Loop transfer recovery for systems under sampled measurements

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Abstract: The paper addresses the problem of loop transfer recovery (LTR) of continuous-time systems with sampled output measurements, that is, given an ideal (desired) continuous-time linear state feedback controller, the authors seek for a dynamic output feedback controller based on sampled measurements, such that the state feedback control is best approximated in a certain sense for robustness reasons. They first point out a simple fact that the so-called exact or asymptotic LTR is not possible for such sampled-data systems when the intersampling response is taken into account, regardless of the relative degree and mininium-phase properties and the sampling rate of the system. Based on this observation, the authors proceed to formulate a generalised loop transfer recovery problem which searches for an optimal dynamic output feedback controller which minimises the difference between the target loop transfer function and the output feedback based one in some H_∞ sense. The main result then is to show that this generalised LTR problem is equivalent to a known filtering problem for sampled-data systems, which is solved in terms of a pair of differential and difference Riccati equations.

1 Introduction

A considerable amount of attention has been paid to the theory and application of loop transfer recovery (LTR) in the past decade; see, for example [1–10]. The standard loop transfer recovery problem is as follows: Given a plant $G(s)$ as in Fig. 1 and a target loop $L_d(s)$, designed using state feedback control, find a dynamic output feedback controller (see Fig. 1), $v(s) = M(s)y(s) + N(s)u(s)$, where $N(s) + I$ is invertible, such that the following two properties are satisfied:

(i) the closed-loop input-output response is the same as in the state feedback case, and

(ii) the target loop is 'recovered' in some sense.

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The state feedback control is normally designed by using a linear optimal quadratic regulation procedure. A Luenberger observer-based output feedback is often used to ensure the first property above. Two loop transfer recovery problems have been widely studied: exact LTR and asymptotic LTR. The exact LTR problem is to find a suitable controller, u, such that the actual loop transfer function $L(s)$ is exactly equal to a target loop $L_d(s)$, i.e. $L(s) = L_d(s)$. The asymptotic LTR problem is to find a parameterised controller, u_σ, such that the actual loop transfer function $L_\sigma(j\omega)$ pointwisely converges to the target loop $L_d(j\omega)$ as $\sigma \to 0$ for almost all $-\infty < \omega < \infty$.

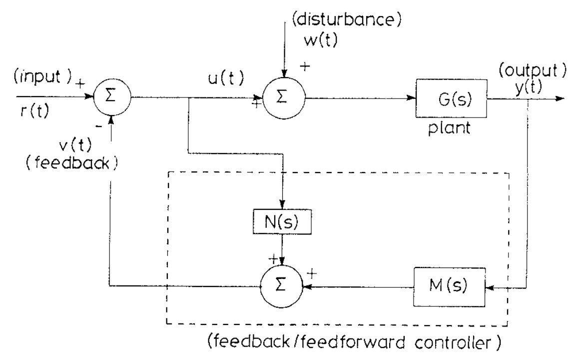

Fig. 1 *Control system for loop transfer recovery*

The research on LTR was originated by the work of [11] for continuous-time systems which demonstrated that any target loop designed via LQR can be arbitrarily closely approximated by using an observed-based output feedback controller, provided that the plant is minimum-phase. That is, the asymptotic LTR problem is always solvable for minimum-phase continuous-time plants. The design procedure for asymptotic LTR has been enhanced by a number of authors; see [3, 9, 10]. The problem of LTR for nonminimum-phase systems was studied in [3, 6, 12], and necessary and sufficient conditions for asymptotic LTR are established. It is realised that these conditions are very severe. Therefore, alternative LTR problems have been studied. In [6], an H_∞/H_2 sensitivity recovery problem is proposed and shown to be solved via the standard H_∞ optimisation techniques. In [3], an optimal LTR problem is proposed aiming at minimising a function, in an H_∞ sense, which represents the difference between the target loop and the actual loop. In particular, the LTR problem in [6] is a special case of that in [3]. Once again, this optimal LTR problem is shown to be solvable via standard

H_∞ optimisation techniques. The results mentioned above have also been generalised to discrete-time systems; see, for example, [7]. However, to the authors' knowledge there are no LTR results available for sampled-data systems.

