MODEL REFERENCE ROBUST CONTROL[1]

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Abstract

Classical model reference adaptive control schemes require the following assumptions on the plant: A1) minimum phase; A2) known upper bound of the plant order; A3) known relative degree; and A4) known sign of high frequency gain. It is well-known that the robustness of the adaptive systems is a potential problem, and it requires many sophisticated techniques to fix it. In this paper, we consider the same model reference control problem via robust control. By further assuming that the boundedness of the parameter uncertainties of the plant (which is a very weak assumption), we show that a linear **time-invariant dynamic output feedback controller** can be constructed to give the following property: the closed-loop system is internally stable and its transfer function is arbitrarily close to the reference model. This method provides simple controllers and good robustness. It also has potential to cope with large size fast time-varying uncertainties.

1 Introduction

Both adaptive control theory and robust control theory have been developed to accommodate a wide range of uncertainties. Although experts have not agreed on the distinction between these two theories, one of their important differences is that adaptive controllers are nonlinear and time-varying (NLTV) while robust controllers are *usually* linear and time-invariant (LTI). Despite of the overwhelming progress made recently on the robust control theory, we find ourselves constantly "bothered" with the following fundamental question:

Q1. Can an LTI controller really compete with an NLTV controller?

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A positive answer to the above question has been given to the important case of quardratic stabilization of linear systems which are subject to a type of norm-bounded uncertainty. It is shown in this case that NLTV controllers offer no advantage over their LTI partners for the (see, e.g., [1]). However, since the adaptive control is often concerned with performance (such as model matching) and structured uncertainty in the parameter (coefficient) space, the above result does not apply and the question Q1 still deserves serious attention.

In this paper, we consider the problem of model reference control of uncertain linear systems and address the question Q1 from a different perspective:

Q2. Under what conditions can we find an LTI controller such that the closed-loop system is stable and "matches" a given reference model in certain sense for all admissible uncertainties?

Note that this is exactly the question asked by the adaptive control theory except that an NLTV controller is allowed there. It is well known that all the classical model reference adaptive control (MRAC) schemes (developed prior to the 1980's) require the following standard assumptions:

A1. The plant is of minimum phase;

A2. The upper bound of the plant order is known;

A3. The relative degree of the plant is known;

A4. The sign of the high-frequency gain is known;

B1. The reference model has the same relative degree as the plant; and

B2. The reference input is bounded and piecewise continuous.

With these assumptions, the MRAC schemes can guarantee that the closed-loop system becomes stable and converges to the reference model. However, the robustness of the classical adaptive schemes is known to be a serious problem, and it requires sophisticated techniques to fix it (see, e.g., [2]). Moreover, besides the complexity problem of the "robustified" adaptive controllers, the degree of robustness is often small, and additional information on the system (such as the size of the unmodeled dynamics and a bound on the input disturbance) is usually required.

The focal point of this paper is to answer the question Q2 by showing that a result similar to that given by MRAC can be achieved by a model reference robust control (MRRC) technique, provided there are some additional very mild assumptions. More precisely, suppose assumptions A1)-A4) and the following additional ones are satisfied:

A5. The set of admissible uncertain parameters is compact and known;

A6. The transfer function of the reference model is stable and strictly proper.

Then, we provide a technique for constructing a simple LTI controller which guarantees the robust stability of the closed-loop system and that the H_∞ norm of the difference in the transfer functions of the closed-loop system and the reference model can be made to be arbitrarily small. Several advantages of this MRRC method are obvious: an LTI controller allows simple analysis of the system performance and robust stability; it provides easy implementation and simulation; furthermore, robustness margins for input and output disturbances and additional unstructured perturbations can be computed by using the sensitivity and complementary sensitivity of the closed-loop system.

The result explained above is achieved by using feedback which possibly involves high gains. But we find in simulations that feedback gains are usually moderate when the plant uncertainties are not large and the requirement on model matching is not severe. The feedback gains need to be high when the plant is subject to large uncertainties and/or the plant is very different from the reference model.

Having established the MRRC method for time-invariant uncertainty, we look into the problem of time-varying uncertainty which the adaptive control theory has difficulty with. We find that the MRRC approach may also be suitable for accommodating a certain class of time-varying uncertainties. This point will be made via some discussions and a conjecture.

The endeavor of this paper should not be interpreted as an de-emphasis of adaptive control, it should rather be viewed as an attempt to have a better understanding of both the adaptive and robust control theories and as an exercise in our course of searching for better adaptive control schemes. More investigation on this subject is needed.

