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Stability and Passivity Analysis of Systems with Time Varying Parameters

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Abstract

This paper concerns Passivity and Stability analysis of Linear Time Varying (LTV) systems characterized by parametric time variations. The first set of results quantify a trade off between the degree of passivity of the frozen systems and the rate of parameter variations, so that passivity of certain classes of such LTV systems is preserved. One class of systems examined has frozen systems whose transfer functions are multiaffinely parametrized. Another class considered includes linear circuits with time varying resistors, inductors capacitors and mutual inductors. The second set of results exposes the utility of these passivity results in stability analysis.

1 Introduction

This paper considers the passivity of Linear Time Varying (LTV) circuits and systems where the time variation is restricted to certain parameters. For circuits, these parameters directly reflect the circuit element values. We also show that these passivity results hold the key to the stability analysis of related LTV systems.

Our basic approach is Lyapunov/Riccati based. In particular the celebrated Kalman-Yakubovic-Popov (KYP) lemma, [1], is extensively invoked. The results fall within the following broad category: Suppose the LTV systems are such that their frozen Linear Time Invariant (LTI) values enjoy a prescribed degree of passivity/stability. Then quantify the *rate* of parameter variations that preserve its passivity/stability.

To appreciate the connection between passivity and stability analysis consider the following facts.

- The zero input dynamics of every strictly passive, uniformly completely controllable and uniformly completely observable LTV system and that of its inverse are exponentially asymptotically stable (eas).
- Given any square $A(t)$, for which

$$\dot{x}(t) = A(t)x(t) \tag{1.1}$$

is eas, one can find $B(t), C(t), D(t)$ such that the LTV system with state variable realization (SVR)

$$\Sigma = \{A(t), B(t), C(t), D(t)\} \tag{1.2}$$

is strictly passive.

Thus, to demonstrate the eas of an LTV system as in (1.1), one may use the following approach. Find if possible, a second system that is strictly passive and whose stability, or the stability of whose inverse, implies that of (1.1). As will be shown in this paper such related systems can be constructed in a number of important problems.

Having dispensed with preliminaries in Section 2, in Section 3, we present algebraic properties of certain LTI passive systems admitting parametric uncertainty. In particular we show that for three classes of uncertain LTI passive systems, the Riccati matrix solving the KYP lemma is multiaffine in the uncertain parameters. The first two of these results are from [2] and [5] and involve systems whose transfer functions are multiaffine in the uncertain parameters. The last category comprises linear circuits with uncertain resistors, inductors, capacitors and mutual inductors. Observe, that the presence of mutual inductors, destroys the multiaffine nature of the transfer function, [3].

Section 4 considers the above systems but now with the parameters allowed to be time varying. It derives hard bounds on the parameter variation rates that preserve passivity. Section 5 gives average bounds on these rates that allow a related "less stable" system to be passive. It then shows how to use this result to undertake stability analysis. Section 6 is the conclusion.

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2 Preliminaries

This Section provides certain preliminary results. First the notion of passivity.

Definition 2.1 *A linear causal system with input $u(t)$ and output $y(t)$ is (strictly) passive, if there exist constants $K_1 \geq (>)0$ and K_2 such that for all bounded $u(t)$ and time T*

$$\int_0^T u'(t)y(t)dt \geq K_1 \int_0^T u'(t)u(t)dt + K_2.$$

Further if the system has a SVR as in (1.2), it is said to be passive with degree of passivity $\sigma \geq 0$, or σ-passive if the system with SVR

$$\Sigma_\sigma = \{\sigma I + A(t), B(t), C(t), D(t)\} \tag{2.1}$$

is strictly passive.

