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Formation Maneuvering with Collision Avoidance and Connectivity Maintenance

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***Abstract*— This paper proposes a switching control strategy for formation maneuvering control of multi-agent systems with guaranteed collision avoidance and connectivity maintenance properties. The control strategy consists of three parts: formation maneuvering, collision avoidance, and connectivity maintenance control. We adopt the idea of using complex Laplacian for formation maneuvering control, which gives rise to one degree of freedom in scaling the size of the achievable formations and thus enables collision avoidance and connectivity maintenance possible in the meantime. Simulation results are provided to demonstrate the effectiveness of the control strategy.**

I. INTRODUCTION

Formation control of multi-agent systems has attracted significant attention from various fields and some progresses have been made in recent years [1]–[4]. As a multi-agent system evolves towards a desired formation shape, collision may occur and the sensing or communication links may become lost. To facilitate practical applications of formation control, it is important to guarantee that collisions are avoided and connectivity is maintained in formation maneuvering.

Collision avoidance is formulated as one of the important control specifications in multi-agent systems to guarantee that the control actions of the agents will keep the system trajectories out of the prescribed unsafe set no matter what control strategies are taken by the others. For the collision avoidance issue, it is common to use the gradient-type algorithm based on a variety of potential functions [5]–[9]. However, existing potential-based collision avoidance control laws may lead to unbounded actuation, which are not practically applicable. For the connectivity maintenance issue, one of the simplest methods is to specify a finite number of time switching topologies for the network [10]. Since this scenario is not practically applicable, a joint connectivity condition is proposed in [11] and [12], i.e., there exist infinitely many consecutive bounded time intervals such that the union of graphs over every interval is connected. However, all of these results consider the connectivity maintenance problem under certain topology conditions. But for physically limited sensing ranges, it may not be possible to satisfy the joint connectivity, too.

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Cooperative control problems integrating simultaneously formation maneuvering, collision avoidance, and connectivity maintenance are usually difficult as multiple control specifications may contradict each other, resulting in unexpected results such as multiple equilibria, deadlock or instability. Following our previous work on formation maneuvering control [13], we aim to find solutions for collision avoidance and connectivity maintenance while achieving formation maneuvering. Due to the merit of having one degree of freedom in scaling the formation for the formation maneuvering control strategy developed in [13], we are able to design extra control to realize the goal of collision avoidance and connectivity maintenance. A switching control law is then proposed based on the potential function approach. We show that with the proposed switching control, any two agents will become to keep a safe distance away from each other in finite time and any two neighboring agents will remain in the sensing range. The main contribution of the paper is that we develop a simple, distributed switching control strategy that solves the formation maneuvering control problem with guaranteed collision avoidance and connectivity maintenance properties. The control input is bounded, which is more feasible than other potential-based control strategies. Moreover, this switching control law can also be used to alter the formation size for better adaptivity to environment changes.

Notation: $\mathbb{C}$ and $\mathbb{R}$ denote the set of complex and real numbers, respectively. $\mathbf{1}_n$ represents the n-dimensional vector of ones and I_n denotes the identity matrix of order n. $\iota = \sqrt{-1}$ denotes the imaginary unit. For a complex number c, the notations c^*, $|c|$, $\mathrm{Re}(c)$, and $\mathrm{Im}(c)$ denote its conjugate, modulus, real part, and imaginary part, respectively. For a complex-valued matrix L, L^* denotes its conjugate transpose. For real numbers, the sign function is defined as $\mathrm{sgn}(x) = \begin{cases} 1, & x>0 \\ 0, & x=0 \\ -1, & x<0 \end{cases}$ and for complex numbers, the sign function is defined as $\mathrm{sgn}(a+b\iota) = \mathrm{sgn}(a) + \mathrm{sgn}(b)\iota$. In a similar way, the saturation function of real numbers is defined as $\mathrm{sat}(x) = \begin{cases} x, & |x| \le 1 \\ x/|x|, & |x|>1 \end{cases}$ and for complex numbers, the saturation function becomes $\mathrm{sat}(a+b\iota) = \mathrm{sat}(a) + \mathrm{sat}(b)\iota$.