In this paper we design an observer-based state feedback controller for a linear continuous-time system using sampled measurements such that the input-output mapping of the closed-loop systems is the same as given by some ideal state feedback and the target loop given by the state feedback is best approximated in some H_∞ sense. We first point out that the exact LTR or asymptotic LTR is not possible in sampled-data systems. One reason is that the discrete-time LTR requires the system to be minimum-phase and of small relative degree, usually equal to 1, which are not possible in general due to sampling. Another reason is that the discrete-time LTR results cannot deal with the intersampling behaviour of the system. The main contributions of this paper are to set up the concept of generalised LTR in sampled-data systems and to obtain necessary and sufficient conditions for it, i.e. to find a controller, if possible, such that the system under this controller satisfies a required H_∞-like performance. This performance contains both continuous and discrete time signals. It will be shown that, similar to the continuous-time and discrete-time cases [3, 4, 6, 7, 12], the generalised LTR problem is equivalent to an H_∞ filtering problem for sampled-data systems, which can then be solved via the technique developed in [13–15].

Notation: Throughout this paper the superscript 'T' denotes matrix transposition and the notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). $L_2[0,T]$ stands for the space of square integrable vector functions over the interval $[0,T]$, while $l_2(0,T)$ is the space of square summable vector sequences over $(0,T)$. $\|\cdot\|$ refers to the Euclidean vector norm, whereas $\|\cdot\|_{[0,T]}$ denotes the $L_2[0,T]$-norm over $[0,T]$ and $\|\cdot\|_{(0,T)}$ is the $l_2(0,T)$-norm over $(0,T)$. $F(\theta^-)$ stands for the left limit of a function $F(\theta)$.

2 Problem Formulation

Let the plant model be represented by a state-space realisation,

$$(\Sigma_I): \quad \dot{x}(t) = Ax(t) + Bw(t) + Bu(t), \; x(0) = x_0 \quad (1)$$

$$y(ih) = Cx(ih) + Dv(ih) \quad (2)$$

where $x(t) \in \mathbf{R}^n$ is the state, x_0 an unknown initial condition, $u(t) \in \mathbf{R}^l$ is the control input, $w(t) \in \mathbf{R}^p$ is the input disturbance, $y(ih) \in \mathbf{R}^m$ is the sampled output measurement, $h > 0$ is the sampling period, i is a positive integer, and A, B, C and D are known real time-varying bounded matrices of appropriate dimensions, with A and B being piecewise continuous.

We shall adopt the following assumption for system (Σ_I).

Assumption 1: $R_D = DD^T > 0$.

Remark 1: The above assumption means that the sampled-data control problem considered here is 'nonsingular'.

Suppose a desired state feedback control law be

$$u(t) = r(t) + Kx(t) \quad (3)$$

where $r(t) \in \mathbf{R}^l$ is the reference input and $K \in \mathbf{R}^{l \times n}$ denotes the feedback gain. Hence, the closed-loop system of eqns. 1 and 2 with the controller eqn. 3 is of the form:

$$\dot{x}_s(t) = (A + BK)x_s(t) + Br(t) + Bw(t) \quad (4)$$

$$y_s(ih) = Cx_s(ih) + Dv(ih) \quad (5)$$

When the state is not measureable, the control law, eqn. 3, needs to be replaced by an observer-based compensator (Σ_c) in the following form:

$$(\Sigma_c): \quad u(t) = r(t) + K\hat{x}(t) \quad (6)$$

$$\dot{\hat{x}}(t) = A_0\hat{x}(t) + B_0u(t), t \neq ih, \hat{x}(0) = \hat{x}_0 \quad (7)$$

$$\hat{x}(ih) = \hat{x}(ih^-) + L[y(ih) - C_0\hat{x}(ih^-)] \quad (8)$$

where $\hat{x}(t)$ is the estimate of $x(t)$, $\hat{x}_0$ is the best estimate of x_0, and A_0, B_0, C_0 and the observer gain matrix L are to be chosen.

We know that when the state $x(t)$ is measurable, the ideal feedback is

$$z(t) = Kx(t) \quad (9)$$

The corresponding estimated feedback is

$$\hat{z}(t) = K\hat{x}(t) \quad (10)$$

The standard LTR problem, either the continuous-time or discrete-time case, is to find an observer-based controller such that the following two conditions are satisfied:

(1) The closed-loop transfer function from r to y is the same as in the state feedback.