We now give the time varying version of the KYP lemma. In the sequel, the *realization matrix* of the system in (1.2) is defined as

$$\Omega(t) = \begin{bmatrix} -A(t) & -B(t) \\ C(t) & D(t) \end{bmatrix}. \tag{2.2}$$

Theorem 2.1 *Consider an eas LTV system with SVR as in (1.2) and $A(t)$ $n \times n$. Then this system is passive with degree of passivity $\sigma \geq 0$ iff there exists a $n \times n$, uniformly positive definite symmetric matrix $P(t)$ such that [4],*

$$\begin{bmatrix} P(t) & 0 \\ 0 & I \end{bmatrix} \Omega(t) - \begin{bmatrix} \frac{1}{2}\dot{P}(t) + \sigma P(t) & 0 \\ 0 & 0 \end{bmatrix}, \tag{2.3}$$

is uniformly positive definite.

Henceforth $P(t)$ will be called the Riccati matrix solving the KYP lemma. Should, the system be LTI rather than LTV, then the Riccati matrix in question becomes time invariant, whence the $\dot{P}(t)$ term in (2.3) dissapears.

Finally given two square matrices P and Q, there direct sum is defined as

$$P \oplus Q = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}.$$

3 Frozen Systems

This Section has the following objectives. First, it introduces the three classes of time varying systems whose passivity is to be studied in Section 4. Second, it provides certain algebraic properties of the underlying frozen systems. Sections 3.1 defines the first two types of systems. Section 3.2 considers the third category.

Throughout the time variations will be confined to a parameter vector $\Theta(t) = [\theta_1(t), \cdots, \theta_N(t)]'$. In the sequel, if an SVR

$$\{A(\Theta(t)), B(\Theta(t)), C(\Theta(t)), D(\Theta(t))\}$$

defines the LTV system, the SVR

$$\{A(\Theta), B(\Theta), C(\Theta), D(\Theta)\}$$

defines its frozen version.

3.1 System Types 1 and 2

For simplicity we will restrict attention to Single Input Single Output (SISO) systems here, though the ideas readily extend to Multiple Input Multiple Output (MIMO) systems as well.

In this subsection we will use a somewhat nonstandard notion of frozen systems. The set of what we call the frozen systems is in fact a slightly expanded version of the standard definition. In particular, the vector of time varying parameters $\Theta(t)$ will lie in the set

$$\Theta_{\mathcal{T}} = [\theta_1^- + \epsilon, \theta_1^+ - \epsilon] \times [\theta_2^- + \epsilon, \theta_2^+ - \epsilon] \times \cdots [\theta_N^- + \epsilon, \theta_N^+ - \epsilon] \tag{3.1}$$

where $\epsilon > 0$ is arbitrarily small. The set of frozen parameters, on the other hand will correspond to the set

$$\Theta_{\mathcal{F}} = [\theta_1^-, \theta_1^+] \times [\theta_2^-, \theta_2^+] \times \cdots [\theta_N^-, \theta_N^+]. \tag{3.2}$$

Both the classes discussed here, labelled as types 1 and 2 have parameter vectors that obey for some $p < N$,

$$\theta_i = \begin{cases} \lambda_i; & i \in \{1, \cdots p\} \\ \mu_i; & i \in \{p+1, \cdots N\} \end{cases} \tag{3.3}$$

Then, with λ and μ vectors respectively comprising λ_i and μ_i, the type 1 system has SVR

$$\{F + gh_1'(\mu(t)), g, h_1'(\mu(t)) - h_2'(\lambda(t)), 1\}. \tag{3.4}$$

with g a constant $n \times 1$ vector and $h_1(\mu(t))$ and $h_2(\lambda(t))$ also $n \times 1$ vectors, respectively multiaffine in the λ_i and μ_i. The type 2 system has SVR

$$\{F + gh_1'(\mu(t)), g, h_1'(\mu(t)) - h_2'(\lambda(t)), 1 - \alpha\}, \tag{3.5}$$

for some $0 < \alpha < 1$. Each of (3.4) and (3.5) obeys, $\Theta(t) \in \Theta_{\mathcal{T}}$, $\forall t$.

In both cases we will assume that each member of the respective frozen sets as defined above is σ-passive. Then we have the following Theorem from [2] and [5], characterizing the Riccati matrices that solve the KYP lemma for the frozen systems.