2 Related Work on Robust Control

There are three robust control design methods which are most pertinent to our MRRC technique.

The quantitative feedback theory (QFT) by Horowitz and his colleagues [3] provides a first systematic procedure for designing robust controllers. This method uses a two-degree-of-freedom controller to achieve desired frequency response and the stability margin of the closed-loop system. More specifically, a loop compensator is used to reduce uncertainty and assuring closed-loop stability while a prefilter is used to shape the closed-loop input-output transfer function. This design method can be regarded as a model reference control method. However, the reference model is not given in terms of a transfer function, and the phase of the closed-loop input-output transfer function is not paid attention to. The QFT method is somewhat empirical because trials and errors are needed for a proper design. There is also a concern about the effectiveness of this design method for multi-input-multi-output systems.

Barmish and Wei [4] employs a unity-feedback controller for robust stabilization of an uncertain linear time-invariant plant satisfying assumptions A1-A4 and some mild conditions. They show that an LTI stable and minimum-phase stabilizer can always be constructed for the uncertain plant. However, the model reference control problem is not treated in [4] because only one degree of freedom is used.

The problem of model reference robust control was recently studied by Sun, Olbrot and Polis [5]. They consider linear single-input-single-output plants satisfying Assumptions similar to A1-A6, and use the so-called "modeling error compensation" technique to show that a stabilizing controller can be constructed such that the closed-loop input-output transfer function is made to be arbitrarily close to the reference model over a finite bandwidth. However, the construction and analysis of the controller seems very complicated. Furthermore, model matching is done only on a finite bandwidth, this restriction, as we shall see, can be lifted.

3 A New Approach to MRRC

In this section, we use a two-degree-of-freedom controller to solve the MRRC problem for plants and reference models satisfying assumptions A1-A6. The

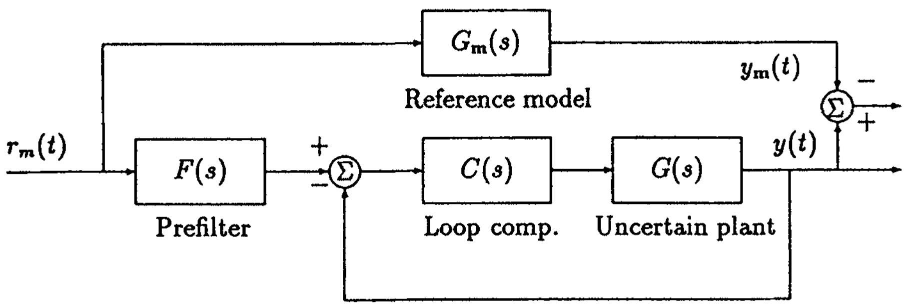

Figure 1: Model Reference Robust Control

schematic diagram of the closed-loop system is shown in Figure 1. The transfer function $G(s)$ of the plant takes the following form:

$$G(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{m} b_i s^i}{s^n + \sum_{i=0}^{n-1} a_i b_i}, \quad b_m \neq 0 \ . \tag{1}$$

The nominal model of the plant is denoted by $G_0(s)$, and it can be chosen arbitrarily although its choice might effect the controller. The closed-loop input-output transfer function will be denoted by $G_c(s)$, i.e.,

$$G_c(s) = \frac{F(s)C(s)G(s)}{1 + C(s)G(s)} \ . \tag{2}$$

The main result is presented as follows:

Theorem 1. *Consider an SISO uncertain linear plant $G(s)$ and a linear reference model $G_m(s)$ satisfying assumptions A1-A6. Then, given any (arbitrarily small) $\varepsilon > 0$, there exist a stable transfer function $F(s)$ and a stable and minimum-phase transfer function $C(s)$ such that the closed-loop system given in Figure 1 is robustly stable and that the difference between the transfer functions of the closed-loop system and the reference model has H_∞ norm less than ε.*

To assist this theorem, we provide a simple algorithm for constructing the controller. The design of $C(s)$ is simplified from the robust stabilization technique in [4], and that of $F(s)$ is motivated by the QFT [3].