II. PRELIMINARIES AND PROBLEM SETUP

This section presents some basic notions from graph theory and several preliminary results, and then formulates the problem we study.

A. Basic notions in graphs

A *directed graph* (digraph for short) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a non-empty node set $\mathcal{V} = \{1, 2, \cdots, n\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An edge of $\mathcal{G}$ is denoted by an ordered pair of nodes (j, i), for which node j is called an *in-neighbor* of node i and node i is called an *out-neighbor* of node j.

For a directed graph $\mathcal{G}$, a node v is said to be *reachable* from another node u if there exists a path from u to v. A directed graph $\mathcal{G}$ is said to be *rooted* if there exists a node, from which every other node is reachable. For a directed graph $\mathcal{G}$, a node v is said to be *2-reachable* from a non-singleton subset of nodes $\{u_1, \ldots, u_k\}$ if there exists a path from a node in $\{u_1, \ldots, u_k\}$ to v after removing any one node except v. A directed graph $\mathcal{G}$ is said to be *2-rooted* if there exists a subset of two nodes, from which every other node is 2-reachable.

For a digraph $\mathcal{G}$, we associate to each edge (j, i) a weight $w_{ij} \neq 0$. Then the Laplacian L of $\mathcal{G}$ is defined in the way that its (i, j)th off-diagonal entry is the negative weight $-w_{ij}$ if (j, i) is an edge and 0 otherwise, and its (i, i)th diagonal entry is the negative row sum of all the off-diagonal entries in the same row. The weights w_{ij}'s can be real or complex numbers, for which the Laplacian is called real-valued Laplacian and complex-valued Laplacian, respectively. Certainly, any Laplacian matrix L satisfies $L\mathbf{1}_n = 0$.

B. Problem setup

Consider a group of agents labeled as $1, \ldots, n$. The agents are governed by a single-integrator kinematic model

$$\dot{z}_i = u_i,$$

where $z_i \in \mathbb{C}$ represents the position of agent i and $u_i \in \mathbb{C}$ represents the control input of the agent i. For convenience, we use a complex variable to represent a 2-dimensional vector.

Each agent is assumed to have the following sensing and communication capabilities.

- **Long-range sensors.** Each agent i carries an onboard long-range sensor, to make them able to sense the relative positions (namely, $z_j - z_i$) of a few others. In general, mutual sensing may not be possible. Thus, we use a directed graph $\mathcal{G}$ of n nodes to represent the *sensing graph* of the system for which each node represents an agent and an edge (j, i) indicates the availability of relative position measurement $(z_j - z_i)$ by agent i.
- **Proximity sensor for collision avoidance.** In addition to a long-range sensor for multi-agent interaction, each agent also carries a proximity sensor, which is used to avoid collisions with others that may not be detected by its long-range sensor. Usually, the proximity sensor is omnidirectional with a very short sensing radius R_1. We assume that the proximity sensors of all the agents have the same sensing radius R_1.
- **Communication.** Each agent is supposed to be able to communicate with a few others, called *communication neighbors*, which may be different from the neighbors sensed by its long-range sensor. Usually, communication is bidirectional, meaning that if agent i can talk to agent j, then agent j can also talk to agent i. We use a bidirectional graph $\mathcal{H}$ of n nodes to represent the *communication graph*, for which each node represents an agent and a bidirectional edge (j, i) indicates that agent j can communicate to agent i each other.