Theorem 3.1 *Consider the set of LTI systems with SVR's given in (3.6) ((3.7)) below.*

$$\{\{F + gh_1'(\mu), g, h_1'(\mu) - h_2'(\lambda), 1\}; \quad \Theta \in \Theta_{\mathcal{F}}\} \quad (3.6)$$

$$\{\{F + gh_1'(\mu), g, h_1'(\mu) - h_2'(\lambda), 1 - \alpha\}; \quad \Theta \in \Theta_{\mathcal{F}}\}, \quad (3.7)$$

$0 < \alpha < 1$. *Then every member (3.6) ((3.7)) is σ-passive only if the corresponding Riccati matrix is multiaffine in the θ_i.*

Note that the set of transfer functions in (3.6) has numerator and denominator coefficients that are multiaffine in the parameters λ_i and μ_i, respectively. There is at the same time decoupling between the numerator and denominator uncertainties. Likewise in (3.7) also, numerator and denominator coefficients are multiaffine in the parameters λ_i and μ_i. Now however they are *not decoupled.*

3.2 Type 3 Systems

The third class under investigation comprises, square MIMO circuits with mutiple external ports obeying the following assumptions. We make a number of assumptions, the first of which applies to the circuits in the next section as well.

Assumption 3.1 *The circuit comprises entirely of, possibly time varying, resistors, inductors, capacitors and mutual inductors. The input and output vectors consist of the external port voltages and currents, with the restriction that:*

- *The input/output dimension equals the number of external ports.*
- *If the i-th input is a voltage (current) at a particular external port, then the i-th output is the current (voltage) at the same port.*

Further, all the capacitors $c_i(t)$, the resistors $r_i(t)$ and the inductors $l_i(t)$ are positive at all times, and the mutual inductors are described by

$$M_i(t) = m_i(t)\Gamma_i, \quad (3.8)$$

where $\Gamma_i = \Gamma_i' > 0$ is constant, with unit diagonal elements, and $m_i(t) > 0$ for all t. With the vectors $V_i(t)$ and $I_i(t)$ comprising the voltages and currents at the mutual inductor ports, one has

$$V_i(t) = \frac{d}{dt}[M_i(t)I_i(t)]. \quad (3.9)$$

The following assumption also applies.

Assumption 3.2 *All the reactive elements are time varying.*

As before $\Theta(t)$ comprises all these time varying elements. We say that a network has been obtained by *extracting a given element* from this network, if in it this circuit element is replaced by an open circuit, and the input/ output dimension over the original network has each increased by one, by augmenting the original input/output vectors by the voltage and current at this newly created port. We have the following theorem proved on the basis of ideas given in [1].

Theorem 3.2 *Consider an LTV MIMO circuit obeying Assumptions 3.1 and 3.2. Suppose, the network obtained by extracting the time varying elements is LTI passive. Suppose the number of inductors, capacitors and mutual imductors are respectively p, q and ν. Then there exist a subset of inductors, capacitors and mutual inductors, labelled with out loss of generality as $l_1(t), \cdots, l_{p_1}(t)$, $c_1(t), \cdots, c_{q_1}(t)$ $M_1(t), \cdots, M_{\nu_1}(t)$, respectively, $A(\Theta(t))$, $B(\Theta(t))$, $C(\Theta(t))$, $D(\Theta(t))$ all independent of l_i, c_i and m_i, and a constant matrix H, such that the SVR of the LTV circuit is*

$$\{\mathcal{M}^{-1}(\Theta(t))\left[-\dot{\mathcal{M}}(\Theta(t)) + A(\Theta(t))\right], B(\Theta(t)), C(\Theta(t)), D(\Theta(t))\}, \quad (3.10)$$

where

$$\mathcal{M}(\Theta(t)) = \mathcal{M}_1(\Theta(t)) + H'\mathcal{M}_2(\Theta(t))H, \quad (3.11)$$

