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Brief Paper

Further results on localization-based switching adaptive control[☆]

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Abstract

We investigate a switching adaptive control scheme based on falsification which is conceptually different from existing switching adaptive control schemes. A feature of the proposed localization method is its fast model falsification capability. In the LTI case this is manifested as the rapid convergence of the switching controller. By analysing the geometry of localization we give a complete solution to the problem of optimal localization. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Adaptive control; Discrete-time systems; Switching algorithms

1. Introduction

Conventional switching adaptive control techniques (Bai, 1988; Chang & Davison, 1995; Fu & Barmish, 1986; Martensson, 1985; Morse, 1993) are all based on some mechanism of an exhaustive search over the entire set of potential controllers (either a continuum set (Martensson, 1985) or a finite set (Fu & Barmish, 1986). A major drawback of such approaches is that the search may converge very slowly, resulting in excessive transients which renders the system "unstable" in a practical sense. To alleviate this problem, several new switching control schemes have been proposed recently. Supervisory control for adaptive set-point tracking of LTI systems is proposed in Morse (1993) to improve the transient response. Similar supervisory control schemes were analysed in Hocherman-Frommer, Kulkarni and Ramadge (1995) and Narendra and Balakrishnan (1994). However, several issues still remain unresolved, in particular, without a simpler proof and better understanding of the mechanisms of supervisory switching control its design will remain mainly a matter of trial and error.

Our primary objective is to analyse a new class of adaptive switching controllers applicable to a wide range of LTI/LTV systems. Our approach is based on a localization method, which incorporates simultaneous falsification of a large number of models (Zhivoglyadov, Middleton & Fu, 1997, 2000) The main contribution of this paper is a complete solution to the problem of optimal localization. The potential advantages of localization-based switching control include finite convergence of switching for LTI systems, fast model falsification capabilities of the controller and simplicity of the stability analysis and realization.

The rest of the paper is organized as follows. In Sections 2 and 3 we introduce the class of uncertain systems to be studied and review some recent results on localization-based switching control (Zhivoglyadov, Middleton & Fu, 1997, 2000) adding an in-depth description of the basic localization scheme. In Section 4 we solve the problem of optimal localization aimed at minimizing the worst-case number of controller switchings. A simulation example demonstrating the rapid falsification capabilities of the localization method and conclusions are given in Sections 5 and 6, respectively.

2. Problem statement

We consider a general class of SISO discrete-time plants in the following form:

$$D(t,z^{-1})y(t) = N(t,z^{-1})u(t) + \xi(t-1) + \eta(t-1), \qquad (1)$$

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where $u(t)$ is the input, $y(t)$ is the output, $\xi(t)$ is the exogenous disturbance, $\eta(t)$ represents the unmodelled dynamics (see Assumption (A5) below), z^{-1} is the unit delay operator, and

$$N(t,z^{-1}) = n_1(t)z^{-1} + n_2(t)z^{-2} + \cdots + n_n(t)z^{-n},$$
$$D(t,z^{-1}) = 1 + d_1(t)z^{-1} + \cdots + d_n(t)z^{-n}. \qquad (2)$$

We will denote by $\theta(t) = (n_n(t), \ldots, n_2(t), -d_n(t), \ldots, -d_1(t), n_1(t))^{\mathrm{T}}$ the vector of unknown time-varying parameters. Throughout the paper, we will use the following nonminimal state-space description of plant (1):

$$x(t+1) = A(\theta(t))x(t) + B(\theta(t))u(t) + E(\xi(t) + \eta(t)), \qquad (3)$$

where

$$x(t) = [u(t-n+1) \ldots u(t-1) | y(t-n+1) \ldots y(t)]^{\mathrm{T}}$$

and the matrices $A(\theta(t))$, $B(\theta(t))$ and E are constructed in a standard way. We also define the regressor vector as $\phi^{\mathrm{T}}(t) = [x(t) | u(t)]$. Then, (1) can be rewritten as $y(t) = \phi^{\mathrm{T}}(t-1)\theta(t-1) + \xi(t-1) + \eta(t-1)$.