(2) The loop transfer function from w to $\hat{z}$ without closing the loop best approximates the transfer function from w to z.

Now we can formulate the loop transfer recovery problem for the system (Σ_I) as follows.

Design a controller (Σ_c) such that:

(i) **(Separation principle):** When we use the estimate of $\hat{x}(t)$, the input-output mapping, i.e. the mapping from $r(t)$ to $y(ih)$ when $w(t) \equiv 0$, $v(ih) \equiv 0$ and $\hat{x}_0 = x_0 = 0$, is the same as in the state feedback case, and

(ii) **(Loop transfer recovery performance):** The feedback error $z(t) - \hat{z}(t)$ is as small as possible in some sense. For convenience, we define the state estimation error to be

$$e(t) = x(t) - \hat{x}(t) \quad (11)$$

In order to satisfy the separation principle we choose $A_0 = A$, $B_0 = B$ and $C_0 = C$. Then, the closed-loop system of eqns. 1 and 2 with eqns. 6–8 is given by:

$$(\Sigma_{cl}): \begin{pmatrix} \dot{x}(t) \\ \dot{e}(t) \end{pmatrix} = \begin{pmatrix} A + BK & -BK \\ 0 & A \end{pmatrix} \begin{pmatrix} x(t) \\ e(t) \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} r(t) + \begin{pmatrix} B \\ B \end{pmatrix} w(t), t \neq ih \quad (12)$$

$$\begin{pmatrix} x(ih) \\ e(ih) \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I - LC \end{pmatrix} \begin{pmatrix} x(ih^-) \\ e(ih^-) \end{pmatrix} + \begin{pmatrix} 0 \\ -LD \end{pmatrix} v(ih) \quad (13)$$

$$y(ih) = (C \quad 0) \begin{pmatrix} x(ih) \\ e(ih) \end{pmatrix} + Dv(ih) \quad (14)$$

From eqns. 12 and 13, we can see that, if $w(t) \equiv 0$, $v(ih) \equiv 0$ and $x_0 - \hat{x}_0 = 0$, then

$$e(t) \equiv 0$$

This implies that eqns. 12–14 will reduce to:

$$\dot{x}(t) = (A + BK)x(t) + Br(t) \quad (15)$$

$$y(ih) = Cx(ih) \quad (16)$$

Comparing the system of eqns. 15 and 16 with the ideal closed-loop system of eqns. 4 and 5, we conclude that the controller of (Σ_c) of the form eqns. 6–8, with $A_0 = A$, $B_0 = B$ and $C_0 = C$, guarantees that the input-output mapping of the system (Σ_{cl}) is exactly the same as in the state feedback case.

For convenience, we introduce the ideal feedback in eqn. 9 at the sampling instants *ih*:

$$z_d(ih) = Kx(ih) \quad (17)$$

Also, the corresponding estimated feedback in eqn. 10 is denoted by

$$\hat{z}_d(ih) = K\hat{x}(ih) \quad (18)$$

To measure the loop transfer recovery performance, we define the following index:

$$J_r(\lambda, R, T) = \sup_{w,v,x_0} \left\{ \left[\frac{\|z-\hat{z}\|^2_{[0,T]} + \lambda\|z_d - \hat{z}_d\|^2_{(0,T)}}{\|w\|^2_{[0,T]} + \|v\|^2_{(0,T)} + x_0^T R x_0} \right]^{\frac{1}{2}} \right.$$

$$(w, v, x_0) \in L_2[0,T] \oplus l_2(0,T) \oplus \Re^n : \|w\|^2_{[0,T]} + \|v\|^2_{(0,T)}$$

$$\left. + x_0^T R x_0 \neq 0 \right\} \quad (19)$$

where T defines the time-horizon for the LTR performance, $R = R^T > 0$ is a weighting matrix for x_0, and $\lambda \geq 0$ is a weighting parameter for the discrete-time error $z_d - \hat{z}_d$. The input disturbance $w(t)$, measurement noise $v(ih)$ and the initial condition x_0 are all considered in the performance index due to the obvious reason that the feedback errors $z - \hat{z}$ and $z_d - \hat{z}_d$ are influenced by all of them. Both the continuous-time feedback error $z(t) - \hat{z}(t)$ and the discrete-time feedback error $z_d(ih) - \hat{z}_d(ih)$ are considered, depending on the designer's emphasis on the intersampling behaviour or sampling point performance.