and $\mathcal{M}_1(\Theta(t))$ and $\mathcal{M}_2(\Theta(t))$ are respectively defined by

$$\left(\bigoplus_{i=1}^{p_1} l_i(t)\right) \oplus \left(\bigoplus_{i=1}^{q_1} c_i(t)\right) \oplus \left(\bigoplus_{i=1}^{\nu_1} M_i(t)\right), \quad (3.12)$$

and

$$\left(\bigoplus_{i=p_1+1}^{p} l_i(t)\right) \oplus \left(\bigoplus_{i=q_1+1}^{q} c_i(t)\right) \oplus \left(\bigoplus_{i=\nu_1+1}^{\nu} M_i(t)\right). \quad (3.13)$$

Further, for all t,

$$\begin{bmatrix} -A(\Theta(t)) & -B(\Theta(t)) \\ C(\Theta(t)) & D(\Theta(t)) \end{bmatrix} \geq 0. \quad (3.14)$$

Observe a few facts. First, the rate of variation in the resistive elements does not at all affect the circuit behaviour. Further, the frozen system has SVR

$$\{\mathcal{M}^{-1}(\Theta)A(\Theta), B(\Theta), C(\Theta), D(\Theta(t))\}, \quad (3.15)$$

Consequently, the passivity of (3.15) is trivially demonstrated with $\mathcal{M}(\Theta)$ as the Riccati matrix. Notice this matrix is affine in the reactive parameters, and independent of the resistive parameters, despite the fact that unlike the case in Section 3.1, the transfer function, is not multiaffine in these parameters.

A natural question to ask is whether this same $\mathcal{M}(\Theta)$ acts as a Riccati matrix for demonstrating σ-passivity. The answer in general is no. However, below we give conditions under which $\mathcal{M}(\Theta)$ does act as the appropriate Riccati matrix even for σ-passivity.

Define

$$\Theta(t) = [\lambda(t), r(t)]' \tag{3.16}$$

where $r(t)$ contains the resistive elements and

$$\lambda(t) = [l_1(t), \cdots l_p(t), c_1(t), \cdots c_q(t), m_1(t), \cdots m_\nu(t)]' \tag{3.17}$$

Assume the frozen parameter set obeys

$$0 < l_i^- \leq l_i(t) \leq l_i^+, \tag{3.18}$$

$$0 < c_i^- \leq c_i(t) \leq c_i^+, \tag{3.19}$$

$$0 < m_i^- \leq m_i(t) \leq m_i^+ \tag{3.20}$$

and

$$0 < r_i^- \leq r_i(t) \leq r_i^+. \tag{3.21}$$

Then the following additional assumption is made.

Assumption 3.3 *The frozen systems (3.15) are σ-passive for all parameters as in (3.18-3.21). Further for each $i \in \{1, \cdots, p_1 + q_1 + \nu_1\}$, there exists a combination of λ_j, $j \in \{\{1, \cdots, p_1 + q_1 + \nu_1\} - \{i\}\}$ and r_i, obeying (3.18 - 3.21), and an $\epsilon > 0$ such that for all*

$$\lambda_i \in (\lambda_i^+, \lambda_i^+ + \epsilon), \tag{3.22}$$

the system in (3.15) is not σ-passive, where λ_i^+ is obviously defined.

Essentially this assumption imposes a maximality requirement on the reactive parameter set from the view point of σ-passivity. Then we have the following Theorem.

Theorem 3.3 *Consider the set of frozen values of the LTV system described in Theorem 3.2. Suppose assumption 3.3 holds. Then $\mathcal{M}(\Theta)$ acts as a Riccati matrix for demonstrating σ-passivity of every frozen system.*

4 Hard Bounds for LTV Passivity

The previous section gave algebraic characterizations of the Riccati matrices whenever the set of frozen systems defining the three LTV systems under consideration, is σ-passive invariant. In this question we address the issue of passivity of the LTV system itself. Observe that there are two crucial determinants to whether the LTV systems in question will be passive.