The following assumptions are used throughout the paper:

(A1) The order n of the nominal plant (excluding the unmodelled dynamics) is known.

(A2) There exists a known compact set $\Omega \in \mathbf{R}^{2n}$ such that $\theta(t) \in \Omega$ for all $t \in \mathbf{N}$.

(A3) Plant (1) with frozen parameters and zero unmodelled dynamics (i.e. $\eta(t) \equiv 0$) is stabilizable over Ω. That is, for any $\theta(t) \equiv \theta \in \Omega$, there exists a linear time-invariant controller $C(z^{-1})$ such that the closed-loop system is exponentially stable.

(A4) $\sup_{t \geq t_0} |\xi(t)| \leq \bar{\bar{\xi}}$ for some $\bar{\bar{\xi}} \geq 0$ and for any $t_0 \in \mathbf{N}$.

(A5) The unmodelled dynamics are arbitrary subject to $|\eta(t)| \leq \bar{\eta}(t) = \varepsilon \sup_{0 \leq k \leq t} \sigma^{t-k} \|x(k)\|$ for some $\varepsilon > 0$ and $0 \leq \sigma < 1$ which represent the "size" and "decay rate" of the unmodelled dynamics, respectively.

(A6) The uncertain parameters are allowed to have two types of time variations:

(i) slow parameter drift described by $\|\theta(t) - \theta(t-1)\| \leq \alpha$, $\forall t > t_0$ for some $\alpha > 0$, and
(ii) infrequent large jumps constrained by $\sum_{i=t}^{t+\tau N} s_i \leq \tau$ for all $t \geq 0$, where $\tau > 0$ and $N > 0$ are constants with $1/N$ representing the "frequency" of large jumps, and $s_i = 0$ if $\|\theta(i) - \theta(i-1)\| \leq \alpha$ and $s_i = 1$ otherwise.

We note that the assumptions outlined above are quite standard and have been used in adaptive control to derive stability results for time-varying systems (see, e.g., Ioannou & Sun, 1996; Middleton & Goodwin, 1988 for more details).

The switching controller to be designed will be of the form $u(t) = K_{i(t)}x(t)$ where $K_{i(t)}$ is the controller gain applied at time t, and $i(t)$ is the switching index at time t, taking value in a finite index set I. The objective of the control design is to determine the set of control gains $K_I = \{K_i, i \in I\}$ and an on-line switching algorithm for $i(t)$ so that the closed-loop system is "stable" in some sense.

Definition 2.1. System (1) satisfying (A4)–(A6) is said to be globally $\bar{\bar{\xi}}$-exponentially stabilized by the switching controller if there exist constants $M_1 > 0$, $0 < \rho < 1$, and a function $M_2(\cdot): \mathbf{R}_+ \to \mathbf{R}_+$ with $M_2(0) = 0$ such that $\|x(t)\| \leq M_1 \rho^{(t-t_0)} \|x(t_0)\| + M_2(\bar{\bar{\xi}})$ holds for all $t_0 \geq 0$, $x(t_0)$, $\bar{\bar{\xi}} \geq 0$, and $\xi(\cdot)$ and $\eta(\cdot)$ satisfying (A4) and (A5), respectively.

3. Description of the basic localization scheme

In this section we review some results on localization-based switching control reported in Zhivoglyadov et al. (1997, 2000) adding an in-depth discussion of the "geometry" of localization needed for further development. The localization technique being the key element in the proposed method implies appropriate decomposition of the uncertainty set Ω and an effective online mechanism of discarding incorrect controllers.

Consider any decomposition of the parameter set Ω satisfying the following conditions:

(C1) $\Omega_i \subset \Omega$, $\Omega_i \neq \emptyset$, $i = 1, \ldots, L$;
(C2) $\bigcup_{i=1}^{L} \Omega_i = \Omega$;
(C3) for each $i = 1, \ldots, L$, $\exists \theta_i \in \Omega_i$ ("centre"), $r_i > 0$ ("raduis"), K_i (control gain), $q > 0$ (scalar parameter) and symmetric positive-definite matrices H_i and Q_i such that $(A(\theta) + B(\theta)K_i)^{\mathrm{T}} H_i (A(\theta) + B(\theta)K_i) - H_i \leq -Q_i$, $\forall \|\theta - \theta_i\| \leq r_i + q$, $i = 1, \ldots, L$.