This paper aims to develop control for the agents to achieve a desired formation shape, maneuver with a constant velocity, and meanwhile avoid collisions. By a desired formation shape, we mean that the achieved formation is similar to a target configuration $\xi = [\xi_1, \xi_2, \cdots, \xi_n]^T$ defined in a specific coordinate system. That is, the formation maneuvering problem with four degrees of freedom is formally stated as to make

$$\lim_{t\to\infty} z(t) = c_1\mathbf{1}_n + c_2\xi + v_s t\mathbf{1}_n \tag{1}$$

for some $c_1, c_2 \in \mathbb{C}$, where z is the aggregate position vector of all the agents and $v_s \in \mathbb{C}$ is the group maneuvering velocity, which is not pre-specified.

To be more specific, the following two problems will be studied.

Problem 1: Suppose that the sensing graph $\mathcal{G}$ is static and directed. As shown in [14], a 2-rooted graph is necessary to encode a formation shape with four degrees of freedom in the plane. So we assume that the sensing graph is 2-rooted in this paper. Moreover, the communication range is often greater than the sensing range in many practical applications. Therefore, we assume that the communication graph $\mathcal{H}$ is rooted and contains the sensing graph $\mathcal{H}$ as a subgraph. In addition, assume that each agent has a physical size modelled as a cylinder with radius r. That means, collision with agent i occurs if some agents come into the region

$$\Omega_i^c = \{q \in \mathbb{C} : \ ||z_i - q|| \leq r\}.$$

We call it the *collision region*. An illustration is given in Fig. 1, in which

$$\mathcal{D}_i^c = \{q \in \mathbb{C} : \ r < ||z_i - p|| \leq R_1\}$$

is called the *collision potential region*, where R_1 is the sensing radius of each proximity sensor.

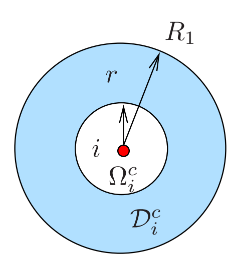


Fig. 1. Collision region and collision potential region.

In such a scenario, how to devise a distributed control law to solve the formation maneuvering problem with guaranteed collision avoidance?

Problem 2: In real applications, the sensing graph $\mathcal{G}$ may not be static due to limited sensing ranges. Therefore, we consider in this problem that the long-range sensors on all the agents have a fixed sensing radius R. That means, the connection with agent i becomes lost if some agents are R distance away from agent i, or equivalently to say, some agents are in the region

$$\Omega_i^d = \{q \in \mathbb{C} : \ ||z_i - q|| > R\}.$$

We call it the *disconnection region*. An illustration is given in Fig. 2, in which another relevant region

$$\mathcal{D}_i^d = \{q \in \mathbb{C} : \ R_2 < ||z_i - q|| \leq R\}$$

is called the *disconnection potential region*, where R_2 is a parameter less than R.

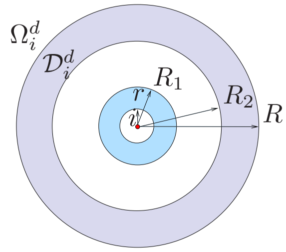


Fig. 2. Disconnection region and disconnection potential region.

In such a scenario, the problem is to devise a distributed control law to solve the formation maneuvering problem with guaranteed collision avoidance and connectivity maintenance.

III. Main Results

This section aims to provide the main results for Problem 1 and 2.

A. Formation maneuvering with collision avoidance

We propose the following distributed control law to solve Problem 1:

$$u_i = u_i^f + u_i^c, \quad i = 1, \ldots, n, \tag{2}$$

which consists of two parts: one (namely, u_i^f) for formation maneuvering and the other (namely, u_i^c) for collision avoidance.