- The extent of parameter variation.
- The rate of parameter variation.

Clearly, the first of these is implicit in the σ-passivity invariance assumption on the frozen systems. We give here, hard bounds on the rate of variation that in combination with this assumption, guarantees passivity.

First we deal with systems described in Section 3.2. To this end we note that the observations made at the end of the last Section indicate that the rate of variation in the resistive elements will not affect the passivity of the LTV circuit. Further, it is known [6], that under all positive rates of variation in positive valued capacitors and inductors, these devices remain passive. Consider for example a time varying capacitor $c(t)$. The net energy delivered to it, over the time interval $[0, t]$ is simply

$$\frac{v^2(t)c(t)}{2} + \frac{1}{2}\int_0^t v^2(\tau)\dot{c}(\tau)d\tau.$$

This is clearly positive whenever $c(t)$ and $\dot{c}(t)$ are positive. Question is what happens under mutual inductance time variation and negative rates of variations in the reactive elements. Theorem 4.1 below answers this question.

Theorem 4.1 *Consider the LTV system described in Theorem 3.2, with assumptions 3.1, 3.2, 3.3 in force. Then this LTV system is strictly passive if for all t,*

$$\sum_{i=1}^{p} \left[\frac{d}{dt}\ln l_i(t)\right]^- + \sum_{i=1}^{q} \left[\frac{d}{dt}\ln c_i(t)\right]^- + \sum_{i=1}^{\nu} \left[\frac{d}{dt}\ln m_i(t)\right]^- > -2\sigma, \tag{4.1}$$

where

$$[a]^- = \begin{cases} a; & a \leq 0 \\ 0; & a > 0 \end{cases}.$$

Further in this case the matrix $\mathcal{M}(\Theta(t))$ described in Theorem 3.2 acts as a Riccati matrix.

Thus passivity is not affected by the resistor variation rates. Further, arbitray positive variation rates can be sustained in the reactive elements without impairing passivity.

For type 1 and two systems however the result is more complicated.

Theorem 4.2 *Consider the LTV system described in (3.4) ((3.5)), (3.3), with $\Theta(t) \in \mathcal{T}$ for all t. Suppose all members of the set in (3.6) ((3.7)) are σ-passive. Then consider the LTV system with SVR given in (4.2) ((4.3)), with $\Theta(t) \in \mathcal{T}$ for all t*

$$\{F + gh_1'(\mu(t)), g, \beta(t)[h_1'(\mu(t)) - h_2'(\lambda(t))], \beta(t)\} \tag{4.2}$$

$$\{F + gh_1'(\mu(t)), g, \beta(t)[h_1'(\mu(t)) - h_2'(\lambda(t))], \beta(t)(1-\alpha)\}, \tag{4.3}$$

where

$$\beta(t) = \prod_{i=1}^{N} \left[\frac{\theta_i - \theta_i^-}{\theta_i^+ - \theta_i} + 1 \right].$$

Then (4.2) ((4.3)) is strictly passive if for all t,

$$\sum_{i=1}^{N} \left[\frac{d}{dt} \ln \frac{\theta_i - \theta_i^-}{\theta_i^+ - \theta_i} \right]^+ < 2\sigma, \qquad (4.4)$$

where

$$[a]^+ = \begin{cases} a; & a \geq 0 \\ 0; & a < 0 \end{cases}$$

Few observations need to be made. First, observe because of the fact that whenever $\Theta(t) \in \mathcal{T}$ for all t, there exist β^-, β^+ such that for all t,

$$0 < \beta^- \leq \beta(t) \leq \beta^+.$$

Thus instead of showing the passivity of the original LTV system, we have shown the passivity of this system *scaled* by a time varying, uniformly positive, bounded signal. The time varying nature of $\beta(t)$ precludes in general the implication that the original LTV system is passive as well. The key reason behind the difference between this result and that in Theorem 4.1 is that the frozen systems in Theorem 4.1 would have retained passivity with their parameters taking values from *any point on the entire positive real axis.* This is not the case with the systems in Theorem 4.2. *Nonetheless, as will be shown in Section 5, the passivity of this scaled system still allows useful conclusions to be drawn about the original LTV system.*

Second, the condition in (4.4) shows that this scaled passivity obtains for *arbitrary* positive rates of variation in the parameters θ_i. Only negative rates of variation cause concern. This is directly opposite to what is the case in (4.1).

