FP3 - 4:40

Proceedings of the
American Control Conference
Baltimore, Maryland • June 1994

LOOP TRANSFER RECOVERY FOR SAMPLED-DATA SYSTEMS[1]

Peng Shi, Minyue Fu and Carlos E. de Souza

Department of Electrical & Computer Engineering
The University of Newcastle
NSW 2308 Australia

Abstract

This paper formulates a loop transfer recovery (LTR) problem for continuous-time systems with sampled output measurements in the H_∞ setting and shows that this LTR problem is equivalent to a known filtering problem for sampled-data systems, which can be solved in terms of a pair Riccati differential and difference equations.

1. Introduction

A consideration amount of attention has been paid to the theory and application of loop transfer recovery in the last decade; see, for example, [1, 2, 3, 4, 5, 6]. The standard loop transfer recovery problem is as follows: Given a plant $G(s)$ and a target loop $L_d(s)$, designed using state feedback control, find a dynamic output feedback controller such that the following two properties are satisfied:
(i) the closed-loop input-output response is the same as in the state feedback case; and
(ii) the target loop is "recovered" in some sense.

In this paper, we will design an observer-based state feedback controller for a linear continuous-time system using sampled measurements such that the input-output mapping of the closed-loop systems is the same as given by some ideal state feedback and the target loop given by the state feedback is best approximated in some H_∞ sense. The main contributions of this paper are to set up the concept of generalized LTR in sampled-data systems and to obtain necessary and sufficient conditions for it. It will be shown that the generalized LTR problem is equivalent to an H_∞ filtering problem for sampled-data systems.

2. Problem Formulation

Let the plant model be represented by a state-space realization:

$$\dot{x}(t) = Ax(t) + Bw(t) + Bu(t), \ x(0) = x_0 \quad (2.1)$$
$$y(ih) = Cx(ih) + Dv(ih) \quad (2.2)$$

[1]This work has been supported by the Australian Research Council.

where $x(t) \in \mathbb{R}^n$ is the state, x_0 is an unknown initial condition, $u(t) \in \mathbb{R}^\ell$ is the control input, $w(t) \in \mathbb{R}^p$ is the input disturbance, $y(ih) \in \mathbb{R}^m$ is the sampled output measurement, $0 < h \in \mathbb{R}$ is the sampling period, i is a positive integer, and A, B, C and D are known real time-varying bounded matrices of appropriate dimensions with A and B being piecewise continuous.

Assumption 2.1 $R_D = DD^T > 0$.

Suppose a desired state feedback control law be

$$u(t) = r(t) + z(t) = r(t) + Kx(t) \quad (2.3)$$

where $r(t) \in \mathbb{R}^\ell$ is the reference input and $K \in \mathbb{R}^{\ell \times n}$ stands for the feedback gain.
Substituting (2.3) into the system (2.1)-(2.2), we have

$$\dot{x}(t) = (A + BK)x(t) + Br(t) + Bw(t) \quad (2.4)$$
$$y(ih) = Cx(ih) + Dv(ih) \quad (2.5)$$

When the state is not measureable, the feedback law (2.3) needs to be replaced by an observer-based compensator, (Σ_c), of the following form:

$$(\Sigma_c) : u(t) = r(t) + z(t) = r(t) + K\hat{x}(t) \quad (2.6)$$
$$\dot{\hat{x}}(t) = A_0\hat{x}(t) + B_0u(t), \ t \neq ih, \ \hat{x}(0) = \hat{x}_0 \quad (2.7)$$
$$\hat{x}(ih) = \hat{x}(ih^-) + L[y(ih) - C_0\hat{x}(ih^-)] \quad (2.8)$$

where $\hat{x}(t)$ is the estimate of $x(t)$, $\hat{x}_0$ is the best estimate of x_0, A_0, B_0, C_0, the observer gain matrix L are to be chosen, and the left limit of $\hat{x}(ih^-)$ is defined as : $\hat{x}(ih) := \lim_{\varepsilon \to 0} \hat{x}(ih - \varepsilon)$ for any $\varepsilon > 0$.

The standard LTR problem, either in the continuous-time or discrete-time case, is to find an observer-based controller such that the following two conditions are satisfied:

1) The closed-loop transfer function from r to y is the same as in the state feedback;

2) The loop transfer function from w to z without closing the loop, best approximates the transfer function from w to z.

Now we can formulate the loop transfer recovery problem for the system (2.1)-(2.2) as follows:

Design a controller (Σ_c) such that

(i) **(Separation principle)**. When we use the estimate $\hat{x}(t)$, the input-output mapping, i.e. the mapping from $r(t)$ to $y(ih)$ when $w(t) \equiv 0$, $v(ih) \equiv 0$ and $\hat{x}_0 = x_0 = 0$ is the same as in the state feedback case; and

(ii) **(Loop transfer recovery performance)**. The feedback error $z(t) - \hat{z}(t)$ is as small as possible in some sense.

It can be easily shown that the separation principle is satisfied by choosing $A_0 = A$, $B_0 = B$ and $C_0 = C$.