The formation maneuvering control u_i^f takes the form from our previous work [13], i.e.,

$$\begin{cases} \dot{\eta}_i = \sum\limits_{j \in \mathcal{M}_i} \alpha_{ij}(\eta_j - \eta_i), \\ \dot{\zeta}_i = -a\zeta_i - \sum\limits_{j \in \mathcal{N}_i^+} w_{ij}(z_j - z_i), \\ u_i^f = \eta_i - \sum\limits_{j \in \mathcal{N}_i^+} w_{ij}^* \zeta_i + \sum\limits_{j \in \mathcal{N}_i^-} w_{ji}^* \zeta_j, \end{cases} \tag{3}$$

where the auxiliary dynamics of η_i is to synchronize the velocity of the agents when they reach the desired formation and the auxiliary dynamics of ζ_i is to help ensure the convergence toward the desired formation shape. The control parameters in (3) should satisfy the following conditions.

1) $\alpha_{ij} \in \mathbb{R}$ and $\alpha_{ij} > 0$;
2) $a \in \mathbb{R}$ and $a > 0$;
3) $w_{ij} \in \mathbb{C}$ satisfying

$$\sum_{j \in \mathcal{N}_i^+} w_{ij}(\xi_j - \xi_i) = 0 \text{ for } i = 1, \ldots, n \tag{4}$$

where $\mathcal{M}_i$ is the neighbor set of agent i in the communication graph $\mathcal{H}$ while $\mathcal{N}_i^+$ is the neighbor set of agent i in the sensing graph $\mathcal{G}$.

The collision avoidance control u_i^c is designed as follows. We consider the following potential function

$$P_{ij} := \begin{cases} \left(\frac{|z_i - z_j|^2 - (2R_1^2 - r^2)}{|z_i - z_j|^2 - r^2}\right)^2 - 1, & \text{if } z_j \in \mathcal{D}_i^c \\ 0, & \text{otherwise.} \end{cases}$$

The function P_{ij} is continuous with respect to $|z_i - z_j|$ for $|z_i - z_j| > r$. An illustrative example is given in Fig. 3 with $r = 10$ and $R_1 = 40$.

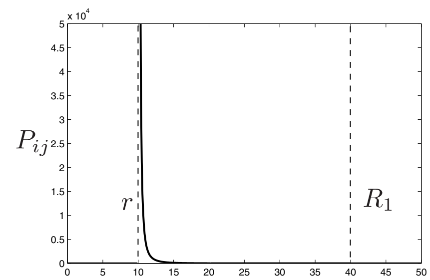


Fig. 3. The Potential function P_{ij} with respect to $|z_i - z_j|$.

It can be known that if $|z_i - z_j| \geq R_1$, then

$$\frac{\partial P_{ij}}{\partial z_i} = 0,$$

and if $r < |z_i - z_j| < R_1$, then

$$\left|\frac{\partial P_{ij}}{\partial z_i}\right| \neq 0.$$

Let

$$s_i = -\sum_{z_j \in \mathcal{D}_i^c} \frac{\partial P_{ij}}{\partial z_i}, \quad i = 1, \ldots, n.$$

Then, the collision avoidance control u_i^c is chosen as

$$u_i^c = (|u_i^f| + b)\operatorname{sgn}(s_i), \quad i = 1, \ldots, n, \tag{5}$$

where b is a positive number.

Remark 3.1: The implementation of u_i^f as shown in (3) requires the following information via the long-range sensor:

- $(z_j - z_i)$ of in-neighbors in the sensing graph $\mathcal{G}$,

and the following information via communications:

- the auxiliary state η_j from all communication neighbors,
- the auxiliary information $w_{ji}^* \zeta_j$ from all out-neighbors in the sensing graph $\mathcal{G}$.

The implementation of u_i^c as shown in (5) needs additionally the relative position information $(z_j - z_i)$ for all $z_j \in \mathcal{D}_i^c$ via the proximity sensor.

Remark 3.2: Since the $\mathrm{sgn}(x)$ in u_i^c is discontinuous, we can replace it by the saturation function $\mathrm{sat}(x)$ to keep continuity.