Given a matrix R, the LTR problem is to design the observer gain matrix L such that $J_r(\lambda,R,T)$ is sufficiently small.

Definition 1: Given the sampled-data system of eqns. 1 and 2, with Assumption 1 and the performance index $J_r(\lambda,R,T)$ in eqn. 19, we say that asymptotic LTR is achievable if, for any $\varepsilon > 0$, there exists an observer matrix L for eqns. 6–8 such that $J_r(\lambda,R,T) < \varepsilon$.

3 Impossibility of asymptotic loop transfer recovery

As pointed out in the Introduction, the asymptotic LTR is always possible for minimum-phase continuous-time systems [11]. LTR results are also available for relative degree one discrete-time systems [16–18]. However, LTR is much more difficult for sampled-data systems. There are several reasons for this. First, it is well known that a minimum-phase continuous-time plant usually becomes nonminimum-phase after sampling [19]. Secondly, relative degree 1 can rarely be guaranteed after sampling [19]. These two points imply that the asymptotic LTR is often impossible even at the sampling points. Finally, we show below that asymptotic LTR is always impossible for sampled-data systems if the intersampling behaviour is considered. More precisely, we show that the performance index $J_r(R,T)$ in eqn. 19 cannot be made arbitrarily small by choosing the observer gain matrix L, no matter whether the continuous-time plant or the discrete-time plant is minimum-phase or not. This problem is due to the fact that L does not help the observer attenuate noises in the intersampling periods.

To see the impossibility of achieving asymptotic LTR for sampled-data systems, we let $v(ih) \equiv 0$, $x_0 = 0$ and $\lambda = 0$, i.e. the performance index eqn. 19 reduces to

$$J_r(T) = \sup_w \left\{ \frac{\|z-\hat{z}\|_{[0,T]}}{\|w\|_{[0,T]}}; w \in L_2[0,T], \|w\|_{[0,T]} \neq 0 \right\}$$

Choosing $w(\cdot)$ as follows:

$$\|w(t)\| = \begin{cases} \frac{1}{\sqrt{h}}, & t \in [0,h) \\ 0 & t \geq h \end{cases}$$

obviously, for any $T \geq h$,

$$\|w\|^2_{[0,T]} = \int_0^T w^T(t)w(t)dt = 1$$

Now,

$$\|z(t) - \hat{z}(t)\|_{[0,T]} = \|Ke(t)\|_{[0,h]}$$

$$= \left\| K exp^{\int_0^t Adt} \int_0^t B \frac{1}{\sqrt{h}} exp^{-\int_0^t Adt} dt \right\|_{[0,h]}$$

$$\triangleq \gamma_0 > 0$$

where γ_0 is independent of the observer gain L. Hence, for any L, we have $J_r(T) \geq \gamma_0 > 0$, i.e. $J_r(T)$ cannot be made arbitrarily small. This shows that the asymptotic LTR is impossible for sampled-data systems.

4 Generalised loop transfer recovery

Due to the impossibility of achieving asymptotic LTR for sampled-data systems, we formulate a generalised loop transfer recovery. To fulfil the 'separation principle' requirements, we shall consider controllers (Σ_c) as in eqns. 6–8 with $A_0 = A$, $B_0 = B$, $C_0 = C$ and $\hat{x}_0 = 0$, i.e. controllers of the form:

$$u(t) = r(t) + K\hat{x}(t) \quad (20)$$

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t), t \neq ih, \hat{x}(0) = 0 \quad (21)$$

$$\hat{x}(ih) = \hat{x}(ih^-) + L[y(ih) - C\hat{x}(ih^-)] \quad (22)$$

The generalised LTR problem we address is as follows.

Given a scalar $\gamma > 0$ and a matrix $R = R^T > 0$, find an observer gain L for the controller eqns. 20–22 such that $J_r(R,T) < \gamma$. We do not weight w and v because such a weighting can be achieved by rescaling w and v in the system eqns. 1 and 2.