For convenience, we denote the ideal feedback at the sampling instants ih and its estimate by

$$z_d(ih) = Kx(ih) \quad (2.9)$$

$$\hat{z}_d(ih) = K\hat{x}(ih) \quad (2.10)$$

respectively. In order to measure the loop transfer recovery performance, we define the following index:

$$J(R,T) = \sup_{w,v,x_0} \left[\frac{\lambda_1 \|z - \hat{z}\|^2_{[0,T]} + \lambda_2 \|z_d - \hat{z}_d\|^2_{(0,T)}}{\|w\|^2_{[0,T]} + \|v\|^2_{(0,T)} + x_0^T R x_0} \right]^{\frac{1}{2}}$$

where T defines the time-horizon, $(w, v, x_0) \in L_2[0,T] \oplus \ell_2(0,T) \oplus \mathbb{R}^n$ is such that $\|w\|^2_{[0,T]} + \|v\|^2_{(0,T)} + x_0^T R x_0 \neq 0$, $R = R^T > 0$ is a weighting matrix for x_0, and $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are weighting parameters for the continuous-time error $z - \hat{z}$ and discrete-time error $z_d - \hat{z}_d$, respectively, satisfying $\lambda_1 + \lambda_2 = 1$. In the above, $\|\cdot\|_{[0,T]}$ and $\|\cdot\|_{(0,T)}$ will refer to the $L_2[0,T]$ norm and $\ell_2(0,T)$ norms, respectively.

The generalized LTR problem is as follows:

Given a scalar $\gamma > 0$ and a matrix $R = R^T > 0$, find observer gain L for the controller (2.6)-(2.8) with $A_0 = A$, $B_0 = B$ and $C_0 = C$ such that $J(R,T) < \gamma$.

3. Solution

In the following, we will show that the generalized LTR problem is equivalent to a filtering problem in sampled-data systems. Note that this equivalence has been demonstrated in both the continuous-time and discrete-time cases [6, 7].

Theorem 3.1 *Consider the system (2.1)-(2.2) satisfying Assumption 2.1, and let $\gamma > 0$ be a given scalar. Then, there exists an observer gain L for the controller (2.6)-(2.8) such that $J(R,T) < \gamma$ if and only if there exists a bounded symmetric matrix function $Q(t) > 0$, $\forall t \in [0,T]$, which satisfies the Riccati differential equation with jumps*

$$\begin{aligned} \dot{Q}(t) &= AQ(t) + Q(t)A^T + \gamma^{-2}Q(t)K^T KQ(t) \\ &\quad + BB^T, t \neq ih, \; Q(0) = R^{-1} \quad (3.11) \\ Q(ih) &= [Q^{-1}(ih^-) - \gamma^{-2}K^T K + C^T R_D^{-1} C]^{-1}, \\ &\quad ih \in (0,T) \quad (3.12) \end{aligned}$$

Under the above condition, a suitable observer gain matrix is given by

$$L(ih) = Q(ih)C^T(ih)R_D^{-1}(ih), \; ih \in (0,T) \quad (3.13)$$

A solution to the generalized loop transfer recovery problem for the system (2.1)-(2.2) over an infinite horizon $[0,\infty)$ is provided in the next theorem.

Theorem 3.2 *Consider the system (2.1)-(2.2) satisfying Assumption 2.1, and let $\gamma > 0$ be a given scalar. Then, there exists an observer gain L for the controller (2.6)-(2.8) such that $J(R,\infty) < \gamma$ if and only if there exists a stabilizing solution $Q(t) = Q^T(t) > 0$, $\forall t \in [0,\infty)$, to the Riccati differential equation with jumps*

$$\begin{aligned} \dot{Q}(t) &= AQ(t) + Q(t)A^T + \gamma^{-2}Q(t)K^T KQ(t) \\ &\quad + BB^T, t \neq ih, \; Q(0) = R^{-1} \quad (3.14) \\ Q(ih) &= [Q^{-1}(ih^-) - \gamma^{-2}K^T K + C^T R_D^{-1} C]^{-1}, \\ &\quad ih \in (0,\infty) \quad (3.15) \end{aligned}$$

When such a solution exists, a suitable observer gain matrix is given by

$$L(ih) = Q(ih)C^T(ih)R_D^{-1}(ih), \; ih \in (0,\infty). \quad (3.16)$$

Remark 3.1 *In view of the results in [8], Theorems 3.1 and 3.2 imply that the generalized LTR problem for the system (2.1)-(2.2) is equivalent to an H_∞ sampled-data filtering problem.*

References

[1] M. Athans. A tutorial on the LQG/LTR method. In *Proc. 1986 American Control Conf*, 1289–1296, Seattle, WA, 1986.

[2] J. C. Doyle. Guaranteed margins for LQG regulators. *IEEE Trans. Automat. Control*, 23(4):756–757, 1978.

[3] A. Saberi and P. Sannuti. Observer design for loop transfer recovery and for uncertain dynamical systems. *IEEE Trans. Automat. Control*, 35(8):878–897, 1990.

[4] G. Stein and M. Athans. The LQG/LTR procedure for multivariable feedback control design. *IEEE Trans. Automat. Control*, 32(2):105–114, 1987.

[5] J. C. Doyle and G. Stein. Robustness with observers. *IEEE Trans. Automat. Control*, 24(4):607–611, 1979.

[6] M. Fu and C. E. de Souza. H_∞ filtering and loop transfer recovery. *Proc. 9th Int. Symp. on Math. Theory of Networks and Systems*, Vol. 2, 311–316, Kobe, Japan, 1991.

[7] Z. Zhang and J. S. Freudenberg. Loop transfer recovery for nonminimum phase plants. *IEEE Trans. Automat. Control*, 35(5):547–553, 1990.

[8] P. Shi, C. E. de Souza, and L. Xie. Robust H_∞ filtering for uncertain systems with sampled-data measurements. *Proc. 32nd IEEE Conf. Decision & Control*, 793–798, San Antonio, Texas, USA, 1993.