Denote $z = [z_1, z_2, \ldots, z_n]^\intercal$, $\zeta = [\zeta_1, \zeta_2, \ldots, \zeta_n]^\intercal$, $\eta = [\eta_1, \eta_2, \ldots, \eta_n]^\intercal$, and $u^c = [u_1^c, u_2^c, \ldots, u_n^c]^T \in \mathbb{C}^n$. Moreover, let L be the complex Laplacian associated to the sensing graph $\mathcal{G}$ with complex weights w_{ij} and let H be the real Laplacian associated to the communication graph $\mathcal{H}$ with real weights α_{ij}. Then, the overall system becomes

$$\begin{bmatrix} \dot{z} \\ \dot{\zeta} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} 0 & -L^* & I_n \\ L & -aI_n & 0 \\ 0 & 0 & -H \end{bmatrix} \begin{bmatrix} z \\ \zeta \\ \eta \end{bmatrix} + \begin{bmatrix} u^c \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

Intuitively, the control u_i^c propels all nearby agents out of the collision potential region $\mathcal{D}_i^c$. Consequently, collision is avoided for the system (6). We present our main result below.

Theorem 3.1: If the initial condition satisfies $|z_i(0) - z_j(0)| > r$ for all i and j, the control law (2) solves the formation maneuvering problem with guaranteed collision avoidance.

The proof of Theorem 3.1 will use the following result.

Theorem 3.2 ([13]): A network of agents achieves the desired formation shape ξ and maneuvers with a constant velocity if each agent takes the distributed control law (3).

Proof of the Theorem 3.1: We consider

$$V = \sum_{i=1}^{n}\sum_{j=1}^{n} P_{ij}.$$

Then it is known that $V \geq 0$ if $|z_i - z_j| > r$ for all i and j and $V = 0$ if $|z_i - z_j| > R_1$. Taking the derivative along the solution of (6), we have

$$\begin{aligned} \dot{V} &= 2\textstyle\sum_{i=1}^n \sum_{j=1}^n \mathrm{Re}((\frac{\partial P_{ij}}{\partial z_i})^* \dot{z}_i) \\ &= -2\textstyle\sum_{i=1}^n \mathrm{Re}(s_i^* \dot{z}_i) \\ &= -2\textstyle\sum_{i=1}^n \mathrm{Re}(s_i^*[u_i^f + u_i^c]) \\ &= -2\textstyle\sum_{i=1}^n \mathrm{Re}(s_i^*[u_i^f + (|u_i^f| + b)\mathrm{sgn}(s_i)]) \\ &= -2\textstyle\sum_{i=1}^n \{\mathrm{Re}(s_i)\mathrm{Re}(u_i^f + (|u_i^f| + b)\mathrm{sgn}(s_i)) \\ &\qquad + \mathrm{Im}(s_i)\mathrm{Im}(u_i^f + (|u_i^f| + b)\mathrm{sgn}(s_i))\}. \end{aligned}$$

Now let us come to look at the sign of $\mathrm{Re}(u_i^f + (|u_i^f| + b)\mathrm{sgn}(s_i))$. We know that $|u_i^f| < |u_i^f| + b$ for $b > 0$. Then it follows that $-(|u_i^f| + b) < \mathrm{Re}(u_i^f) < |u_i^f| + b$. If $\mathrm{Re}(s_i) > 0$, we have

$$\mathrm{Re}(u_i^f + (|u_i^f| + b)\mathrm{sgn}(s_i)) = \mathrm{Re}(u_i^f + (|u_i^f| + b)) > 0.$$

On the other hand, if $\mathrm{Re}(s_i) < 0$, we have

$$\mathrm{Re}(u_i^f + (|u_i^f| + b)\mathrm{sgn}(s_i)) = \mathrm{Re}(u_i^f - (|u_i^f| + b)) < 0.$$