Compared with known existing LTR techniques for both continuous-time and discrete-time systems, our new approach has the following features:

- It allows for both minimum-phase and nonminimum-phase systems in the same framework.
- It also allows for different relative degrees of the discretised system in the same framework.
- It considers not only sampling points for the performance, but also the intersampling behaviour, which cannot be dealt with by discrete-time LTR techniques.
- It applies to both finite and infinite horizon cases.
- It emphasises the effect of non-zero initial condition on the feedback error $z - \hat{z}$, while zero initial conditions are essential for known LTR methods.
- It contains both input and measurement noises in the performance, while the normal LTR performance contains either input noise only or measurement noise only.

Before presenting our main results, we introduce a mixed L_2/l_2 filtering problem for sampled-data systems and two lemmas; see [15] for more details.

Consider the following sampled-data system:

$$(\Sigma_f): \quad \dot{x}(t) = Ax(t)+Bw(t), t \in [0,T], x(0) = x_0 \quad (23)$$

$$y(ih) = Cx(ih) + Dv(ih), ih \in (0,T) \quad (24)$$

$$z(t) = Hx(t), t \in [0,T] \quad (25)$$

$$z_d(ih) = H_d x(ih), ih \in (0,T) \quad (26)$$

where $x(t) \in \mathbf{R}^n$ is the state, x_0 an unknown initial state, $w(t) \in \mathbf{R}^m$ is the process noise, $y(ih) \in \mathbf{R}^m$ is the sampled output measurement, $v(ih) \in \mathbf{R}^q$ is the measurement noise, $z(t)$ and $z_d(ih) \in \mathbf{R}^m$ are linear combinatious of the state variables to be estimated, $h > 0$ is the sampling period $i \geq 0$ is an integer, and A, B, C, D, H and H_d are known, real, time-varying bounded matrices of appropriate dimensions with A, B and H being piecewise continuous. It is also assumed that the matrix D satisfies Assumption 1.

The mixed L_2/l_2 sampled-data filtering problem is to design a causal linear filter $\mathbf{F}$ to estimate $z(t)$ and $z_d(ih)$ based on the sampled measurements $y(ih)$. The filter $\mathbf{F}$ is of the following form:

$$\dot{\hat{x}}(t) = A_e x_e(t), t \in [ih, ih+h), x_e(0) = 0 \quad (27)$$

$$\hat{x}(ih) = A_{de} x_e(ih^-) + B_{de} y(ih) \quad (28)$$

$$\hat{z}(t) = H_e x_e(t), t \in [ih, ih+h) \quad (29)$$

$$\hat{z}_d(ih) = H_{de} x_e(ih) \quad (30)$$

where $\hat{z}(t)$ and $\hat{z}_d(ih)$ are the estimates of $z(t)$ and $z_d(ih)$, respectively. The dimension of the filter and time-varying matrices A_e, A_{de}, B_{de}, H_e and H_{de} are to be chosen. Note that eqns. 27–30 can be regarded as a linear discrete time-varying filter with an interpolation function.

The filtering performance measure is given by

$$J_r(\Sigma_f, \mathbf{F}, R, T) = \sup\left\{\left[\frac{\|z-\hat{z}\|^2_{[0,T]} + \|z_d - \hat{z}_d\|^2_{(0,T)}}{\|w\|^2_{[0,T]} + \|v\|^2_{(0,T)} + x_0^T R x_0}\right]^{\frac{1}{2}} \quad \|w\|^2_{[0,T]} + \|v\|^2_{(0,T)} + x_0^T R x_0 \neq 0\right\} \quad (31)$$

where $R = R^T > 0$ is a given initial state weighting matrix, and $\hat{z}(t)$ and $\hat{z}_d(ih)$ are estimates of $z(t)$ and $z_d(ih)$ over a horizon $[0,T]$, respectively.

Remark 2: The H_∞ filtering problem for a sampled-data system was originally addressed in [13]. The performance index, eqn. 31, which has been used in [15], is indeed generalised from [13], where the discrete term $\|z_d - \hat{z}_d\|_{(0,T)}{}^2$ is not present.