5 Passivity Under Average Bounds

In this Section, we consider the situation when the hard bounds in the previous Section are replaced by average bounds of the type used in the stability analysis in [2]. In Section 5.1 we show that, under these conditions a related LTV system is passive. In Section 5.2 we give the stability implications to the original systems.

5.1 Passivity Results

We first turn to the system in Section 3.2.

Theorem 5.1 *Consider the LTV system described in Theorem 3.2, with assumptions 3.1, 3.2, 3.3 in force. Suppose there exist $\epsilon_1 > 0$ and $T > 0$ such that for all t_0*

$$\int_{t_0}^{t_0+T} \Big[\sum_{i=1}^{p} \left[\tfrac{d}{dt} \ln l_i(t)\right]^- + \sum_{i=1}^{q} \left[\tfrac{d}{dt} \ln c_i(t)\right]^- + \sum_{i=1}^{\nu} \left[\tfrac{d}{dt} \ln m_i(t)\right]^- \Big] dt > -2(\sigma - \epsilon_1). \qquad (5.1)$$

Then with

$$\rho(t) = 2\sigma + \sum_{i=1}^{p} \left[\tfrac{d}{dt} \ln l_i(t)\right]^- + \sum_{i=1}^{q} \left[\tfrac{d}{dt} \ln c_i(t)\right]^- + \sum_{i=1}^{\nu} \left[\tfrac{d}{dt} \ln m_i(t)\right]^- . \qquad (5.2)$$

the LTV system with SVR (see (3.10)),

$$\{0.5\rho(t)I + \mathcal{M}^{-1}(\Theta(t)) \left[-\dot{\mathcal{M}}(\Theta(t)) + A(\Theta(t))\right], B(\Theta(t)), C(\Theta(t)), D(\Theta(t))\}, \qquad (5.3)$$

together with its inverse, is strictly passive and eas.

We will call (5.3) the $\rho(t)$-shifted version of (3.10).

Remark 5.1 *If one assigns the operators G and G_ρ to (3.10) and (5.3), respectively, then one has*

$$G_\rho = exp\left[-0.5 \int_{t_0}^{t} \rho(\tau) d\tau\right] G \left\{ exp\left[0.5 \int_{t_0}^{t} \rho(\tau) d\tau\right] \right\}.$$

Further with the states of these two systems labelled $x(t)$ and $x_\rho(t)$ respectively, one has

$$x_\rho(t) = x(t) exp\left[0.5 \int_{t_0}^{t} \rho(\tau) d\tau\right].$$

The states of their respectve inverses also obey this relationship. It is easy to construct examples where the passivity of G_ρ does not imply the passivity of G.

For type 1 and 2 systems a similar result is possible.

Theorem 5.2 *Consider the LTV system described in (3.4) ((3.5)), (3.3), with $\Theta(t) \in \mathcal{T}$ for all t. Suppose all members of the set in (3.6) ((3.7)) are σ-passive. Suppose there exist $\epsilon_1 > 0$ and $T > 0$ such that for all t_0*

$$\int_{t_0}^{t_0+T} \sum_{i=1}^{N} \left[\frac{d}{dt} \ln \frac{\theta_i - \theta_i^-}{\theta_i^+ - \theta_i} \right]^+ dt < 2(\sigma - \epsilon_1), \qquad (5.4)$$