Conditions (C1) and (C2) basically say that the uncertainty set Ω is presented as a finite union of nonempty subsets while condition (C3) defines each subset Ω_i as being quadratically stabilizable by a single LTI controller K_i. It is well known that such a finite cover can be found under assumptions (A1)–(A3) (see, e.g., Fu & Barmish, 1986; Morse, 1993 for technical details and examples). The complexity of decomposing the uncertainty set, in general, depends on many factors including the "size" of the set, its dimension and "stabilizability" properties.

The key observation used in the localization technique is the following fact: Given any parameter vector $\theta \in \Omega_j$ and a control gain $K_{i(t)}$ for some $i(t)$, $j = 1, \ldots, L$, suppose that $i(t) = j$, then it follows from the description of the plant that

$$|\theta_j^{\mathrm{T}} \phi(t-1) - y(t)| \leq r_j \|\phi(t-1)\| + \bar{\bar{\xi}} + \bar{\eta}(t-1). \qquad (4)$$

If the above inequality is violated, then the switching index $i(t)$ is wrong (i.e. $i(t) \neq j$), so it can be eliminated (falsified; see, e.g., Haber & Unbehauen, 1990). The unique feature of the localization technique comes from the fact that violation of (4) allows us not only to eliminate a single index, $i(t)$, (if $i(t) \neq j$) from the set of possible controller indices, but many others.

Let $I(t)$ denote the set of "admissible" control gain indices at time t and initialize it to be $I(t_0) = \{1, 2, \dots, L\}$. Choose any initial switching index $i(t_0) \in I(t_0)$. For $t > t_0$, define $\hat{I}(t) = \{j\colon (4) \text{ holds}, j \in I(t_0)\}$. Then, the localization algorithm is simply given by

$$I(t) = I(t-1) \cap \hat{I}(t), \quad \forall t > t_0. \tag{5}$$

The switching index is updated by taking[1]

$$i(t) = \begin{cases} i(t-1) & \text{if } t > t_0 \text{ and } i(t-1) \in I(t), \\ \text{any member of } I(t) & \text{otherwise.} \end{cases} \tag{6}$$

One possible way to view the localization technique is to interpret it as family set identification conducted on a finite set of elements. The strip depicted in Fig. 1 contains only those elements which are consistent with the measurement of the input/output pair $\{y(t), u(t-1)\}$. The high falsifying capability of the proposed algorithm observed in simulations can informally be explained in the following way. Let the index $i(t)$ be falsified, then the discrete set $\{\theta_i\colon i \in I(t)\}$ consistent with all the past measurements is separated from the point $\theta_{i(t)}$ by one of the hyperplanes $\theta_j^{\mathrm{T}}\phi(t-1) = y(t) + r_j\|\phi(t-1)\| \pm \bar{\xi} \pm \bar{\eta}(t-1)$ dividing the parameter space into two half-spaces. Moreover, every element belonging to the half-space containing $\theta_{i(t)}$ is falsified by the algorithm of localization (5) at the instant t.

Theorem 3.1 (Zhivoglyadov et al., to appear). *Any localization algorithm given in* (5) *and* (6) *guarantees the following properties when* ε (*i.e., the "size" of unmodelled dynamics*) *is sufficiently small*:

(i) *The closed-loop system is globally* $\bar{\xi}$*-exponentially stable, i.e., there exists constants* $M_1 > 0$, $0 < \rho < 1$, *and a function* $M_2(\cdot)\colon \mathbf{R}_+ \to \mathbf{R}_+$ *with* $M_2(0) = 0$ *such that*

$$\|x(t)\| \leq M_1 \rho^{(t-t_0)} \|x(t_0)\| + M_2(\bar{\xi}) \tag{7}$$

holds for all $t \geq t_0$ *and* $x(t_0)$;

(ii) *The switching sequence* $\{i(t_0), i(t_0+1), \dots\}$ *is finitely convergent.*

The proof of the theorem is based on the observation that between any two consecutive switchings the closed-loop system behaves as an exponentially stable LTI system subject to small parametric perturbations and bounded exogenous disturbance. Moreover, this property does not depend on the possible evolution of the parameters in time. This is the key point offering a clear understanding of the control mechanisms.