Lemma 1: [15] Consider the system of eqns. 23–26 and let $\gamma > 0$ be a given scalar. Then there exists a filter $\mathbf{F}$ of the form of eqns. 27–30 such that $J_f(\Sigma_f, \mathbf{F}, R, T) < \gamma$ if and only if there exists a bounded symmetric solution $Q(t) > 0$, $\forall t \in [0,T]$, to the following Riccati differential equation with jumps:

$$\dot{Q}(t) = AQ(t) + Q(t)A^T + \gamma^{-2}Q(t)H^T HQ(t) + BB^T, \quad t \neq ih \quad (32)$$

$$Q(ih) = [Q^{-1}(ih^-) - \gamma^{-2}H_d^T H_d + C^T R_D^{-1} C]^{-1}, \quad ih \in (0,T) \quad (33)$$

$$Q(0) = R^{-1} \quad (34)$$

In this situation, a suitable filter is given by

$$(\Sigma): \dot{\hat{x}}(t) = A\hat{x}(t), t \neq ih, \hat{x}(0) = 0 \quad (35)$$

$$\hat{x}(ih) = \hat{x}(ih^-) + Q(ih)C^T R_D^{-1}[y(ih) - C\hat{x}(ih^-)], \quad ih \in (0,T) \quad (36)$$

$$\hat{z}(t) = H\hat{x}(t), t \neq ih \quad (37)$$

$$\hat{z}_d(ih) = H_d \hat{x}(ih), ih \in (0,T) \quad (38)$$

For the infinite horizon case, the performance index, eqn. 31, becomes

$$J_f(\Sigma_f, \mathbf{F}, R, \infty) = \sup\left\{\left[\frac{\|z-\hat{z}\|^2_{[0,\infty]} + \|z_d - \hat{z}_d\|^2_{(0,\infty)}}{\|w\|^2_{[0,\infty]} + \|v\|^2_{(0,\infty)} + x_0^T R x_0}\right]^{\frac{1}{2}} \quad \|w\|^2_{[0,\infty]} + \|v\|^2_{(0,\infty)} + x_0^T R x_0 \neq 0\right\} \quad (39)$$

and we require the asymptotic stability of the filter, eqns. 27–30. We have the following result.

Lemma 2: [15] Consider the system of eqns. 23–26 and let $\gamma > 0$ be a given scalar. Then there exists a filter $\mathbf{F}$ of the form of eqns. 27–30 such that $J_f(\Sigma_f, \mathbf{F}, R, \infty) < \gamma$ if and only if there exists a bounded symmetric solution $Q(t) > 0$, $\forall t \in [0,\infty)$ to the following Riccati differential equation with jumps:

$$\dot{Q}(t) = AQ(t) + Q(t)A^T + \gamma^{-2}Q(t)H^T HQ(t) + BB^T, \quad t \neq ih \quad (40)$$

$$Q(ih) = [Q^{-1}(ih^-) - \gamma^{-2}H_d^T H_d + C^T R_D^{-1} C]^{-1}, \quad ih \in (0,\infty) \quad (41)$$

$$Q(0) = R^{-1} \quad (42)$$

and the system

$$\dot{q}(t) = (A + \gamma^{-2}QH^T H)q(t), t \neq ih \quad (43)$$

$$q(ih) = [I + Q(ih)(\gamma^{-2}H_d^T H_d - C^T R_D^{-1} C)]q(ih^-), ih \in (0,\infty) \quad (44)$$

is exponentially stable. Moreover, if the above solution $Q(t)$ exists, the filter given by eqns. 35–38 with T approaching infinity is stable and achieves $J_f(\Sigma_f, \mathbf{F}, R, \infty) < \gamma$.

In the following, we call the solution $Q(t)$ to eqns. 40–42 which renders the system of eqns. 43 and 44 exponentially stable a stabilising solution. In the following, we show that the generalised LTR problem is equivalent to a H_∞ filtering problem for systems with sampled measurements. Note that this equivalence has been demonstrated in both the continuous-time and discrete-time cases [3, 4, 6, 7, 12]. First, we give a solution to the generalised loop transfer recovery problem of eqns. 1 and 2 over a finite horizon $[0,T]$.