Then the LTV system with SVR given in (5.5) ((5.6)), together with its inverse, with $\Theta(t) \in \mathcal{T}$ for all t, is strictly passive and eas

$$\{0.5\rho(t)I + F + gh_1'(\mu(t)), g, \beta(t)[h_1'(\mu(t)) - h_2'(\lambda(t))], \beta(t)\} \qquad (5.5)$$

$$\{0.5\rho(t)I + F + gh_1'(\mu(t)), g, \beta(t)[h_1'(\mu(t)) - h_2'(\lambda(t))], \beta(t)(1-\alpha)\}, \qquad (5.6)$$

where $\beta(t)$ is given in Theorem 4.2 and

$$\rho(t) = \sigma - \left[\frac{d}{dt}\ln\frac{\theta_i - \theta_i^-}{\theta_i^+ - \theta_i}\right]^+$$

Observe that remark 5.1 applies to this setting as well. Further (4.4) is a special case of (5.4). Consequently, the results in the next subsection, though stated in terms of Theorem 5.4 apply to (4.4) as well.

5.2 Stability Results

It is clear that under the hypothesis of the results in Section 5.1, $\rho(t)$ is *on the average* positive. Thus, because of Remark 5.1 we have the following stability result on the *original* LTV systems in Section 3.

Theorem 5.3 *Under the hypothesis of Theorem 5.1, (Theorem 5.2), the system in (3.10) ((3.4)) and its inverse is eas.*

This result can in fact be applied in a wider stability verification context. For example, it provides an interpretation of the result in [2], on the stability of (1.1) with

$$A(t) = F + gh'(\Theta(t)), \tag{5.7}$$

where $h(\Theta(t))$ is affine in $\theta(t)$, for all t, $\Theta(t) \in \mathcal{T}$. Then [2] shows the eas of this system under (i) $Re[F + gh'(\Theta)] < -\sigma$, for all $\Theta \in \mathcal{F}$ and (ii) (5.4) holding. One can show that (i) holds iff one can find a LTV system as in (3.4) whose frozen SVR's are all σ-passive and stability of whose inverse implies the stability of (1.1), (5.7). Thus Theorem 5.3 proves this result.

The next two results concern the closed loop of fig. 1 with $f(\cdot,\cdot)$ a memoryless nonlinear time varying (NLTV) block. If G and F were passive then the closed loop would have been stable. However, as the next theorem shows, the passivity of a shifted version of G, suffices for stability.

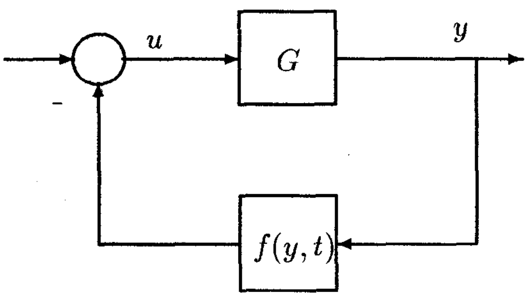

Figure 1: A closed-loop configuration

Theorem 5.4 *Suppose G in fig. 1 is as in (3.4), ((3.10)) with the assumptions in Theorem 5.2, (Theorem 5.1) in force. Then the closed loop is L_2 stable if for all t,*

$$f(y,t)y > 0$$

The next result is similar in nature.

Theorem 5.5 *Suppose G in fig. 1 has SVR*

$$\{F + gh_1'(\mu(t)), g, h_1'(\mu(t)) - h_2'(\lambda(t))\}$$

and that all members of the LTI set (3.7) are σ-passive. Suppose, with $\Theta(t) = [\lambda(t), \mu(t)]'$, $\Theta(t) \in \mathcal{T} \forall t$. Then the closed loop is L_2 stable if for all t,

$$0 < f(y,t)y < \frac{1}{1-\alpha}$$

and (5.4) holds.

6 Conclusions

We have presented a number of results on the passivity of LTV systems and its utility to stability analysis. All these results exploit the passivity of a frozen version of the system and on the solution of a Riccati matrix being multiaffine in the time varying parameters.

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