Thus, it is concluded that $\mathrm{Re}(u_i^f + (|u_i^f| + b)\mathrm{sgn}(s_i))$ has the same sign with $\mathrm{Re}(s_i)$. By the same argument, we can conclude that the imaginary part has the same feature. Therefore, $\dot{V} < -\varepsilon$ for a positive constant ε if at least one agent is in the collision potential region $\mathcal{D}_i^c$ of another agent i. With the negative derivative of potential function V, all the agent will move towards edge of the collision potential region of any other agent. In the end, the agents will at least R_1 distance away from each other, which means that collision will not occur. And when u_i^c will vanish in finite time. Then by using Theorem 3.2, it can be concluded that $z(t)$ of system (6) asymptotically converges to a desired formation shape ξ with a constant maneuvering velocity. ■

B. Formation maneuvering with collision avoidance and connectivity maintenance

In this subsection, we come to solve Problem 2. For this problem, as the long-range sensing radius is limited, we have to ensure that all the neighboring agents maintain their links by not departing away from each other too much. For this purpose, we add an extra control term u_i^d when two agents are in the disconnection potential region. Thus, the control law becomes

$$u_i = u_i^f + u_i^c + u_i^d, \quad i = 1, \ldots, n, \tag{7}$$

where u_i^f is the formation maneuvering control given in (3), u_i^c is the collision avoidance control given in (5) given in (3). The connectivity maintenance control u_i^d will be designed in this subsection.

We adopt the following potential function

$$Q_{ij} = \begin{cases} \left(\frac{|z_i - z_j|^2 - (2R_2^2 - R^2)}{|z_i - z_j|^2 - R^2}\right)^2 - 1, & \text{if } z_j \in \mathcal{D}_i^d \\ 0, & \text{otherwise.} \end{cases}$$

It can be known that if $|z_i - z_j| \geq R$ or $|z_i - z_j| < R_2$, then

$$\frac{\partial Q_{ij}}{\partial z_i} = 0.$$

If $R_2 < |z_i - z_j| < R$, then

$$\left|\frac{\partial Q_{ij}}{\partial z_i}\right| \neq 0.$$

An illustrative example is given in Fig. 4 with $R_2 = 70$ and $R = 100$.

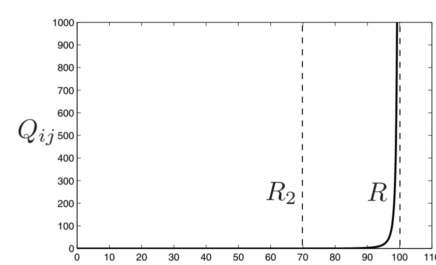


Fig. 4. The potential function Q_{ij} with respect to $|z_i - z_j|$.

Let

$$r_i = -\sum_{z_j \in \mathcal{D}_i^d} \frac{\partial Q_{ij}}{\partial z_i}, \quad i = 1, \ldots, n.$$

Then the connectivity maintenance control u_i^d is designed as follows: Design

$$u_i^d = (|u_i^f| + b)\mathrm{sgn}(r_i), \quad i = 1, \ldots, n, \tag{8}$$

where b is a positive number.

Let $u^d = [u_1^d, u_2^d, \ldots, u_n^d]^T \in \mathbb{C}^n$. Then the whole system has following form

$$\begin{bmatrix} \dot{z} \\ \dot{\zeta} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} 0 & -L^* & I_n \\ L & -aI_n & 0 \\ 0 & 0 & -H \end{bmatrix} \begin{bmatrix} z \\ \zeta \\ \eta \end{bmatrix} + \begin{bmatrix} u^c + u^d \\ 0 \\ 0 \end{bmatrix}. \tag{9}$$

When choosing $R_2 > R_1$, the control u_i^c and u_i^d works in the different regions. So they do not affect each other. The total potential function is

$$T_{ij} = P_{ij} + Q_{ij},$$

given in Fig. 5.