Theorem 1: Consider the system of eqns. 1 and 2 satisfying Assumption 1, and let $\gamma > 0$ be a given scalar. Then there exists an observer gain L for the controller eqns. 20–22 such that $J_r(\lambda, R, T) < \gamma$ if and only if there exists a bounded symmetric matrix function $Q(t) > 0$, $\forall t \in [0,T]$, which satisfies the Riccati differential equation with jumps:

$$\dot{Q}(t) = AQ(t) + Q(t)A^T + \gamma^{-2}Q(t)K^T KQ(t) + BB^T, t \neq ih \quad (45)$$

$$Q(ih) = [Q^{-1}(ih^-) - \gamma^{-2}\lambda K^T K + C^T R_D^{-1} C]^{-1}, ih \in (0,T) \quad (46)$$

$$Q(0) = R^{-1} \quad (47)$$

Under the above condition, a suitable observer gain matrix is given by

$$L(ih) = Q(ih)C^T(ih)R_D^{-1}(ih), ih \in (0, T) \quad (48)$$

Proof: Considering the system of eqns. 1 and 2 and the controller eqns. 20–22, it is easy to see that a state space realisation for $z - \hat{z}$ and $z_d - \hat{z}_d$ is given by

$$\dot{e}(t) = Ae(t) + Bw(t), t \neq ih, e(0) = x_0 \quad (49)$$

$$e(ih) = e(ih^-) - L[Ce(ih^-) + Dv(ih)] \quad (50)$$

$$z(t) - \hat{z}(t) = Ke(t) \quad (51)$$

$$z_d(ih) - \hat{z}_d(ih) = Ke(ih) \quad (52)$$

Hence, it follows immediately from eqns, 49–52 that finding L such that $J_r(\lambda, R, T) < \gamma$ is equivalent to solving the H_∞ filtering problem for the following system:

$$(\Sigma_a): \quad \dot{x}(t) = Ax(t) + Bw(t), \forall t \in [0, T], x(0) = x_0$$
$$z(t) = Kx(t), \forall t \in [0, T]$$
$$z_d(ih) = \sqrt{\lambda}Kx(ih, \forall ih \in (0, T)$$
$$y(ih) = Cx(ih) + Dv(ih), \forall ih \in (0, T)$$

to achieve the performance $J_f(\Sigma_a, \mathbf{F}, R, T) < \gamma$. Finally, in view of Lemma 1, the desired result follows.

A solution to the generalised loop transfer recovery problem for the system of eqns. 1 and 2 over an infinite horizon $[0, \infty)$ is provided in the following theorem.

Theorem 2: Consider the system of eqns. 1 and 2 satisfying Assumption 1, and let $\gamma > 0$ be a given scalar. Then there exists an observer gain L for the controller eqns. 20–22 such that $J_r(R, \infty) < \gamma$ if and only if there exists a stabilising solution $Q(t) = Q^T(t) > 0$, $\forall t \in [0, \infty)$, to the Riccati differential equation with jumps:

$$\dot{Q}(t) = AQ(t) + Q(t)A^T + \gamma^{-2}Q(t)K^TKQ(t) + BB^T, t \neq ih \quad (53)$$

$$Q(ih) = [Q^{-1}(ih^-) - \gamma^{-2}\lambda K^TK + C^TR_D^{-1}C]^{-1}, ih \in (0, \infty) \quad (54)$$

$$Q(0) = R^{-1} \quad (55)$$

When such a solution exists a suitable observer gain matrix is given by

$$L(ih) = Q(ih)C^T(ih)R_D^{-1}(ih), ih \in (0, \infty) \quad (56)$$

Proof: It can be carried out using the same argument as in the proof of Theorem 1 except that now $T \to \infty$ and Lemma 2 is used in lieu of Lemma 1

Remark 3: In view of Lemmas 1 and 2, Theorems 1 and 2 imply that the generalised loop transfer recovery problem for the system of eqns. 1 and 2 is equivalent to an H_∞ filtering problem for a related sampled-data system.

Remark 4: A number of algorithms are available for solving the matrix Riccati differential equation (RDE) eqns. 45–47 [20, 21]. In particular, [22] proposes a new matrix-valued algorithm based on a matrix generalisation of the backward differentiation formulas, which is much faster to compute the solution per time step than the classical approaches [20, 21].

5 Conclusions

The LTR problem for continuous-time systems with sampled output measurements has been studied. It has been shown that exact LTR and asymptotic LTR for sampled-data systems are not possible in general. Consequently, a generalised LTR problem is formulated with the aim of minimising the difference between the target loop and the actual loop in an H_∞ sense. It is shown that the generalised LTR problem is equivalent to an H_∞ filtering problem for a related sampled-data system.

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