[1] In fact, we will see in Section 4 that there are "clever" ways of selecting $i(t)$ when $i(t-1)$ is falsified.

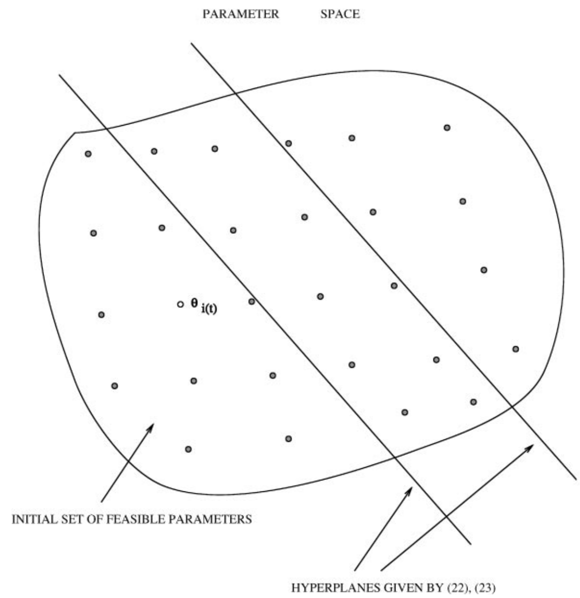

Fig. 1. Localization.

We note that the constant M_1 in (7) is proportional to the total number of switchings while the parameter ρ is dependent on the "stabilizability" property of the set Ω.

The general structure of the switching controller for the LTV case is similar to the time-invariant case except that the localization algorithm needs some modification. More specifically, the switching index set $I(t)$ is initialized as $I(t) = \{1, \dots, L\}$. At each $t > t_0$, the set $\hat{I}(t)$ is computed using (4) where r_j is replaced by $(r_j + q)$, that is,

$$\hat{I}(t) = \{j\colon |\theta_j^{\mathrm{T}}\phi(t-1) - y(t)| \leq (r_j + q)\|\phi(t-1)\| + \bar{\xi} + \bar{\eta}(t-1),\ j = 1, \dots, L\}. \tag{8}$$

By doing this we make the decomposition of the uncertainty set Ω slightly redundant. This avoids rapid switching of the controller caused by the parameters drifting slowly along the boundary of two (or more) neighbouring subsets. The localization set $I(t)$ is updated by

$$I(t) = \begin{cases} I(t-1) \cap \hat{I}(t) & \text{if } I(t-1) \cap \hat{I}(t) \neq \emptyset, \\ \hat{I}(t) & \text{otherwise.} \end{cases} \tag{9}$$

This discrete forgetting scheme eliminates the need for persistency of excitation which is required in many adaptive control schemes.

Theorem 3.2 (Zhivoglyadov et al., to appear). *The localization scheme described above guarantees the following property when* ε (*i.e., the "size" of unmodelled dynamics*) *is sufficiently small*:

The closed-loop system is globally $\bar{\xi}$*-exponentially stable if* $M_1^{(1+[N\alpha/q])l}\rho^N < 1$ *where* M_1 *and* ρ *are constants in* (7), α, N *are constants used in Assumption* (A6) *to describe the "rate" of parameter variations and the "frequency" of large* *parameters jumps, q is given in Condition* (C3), *and l denotes the maximum number of switchings for the LTI case.*

Remark 3.1. We note that the constants M_1, l and ρ corresponding to the LTI case have been deliberately included in the condition of Theorem 3.2 to confirm the intuitive conjecture that the high falsification capability of the algorithm of localization is of great importance.