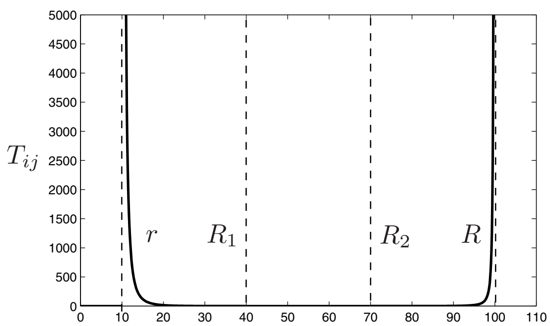


Fig. 5. The total potential function T_{ij}.

We then present our main result for Problem 2.

Theorem 3.3: If the initial condition satisfies $|z_i(0) - z_j(0)| > r$ for any i and j, and the sensing graph $\mathcal{G}$ is 2-rooted such that $|z_i - z_j| < R$ for all $j \in \mathcal{N}_i$, then the control law (7) solves the formation maneuvering problem with guaranteed collision avoidance and connectivity maintenance.

Proof: Take

$$V = \sum_{i=1}^{n} \sum_{j=1}^{n} T_{ij}.$$

Then by the same argument, it can be shown that any two agents will come to satisfy $|z_i - z_j| > R_1$ and any two neighbor agents in $\mathcal{G}$ will come to satisfy $|z_i - z_j| < R_2$ in finite time, and thus both u_i^c and u_i^d vanishes. Therefore, collisions will not occur and the connectivity is maintained, i.e., if two agents are neighbors initially in the sensing graph $\mathcal{G}$, they remain to be neighbors. Thus, by using Theorem 3.2, it is obtained that the agents will reach a desired formation shape ξ while maneuvering with a constant velocity. ■

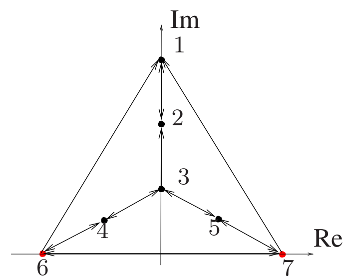
Remark 3.3: From the proof of Theorem 3.3, if we choose the minimal distance d_{min} and the maximal distance d_{max} between any two neighboring agents in the sensing graph $\mathcal{G}$ such that $\frac{d_{max}}{d_{min}} = \frac{R_2}{R_1}$, then the formation size can also be determined. So by changing the values of R_1 and R_2, the formation size can be altered for better adaptivity to environment changes.

IV. SIMULATION

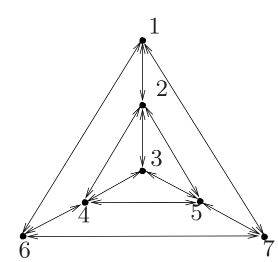
In this section, we present several simulations to demonstrate the effectiveness of our proposed control laws. Consider a system consisting of 7 agents labelled as $1, \ldots, 7$. Suppose that the target configuration is the one shown in Fig. 6(a), i.e., with respect to a coordinate system, the formation vector is

$$\xi = \begin{bmatrix} 6\iota & 4\iota & 2\iota & -\sqrt{3}+\iota & \sqrt{3}+\iota & -2\sqrt{3} & 2\sqrt{3} \end{bmatrix}^T.$$

The simulation example has the sensing graph $\mathcal{G}$ given in Fig. 6(a), which is 2-rooted, and has the communication graph $\mathcal{H}$ given in Fig. 6(b), which is rooted and contains the sensing graph $\mathcal{G}$ as a subgraph. The complex Laplacian L of the sensing graph $\mathcal{G}$ and the real Laplacian H of the communication graph $\mathcal{H}$ are chosen to satisfy the conditions as required.



(a) The desired formation ξ and the sensing graph $\mathcal{G}$.



(b) The communication graph $\mathcal{H}$.

Fig. 6. The desired formation and graph topologies.