Construction of $C(s)$ and $F(s)$:

- Choose $N_c(s)$ to be any (n-m-1)th stable polynomial;

- Choose $1/D_c(s,\sigma)$ to be any parameterized stable unity gain low-pass filter (parameterized in $\sigma > 0$) with relative degree no less than n-m-1, and and the cutoff frequency $\to \infty$ as $\sigma \to 0$. For example, take

$$1/D_c(s) = \frac{1}{(\sigma s + 1)^{n-m-1}}\ ; \tag{3}$$

- Take

$$C(s) = K\frac{N_c(s)}{D_c(s,\sigma)} \tag{4}$$

$$F(s) = L(s,\rho)G_m(s)\left[\frac{C(s)G_0(s)}{1 + C(s)G_0(s)}\right]^{-1} \tag{5}$$

  where $L(s,\rho)$ is either a unity gain or a parameterized unit-gain low-pass filter with cutoff frequency $\to \infty$ when $\rho \to 0$. K, σ and ρ are design parameters to be tuned;

- Choose K sufficiently large and σ and ρ sufficiently small (which are guaranteed to exist) such that

$$||G_c(s) - G_m(s)||_\infty < \varepsilon \tag{6}$$

  for all admissible $G(s)$ satisfying assumptions A1)-A5).

Remark 1. The purpose of the stable polynomial $N_c(s)$ is to assure that $N_c(s)G(s)$ is a minimum phase transfer function with relative degree equal to one so that with $C(s) = KN_c(s)$ and sufficiently large K, the characteristic equation of the closed-loop system is robustly stable and its sensitivity function is sufficiently small. The reason for $1/D_c(s,\sigma)$ to be a low-pass filter of the required form is to guarantee the properness of $C(s)$ while preserving the above property of the closed-loop system when σ is sufficiently small. The function $F(s)$ is used to shape the closed-loop transfer function so that it approximates the reference model. The low-pass filter $L(s,\rho)$ is optional, only for reducing the unnecessary high-frequency gain of the closed-loop system. These points should be considered in tuning the controller. The proof of Theorem 1 is omitted, for the construction of the controller and this remark have explained the validity of the result.

4 Beyond Time-invariant Parameter Uncertainty

The purpose of this section is to look into the possibility of using the MRRC technique to deal with time-varying parameter uncertainty. In certain sense, the discussions in this section might be "the center of gravity" of the paper.

As we pointed out earlier, the adaptive control theory is applicable only to uncertainties which are time-invariant (or varying "sufficiently slowly"). The robust control methods mentioned in Section 2 and the MRRC technique in Section 3 are in general also restricted to time-invariant uncertainties due to their dependency on frequency domain analysis. However, the fact the MRRC technique requires only a simple LTI controller seems to suggest that it might be able to tolerate time-varying uncertainty better the adaptive schemes. This intuition is supported by a number of simulations carried out by the author. Motivated by these simulations, it is suspected by the author that a result similar to Theorem 1 in Section 3 also holds for plants with time-varying uncertainties.

To be more precise, we consider an uncertain plant described by the following input-output differential equation:

$$y^{(n)} + \sum_{i=0}^{n-1} a_i(t) y^{(i)} = \sum_{i=0}^{m} b_i(t) u^{(i)} \tag{7}$$

where $u(t)$ and $y(t)$ are the input and output of the plant, respectively. It is assumed that the assumptions A1-A6 hold. Here, the minimum-phase should be interpreted as that the zero dynamics of the plant:

$$\sum_{i=0}^{m} b_i(t) u^{(i)} = 0 \tag{8}$$

is "stable" in certain sense. Then, the following conjecture is put forward:

Conjecture: *Consider the uncertain plant in (7) and reference model $G_m(s)$ satisfying assumptions A1-A6. Then, under some additional mild assumptions and an appropriate definition for stability, there exists a linear time-invariant controller such that the closed-loop system is stable and the induced norm of the operator from $r_m(t)$ to $y(t) - y_m(t)$ can be made arbitrarily small.*

Possible additional assumptions might be that the coefficients $a_i(t)$ and $b_i(t)$ are differentiable and their derivatives are bounded. And investigation is needed for different types of stability such as Lyapunov, bounded-input-bounded-output, and exponential. Since time-varying systems are considered, frequency domain analysis is no longer valid. Therefore, new techniques need to be developed for proving or disproving the conjecture. A promising avenue is to apply the singular perturbation theory, this will be studied.

5 Conclusion

In this paper, we have demonstrated the potentials of an MRRC technique. This technique provides a non-H_∞ method for designing a low order robust LTI controller to solve the model reference problem, and it is able to handle large size parametric uncertainties. An important feature of this technique is its potential of handling large-size fast time-varying parametric uncertainties, this is a matter which deserves further research.

It is, however, noted that the MRRC technique potentially requires a high-gain feedback controller, when the size of uncertainties is large and/or the plant is vastly different from the reference model. The tradeoffs between the MRAC and MRRC techniques need to be further investigated.

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