4. Optimal localization

The problem of optimal localization addresses the issue of optimal selection of the new switching index at each switching instant so that the set of admissible switching indices $I(t)$ is guaranteed to be pruned down as rapidly as possible. The problem is solved in this paper in terms of the indices of localization defined below. We assume the following for simplicity of notation.[2]

(A7) $r_i = r_j = r$, $\forall i,j = 1,2,\dots,L$, and some $r > 0$.

For any set $I \subset \{1,2,\dots,L\}$, $\Theta = \{\theta_i\colon i \in I\}$, a fixed $j \in I$ and any $z \neq 0$, $z \in \mathbf{R}^{2n}$, define the function $\lambda(z,j,\Theta) = |\{\theta_i\colon (\theta_i - \theta_j)^{\mathrm{T}} z \geq 0,\ i \in I\}|$ where $|\cdot|$ denotes the cardinal number of a set. Then $\operatorname{ind}(\theta_j,\Theta) = \min_{\|z\|=1} \lambda(z,j,\Theta)$ will be referred to as the index of localization of the element θ_j with respect to the set Θ.

Lemma 4.1. *The index of localization* $\operatorname{ind}(\theta_j,\Theta)$ *represents a guaranteed lower bound on the number of indices discarded from the localization set $I(t)$ at the next switching instant provided that $u(t) = K_j x(t)$.*

Proof. Without loss of generality, we assume that $(t+1)$ is the next switching instant, and controller K_j is discarded. From (8) we have $j \notin \hat{I}(t+1)$, equivalently,

$$\theta_j^{\mathrm{T}} \phi(t) > y(t+1) + (r_j + q)\|\phi(t)\| + \bar{\xi} + \bar{\eta}(t) \tag{10}$$

or

$$\theta_j^{\mathrm{T}} \phi(t) < y(t+1) - (r_j + q)\|\phi(t)\| - \bar{\xi} - \bar{\eta}(t). \tag{11}$$

Taking $z = -\phi(t)/\|\phi(t)\|$ for (10), or $z = \phi(t)/\|\phi(t)\|$ for (11) we see that there are $\lambda(z,j,\Theta)$ number of controller indices which do not belong to $\hat{I}(t+1)$. We note that $\phi(t) \neq 0$, because otherwise it is easy to see that there exists no element $\theta_j \in \Theta$ satisfying (10) or (11), and, consequently, switching is not possible. Since $\operatorname{ind}(\theta_j,\Theta) \leq \lambda(z,j,\Theta)$, we conclude that there are at least $\operatorname{ind}(\theta_j,\Theta)$ number of controllers to be discarded at the switching instant $(t+1)$. $\square$

[2] For notational convenience we drop when possible the index t from the description of the set $I(t)$.

Then $\operatorname{ind}\Theta = \max_j \{\operatorname{ind}(\theta_j,\Theta)\colon j \in I\}$ will define the index of localization of the discrete set Θ. That is, $\operatorname{ind}\Theta$, is the largest attainable lower bound on the number of controllers eliminated at the time of switching, assuming that the regressor vector can take any value. Thus the problem of optimal localization reduces to determining the index $i(t) = i_{\mathrm{opt}}(t) = \arg\max_j \{\operatorname{ind}(\theta_j,\Theta(t))\colon j \in I(t)\}$ at each switching instant.

Definition 4.1. Given a finite set $\Theta \subset \mathbf{R}^n$ and a subset $J \subset \Theta$; J is called a separable set of order k if
(i) $|J| = k$;
(ii) $\mathrm{co}\{J\} \cap \mathrm{co}\{\Theta - J\} = \emptyset$ where co $\{\cdot\}$ stands for the convex hull of a set.

Some properties of separable sets are listed below:

(a) a vertex of $\mathrm{co}\{\Theta\}$ is a separable set of order 1;
(b) the order of a separable set $k \leq |\Theta|$;
(c) for each separable set J of order k, $k > 1$, there exists a set $J' \subset J$ such that J' is a separable set of order $(k-1)$.