A. Formation maneuvering with collision avoidance

For the first simulation with only collision avoidance control, the dynamics is given in system (6), and we choose $a = 5$, $b = 10$, $r = 2$ and $R_1 = 6$. The simulation result is shown in Fig. 7. Fig. 7(a) plots the trajectories of seven agents, from which we can see that they asymptotically converge to a desired formation. Fig. 7(b) shows the evolution of the minimal distance between any two agents with and without collision avoidance control. As we can see, with the collision avoidance control, the minimal distance is always greater than $r = 2$ and becomes greater than $R_1 = 6$ in finite time, meaning that no collision occurs. However, collision occurs for the control law without the collision avoidance term as the minimal distance goes below $r = 2$.

B. Formation maneuvering with collision avoidance and connectivity maintenance

The second simulation is concerned with the control law with both collision avoidance and connectivity maintenance control. The resulting closed-loop dynamics is (9), for which we choose $a = 5$, $b = 10$, $r = 2$, $R_1 = 6$, $R_2 = 40$ and $R = 85$. The simulation result is shown in Fig. 8. Fig. 8(a) plots the simulation trajectories of seven agents, from which we can see that they also asymptotically converge to the desired formation. Fig. 8(b) shows the evolution of the maximal distance between any two neighboring agents in

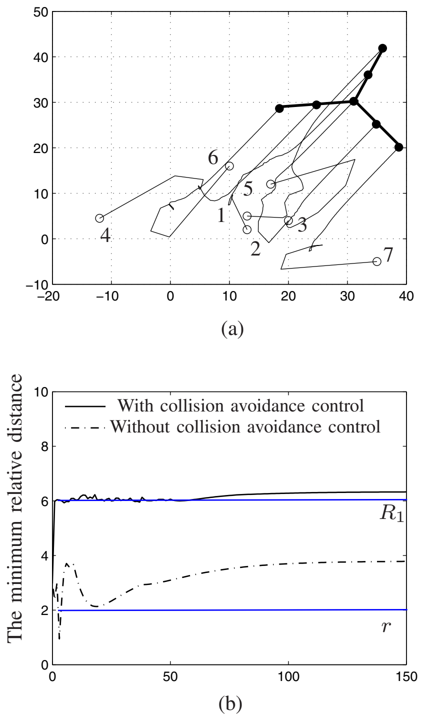


Fig. 7. Simulation results with collision avoidance control.

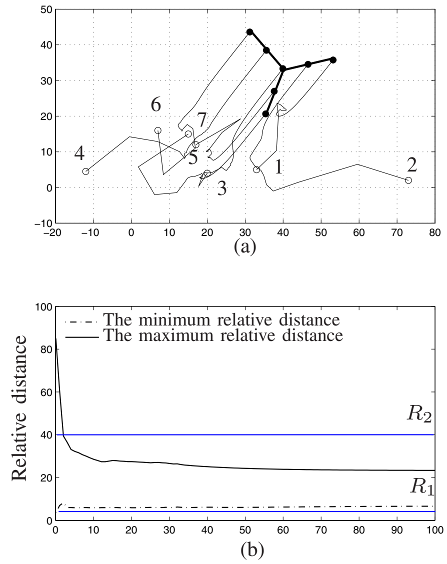


Fig. 8. Simulation results with collision avoidance and connectivity maintenance control.

the sensing graph $\mathcal{G}$ and the minimal distances between any two agents. As we can see, the neighboring agents remain to be neighbors as their distances are kept less than the disconnection radius $R = 85$. Also, no collision occurs, which is the same as the first simulation.

V. Conclusion

This paper develops a simple, distributed switching control approach for the design of extra control to achieve collision avoidance and connectivity maintenance during the convergence of a multi-agent system to a desired formation shape. This switching control is based on a distance-based potential function, and the sign of the negative gradient of this potential function is the switching signal. Thus this control inputs can propel all the agents to leave out the collision potential regions and the disconnection potential regions. Since our previous work on formation maneuvering has one degree of freedom in scaling the size of formation, we can use this distance based control to achieve collision avoidance and connectivity maintenance, but have no influence on achieving formation maneuvering.

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