose appropriate d_i's to minimize the spectral radius of $I - DH$.

In the following, we provide a method on how to find an appropriate complex scaling parameter d_i. Similar as [7] and [8], we apply the continuity property of eigenvalues. Denote by $H^{(i)}$ the i-th leading principal sub-matrix. This method firstly assigns a d_1 such that the eigenvalue of $d_1 H^{(1)}$, i.e., $d_1 H^{(1)}$ itself, lies on the positive real axis. Note that $diag\{d_1,\ 0\}H^{(2)}$ has two eigenvalues, namely, $d_1 H^{(1)}$ and 0. Then according to the continuity of eigenvalues, we can find an appropriate d_2 around zero to make the eigenvalues of $diag\{d_1,\ d_2\}H^{(2)}$ lie on the positive real axis. The process is repeated until all d_i's are found.

Algorithm 1 Find scaling parameters d_i.

1: **for** $i = 1, \cdots, n$ **do**
2: Find d_i such that the eigenvalues of $diag\{d_1, \cdots, d_i\}H^{(i)}$ lie on the real axis between $(0, 2)$.
3: **end for**
4: **return** $d_i, i = 1, \ldots, n$.

5. SIMULATIONS

In this section we provide a simulation example to illustrate our results. To demonstrate the idea clearly, we consider a very simple example with two anchor nodes and three sensor nodes to be localized as shown in Fig. 3(a). The two triangles in the figure indicate the anchor nodes, say node 1 and 2. The other three filled circles represent the true positions of the sensor nodes. The lines with arrows indicate the sensing graph topology. In other words, the arrow pointed from i to j means that node j can measure the relative position of node i. It can be known that for the sensing graph, it satisfies Assumption 1 and every sensor node in $\mathcal{G}$ is 2-reachable from the set of anchor nodes. So it is localizable by our results. Based on the local relative position measurements in the sensing graph $\mathcal{G}$, each node, $i = 3, 4, 5$, can have their weights

$$w_{31} = -1 + \iota 2,\ w_{32} = 2 + \iota,$$

$$w_{41} = -2 + \iota,\ w_{43} = 3,$$

$$w_{52} = 3 - \iota,\ w_{53} = -3 + \iota 3.$$

According to Algorithm 1, the complex scaling parameters are obtained as follows

$$d_3 = 0.05 - \iota,\ d_4 = 0.25(1 - \iota),\ \text{and}\ d_5 = -\iota 0.5.$$

Thus, the distributed and iterative localization algorithm provides a convergent trajectory to the true value for each sensor node, $i = 3, 4, 5$. That is,

$$z_i(k+1) = z_i(k) + \sum_{j \in \mathcal{N}_i} d_i w_{ij}(z_j(k) - z_i(k)), \tag{16}$$

where $z_1(k) = 1$ and $z_2(k) = -1$ are the locations of the two anchor nodes, which are known. With the iterative algorithm (16), the trajectories of the estimates converging to the true values ($p_3 = \iota,\ p_4 = 1 + \iota 3,\ p_5 = -1 + \iota 3$) are plotted in Fig. 3(a). The estimation error $\|z(k) - p\|$ is shown in Fig. 3(b), which asymptotically converges to zero.

6. CONCLUSIONS

This paper introduces a localization problem for sensor networks based on relative position measurements. Also we assume that the relative position measurements are obtained by local onboard sensors and thus do not have a common reference frame. For this problem, necessary and sufficient conditions are developed for localizability in terms of 2-reachability of the sensing graph. Also, necessary and sufficient algebraic conditions are obtained in terms of the rank of graph Laplacians. In addition, we provide a distributed and iterative algorithm to compute the true position of each sensor node, which uses only the local measurements and exchanged information from neighbors, yet globally asymptotic convergence is assured. The minimum sensing graph is studied in the paper. That is, each sensor node is assumed to have only relative position measurements of two neighbors. Our results can be extended to a more general topology where every node may have more than two neighbors. In that case, sensitivity to measurement noises can also be reduced. This is left for our future work.

7. ACKNOWLEDGEMENT

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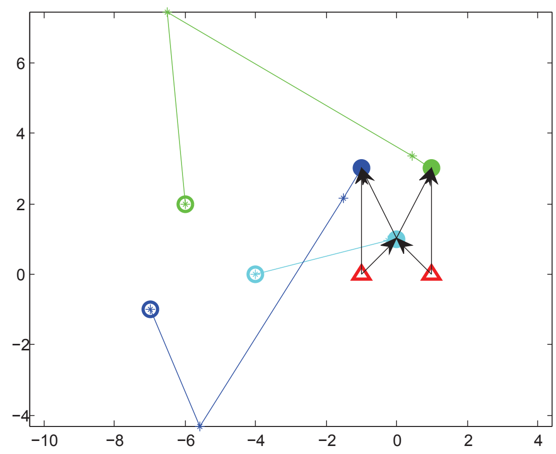
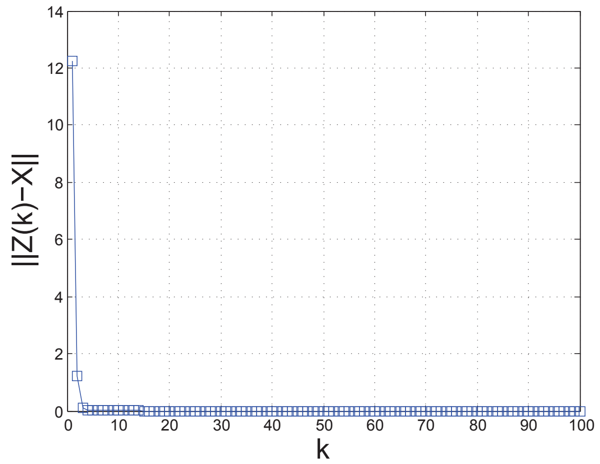


(a) The localization trajectories of $z_i(k)$, $i = 3, 4, 5$.



(b) $||z(k) - p||$ with respect to k.

Fig. 3. A simulation of a sensor network with two anchor nodes and three sensor nodes.

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