Lemma 4.2. *Let $\mathbf{\Theta}^k$ be the set of all separable sets of order k and $\mathbf{\Xi}^k = \bigcup_{J_k \in \mathbf{\Theta}^k} J_k$. Then,*

$$\operatorname{ind}\Theta = 1 + \arg\max_k \{k\colon \mathbf{\Xi}^k \neq \Theta\}. \tag{12}$$

Proof. Follows immediately from Definition 4.1 and the property of separable sets (c). Indeed, suppose that the index of localization satisfies the relation

$$\operatorname{ind}\Theta = m > 1 + \arg\max_k \{k\colon \mathbf{\Xi}^k \neq \Theta\}, \tag{13}$$

then there must exist an element $\theta_j \in \Theta$, such that $\operatorname{ind}(\theta_j,\Theta) = m$, moreover, $\theta_j \notin \mathbf{\Xi}^{m-1}$, $\theta_j \in \Theta - \mathbf{\Xi}^{m-1}$ since otherwise, by definition of separable sets $\operatorname{ind}(\theta_j,\Theta) \leq m-1$. But it follows from (13) that $\Theta - \mathbf{\Xi}^{m-1} = \emptyset$. On the other hand by Definition 4.1 and the properties of separable sets (b) and (c) the index of localization of the set Θ cannot be smaller than that given by (12). This concludes the proof. $\square$

Denote by $V(\cdot)$ the set of vertices of co $(\cdot)$. The complete solution to the problem optimal localization is given by the following iterative algorithm.

Algorithm A (*Optimal localization*). *Step* 1: Initialize $k = 1$. Compute $\mathbf{\Theta}^1 = \{\{\theta\}\colon \theta \in V(\Theta)\}$

Step 2: Set $k = k + 1$. Compute

$$\mathbf{\Theta}^k = \{J_{k-1} \cup \theta_i\colon J_{k-1} \in \mathbf{\Theta}^{k-1},$$

$$\theta_i \in V(\Theta - J_{k-1}),\ J_{k-1} \cup \theta_i \text{is separable}\}.$$

Step 3: If $\mathbf{\Xi}^k = \Theta$, then $\operatorname{ind}\Theta = k$, and stop, otherwise go to Step 2.

This can be formulated as

Theorem 4.1. (i) *The solution to the problem of optimal localization may not be unique and is given by the set* $\mathbf{I}_{\text{opt}} = \text{sub}\{\Theta - \Xi^{m-1}\}$ *where*

$$m = \text{ind}\,\Theta = 1 + \arg\max_{k} \{k\colon \Xi^k \neq \Theta\} \tag{14}$$

and $\text{sub}\{\cdot\}$ *denote the set of subscripts of all the elements in* $\{\cdot\}$.

(ii) *For* $\bar{\xi} \geq 0, \varepsilon \geq 0$, *the total number of switchings* l *made by the optimal switching controller applied to the LTI plant* (1) *satisfies the relation* $\sum_{p=0}^{l-1} \text{ind}\,\Theta(t_p) \leq L - 1$ *where* t_p, $p = 0,1,\dots,l-1$ *denote the switching instants.*

Proof. The proof of (i) follows directly from Lemma 4.2. To prove (ii) we note that

$$|\Theta(t_1)| \leq L - \text{ind}\,\Theta(t_0),$$

$$|\Theta(t_2)| \leq |\Theta(t_1)| - \text{ind}\,\Theta(t_1)$$
$$\leq L - \text{ind}\,\Theta(t_0) - \text{ind}\,\Theta(t_1), \dots$$

then

$$|\Theta(t_l)| \leq v_l = L - \sum_{i=0}^{l-1} \text{ind}\,\Theta(t_i).$$

Since $v_l \geq 1$ the result follows. □

Algorithm A applied to an arbitrary localization set Θ indicates that except for a very special case, namely, $\{\theta_j\}_{j \in I} = V(\Theta)$, localization with any choice of the switching index $i(t)$ such that $\theta_{i(t)} \notin V(\Theta)$ will always result in elimination of more than one controller at any switching instant. This feature distinguishes localization-based switching controllers from conventional switching controllers. Moreover, a simple geometrical analysis (see, e.g., Fig. 1) indicates that for "nicely" shaped uncertainty sets (for example, a convex Ω) and large L the index of localization is typically large, that is, $\text{ind}(\theta) \gg 1$. To alleviate potential computational difficulties we propose a suboptimal localization scheme.

Algorithm B (*Suboptimal localization*). *Step* 1: Initialize $k = 1$. Compute $\mathbf{\Gamma}^1 = V(\Theta)$.

Step 2: Set $k = k + 1$. Compute $\mathbf{\Gamma}^k = \mathbf{\Gamma}^{k-1} \cup V(\Theta - \mathbf{\Gamma}^{k-1})$.

Step 3: If $\mathbf{\Gamma}^k = \emptyset$, then $\text{ind}\,\Theta \geq k$, and stop, otherwise go to Step 2.

Algorithm B allows for a simple geometrical interpretation, namely, at each step a new set $\mathbf{\Gamma}^k$ is obtained recursively by adding the set of vertices of $(\Theta - \mathbf{\Gamma}^{k-1})$. The simplicity of the proposed algorithm is explained by the fact that we no longer need to check the property of separability (see Step 2 in Algorithm A).

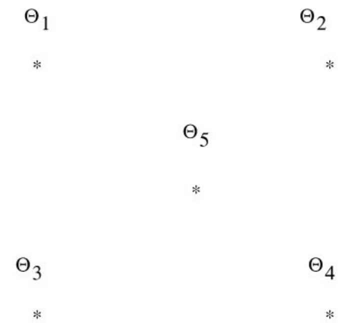

Fig. 2. Example of optimal localization.

Proposition 4.2. *The index of localization* $\text{ind}\,\Theta$ *satisfies the inequality*

$$\text{ind}\,\Theta \geq 1 + \arg\max_{k} \{k\colon \mathbf{\Gamma}^k \neq \Theta\}. \tag{15}$$

Proof. The proof is simple and follows from the fact that for any $\theta \in \Theta$, such that $\theta \notin V(\Theta)$ it is true that $\text{ind}(\theta, \Theta) \geq 2$. By applying this rule recursively we obtain (15). □

Example 4.1. To illustrate the idea of optimal (suboptimal) localization we consider a simple localization set $\Theta = \{\theta_j\}_{j=1}^5$ in Fig. 2. We note that the point θ_5 is located exactly in the centre of the square $(\theta_1, \theta_2, \theta_4, \theta_3)$. Applying Algorithm A to the set Θ we have $\mathbf{\Theta}^1 = \{\{\theta_1\}, \{\theta_2,\}, \{\theta_3\}, \{\theta_4\}\}$, $\mathbf{\Theta}^2 = \{\{\theta_1, \theta_2\}, \{\theta_1, \theta_3\}, \{\theta_2, \theta_4\}, \{\theta_3, \theta_4\}\}$, and $\mathbf{\Theta}^3 = \{\{\theta_1, \theta_2, \theta_5\}, \{\theta_1, \theta_3, \theta_5\}, \{\theta_3, \theta_4, \theta_5\}, \{\theta_2, \theta_4, \theta_5\}\}$. Since $\bigcup_{J \in \mathbf{\Theta}^3} J = \Theta$ we conclude that $\text{ind}\,\Theta = 3$ and the optimal switching index is given by $i(t) = 5$. To compute a lower bound on the index of localizaton ind Θ Algorithm B is used. We have $\mathbf{\Gamma}^1 = \{\theta_1, \theta_2, \theta_3, \theta_4\}$, $\mathbf{\Gamma}^2 = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\} = \Theta$, therefore, $\text{ind}\,\Theta \geq 2$.

Remark 4.1. To deal with the problem of suboptimal localization different heuristic procedures can be envisioned. For example, the following "geometric mean" algorithm of computing a new switching index $i(t) = \arg\min_j \|\theta_j - \sum_{i \in I(t)} \theta_i / |\Theta(t)|\,\|$ is likely to perform well, though it is quite difficult, in general, to theoretically substantiate it.

5. Simulation example

Some interesting features of the localization technique observed in simulations include the low sensitivity of the speed of localization to the total number of fixed controllers and the switching index update rule, and the ability of the controller to successfully cope with time-varying uncertainty including frequent parameter jumps.

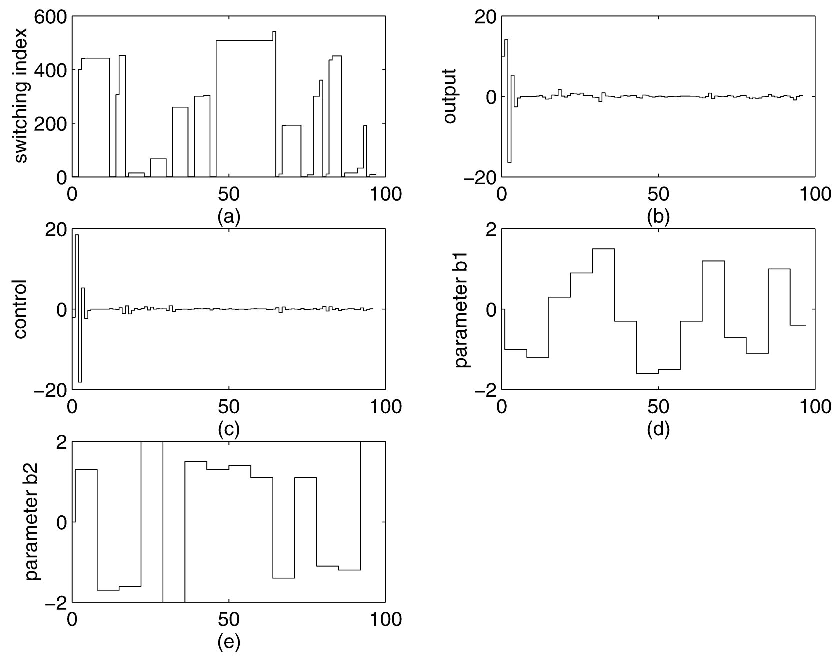

Fig. 3. Example of localization: parameters jump every 7 steps.

Consider the following family of unstable (pointwise nonminimum phase) LTV plants: $y(t)=1.2y(t-1)-1.22y(t-2)+b_1(t)u(t-1)+b_2(t)u(t-2)+\xi(t)$ where the disturbance $\xi(t)$ is uniformly distributed on the interval $[-0.1, 0.1]$, and $b_1(t)$, $b_2(t)$ are uncertain time-varying parameters. The a priori parameter uncertainty bounds are given by $b_1(t)\in[-1.6,-0.15]\cup[0.15,1.6]$; $b_2(t)\in[-2,-1]\cup[1,2]$. With the parameters b_{1i} and b_{2i} taking values from the sets $\{-0.2,-0.3,\ldots,-1.6,0.2,0.3,\ldots,1.6\}$ and $\{-1.1,-1.2,\ldots,-2,1.1,1.2,\ldots,2\}$, respectively, the results of localization on the finite set $\{\theta_i\}_{i=1}^{600}$ are presented in Figs. 3(a)–(e). The algorithm of localization in Section 3 was applied and a pole placement technique was used to compute the set of controller gains $\{K_i\}_{i=1}^{600}$. The poles of the nominal closed-loop system were chosen to be $(0, 0.07, 0.1)$. We note that we are not aware of any successful attempts to develop a switching controller for LTV systems, but for the LTI case the quality of regulation is similar to that produced by supervisory switching control (Morse, 1993)

6. Conclusions

In this paper we have presented a discussion of a new type of adaptive switching control, namely, localization-based switching control (Zhivoglyadov et al., 1997, 2000). In our discussion the emphasis is placed on the "geometry" of localization paving the way for optimal control design.

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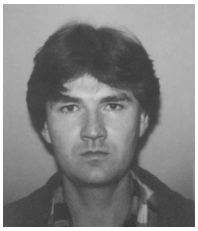

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