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Maceration Control of a Sugar Cane Crushing Mill

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Abstract

Raw sugar is produced from juice in sugar cane crushed by a series of mills. To improve the extraction a liquid bath is applied to the cane between mills. This liquid bath, commonly called maceration, consists of water and some of the produced juice. Although the extraction is improved with higher water content in the maceration, the total juice output is restricted by the storage capacity of the plant. The aim of maceration control is to manipulate the added water within the process limits placed by the storage capacity whilst optimising sugar extraction. In this paper, mathematical models of the processes pertaining to maceration are derived from first principles. A cascaded model predictive controller is then designed using the derived models.

Key words: Model predictive control, modelling, sugar mills, maceration

1 Introduction

The function of a raw sugar factory is to produce crystal sugar from the juice in sugar cane delivered to the factory [1]. The extraction process in Australia is mostly done by crushing mills. The prepared cane is passed through a series of mills called the milling train, see Fig. 1. The mills crush the cane to separate the juice which contains the sugar from its fibrous part. The fibrous material left after the juice is removed is called bagasse. To help the extraction of juice, some of the produced juice is returned to the bagasse between the mills. Water is also added before the last mill to wash out any remaining sugar. Measurements of bagasse mass and fedback juice flow between mills are often not available, making it difficult to estimate mass balance on-line. Bagasse is carried by a fixed speed inter-carrier between any two mills. From the carrier, the bagasse is fed into the next mill. At the exit of a mill the bagasse dives into a boot where it absorbs the feedback juice or water. The liquid level in a boot is controlled by a boot valve which splits the juice from the following mill between that boot and the preceeding boot. The part of the second mill juice which is not returned back to boot no. 1 and the first mill juice are then sent to a screen which filters tiny fibres of bagasse. The screened juice is next sent to downstream processes, e.g. the evaporators. The total capacity of juice storage before the evaporators exerts an upper bound on the nett juice output of the milling train. See Fig. 1 for a schematic of the process.

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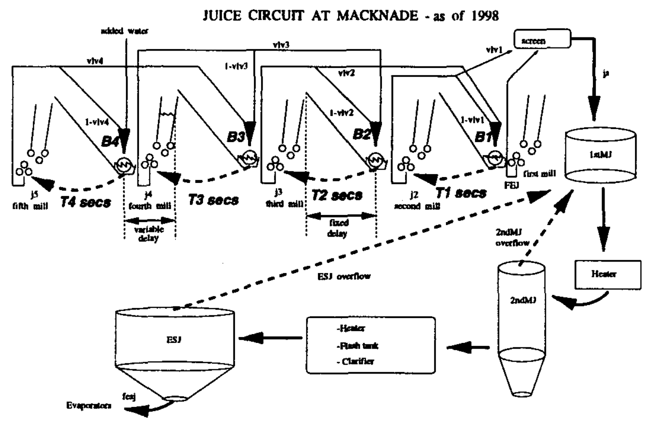

Figure 1: Process Schematics

2 Integration of three subprocesses

Regarding the control and constraints the problem can be considered as an interaction of three subprocesses:

2.1 Juice subsystem

After dry crushing in the first mill, the crushing process can be seen as the replacement of cane juice with maceration, see Fig. 1. The juice subsystem formulates the juice circuit of the milling train where the juice and maceration flows are defined in terms of the added water flow, transport lags and boot valves:

$$\begin{aligned} J_{i+1}(t) &= J_{i+1}(t-T_i)(1-V_i(t-T_i)) + u(t-T_i) \\ u(t) &= \begin{cases} J_{i+2}(t)V_{i+1}(t) & \text{for } i=1,2,3 \\ addw(t) & \text{for } i=4 \end{cases} \end{aligned} \quad (1)$$

where J_i and V_i are the i^{th} mill juice and i^{th} boot valve opening respectively, T_i is the transport lag from the i^{th} boot to the following mill, mill no $(i+1)$, and $addw$ is the volumetric flow of the water added to the last boot. The left side of Eqn. 1 formulates the maceration flows subject to transportation delays. Next, we rewrite Eqn. 1 in discrete-time state space form. To this end, we choose a sampling time T_s and assume that $\frac{T_i}{T_s}$ ($=n_i$) is approximately an integer for all $n = 1, \cdots, 4$. With this assumption, Eqn. 1 is approximated by:

$$X_i[k+1] = A_i[k]X_i[k] + B_i[k]J_{i+2}[k-n_i] \quad (2)$$

where

$$A_i[k] = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 1 \\ 1-V_i[k-n_i] & 0 & \cdots & 0 \end{bmatrix}$$

$$B_i[k] = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ V_{i+1}[k-n_i] \end{bmatrix}, X_i[k] = \begin{bmatrix} J_{i+1}(t-T_i) \\ \vdots \\ J_{i+1}(t-1) \end{bmatrix}$$

$addw(t-T_4)$ can also be represented in discrete-time state space form:

$$X_w[k+1] = A_w[k]X_w[k] + B_w[k]addw[k] \quad (3)$$

Expanding each row of Eqn. 1 and augmenting it with the state space representation for $addw(t-T_4)$, we have:

$$\begin{aligned} \begin{bmatrix} X_1[k+1] \\ \cdots \\ X_4[k+1] \\ X_w[k+1] \end{bmatrix} &= \begin{bmatrix} A_1 & M_1 & \cdots & O \\ O & A_2 & \cdots & O \\ O & \cdots & \cdots & O \\ O & O & \cdots & M_4 \\ O & O & \cdots & A_w \end{bmatrix} \begin{bmatrix} X_1[k] \\ \cdots \\ X_4[k] \\ X_w[k] \end{bmatrix} \\ &+ \begin{bmatrix} O & \cdots & O & B_w \end{bmatrix}^T addw[k] \\ X[k+1] &= A_{jd}X[k] + B_{jd}addw[k] \end{aligned} \quad (4)$$

Outputs of the system are chosen as the maceration flows to all four boots, the juice output of the second mill and immediate macerations. The flow of the second mill juice is chosen because it makes up for the portion of the total juice output which can be controlled by added water. The outputs of the system can be written as follows:

$$\begin{aligned} y[k] &= \begin{bmatrix} O & c_1 & O & c_2 & O & c_3 & O & c_4 & O \end{bmatrix} X[k] \\ &+ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T addw[k] \\ y[k] &= C_{jd}X[k] + D_{jd}addw[k] \end{aligned} \quad (5)$$

where the nonzero columns, c_i are given as follows:

$$\begin{aligned} c_1 &= \begin{bmatrix} 1-V_1[k] & \cdots & 1 & 1-V_1[k] & 0 & 0 & 0 \end{bmatrix}^T \\ c_2 &= \begin{bmatrix} V_2[k] & 1-V_2[k] & \cdots & 1-V_2[k] & 0 & 0 \end{bmatrix}^T \\ c_3 &= \begin{bmatrix} 0 & V_3[k] & 1-V_3[k] & \cdots & 1-V_3[k] & 0 \end{bmatrix}^T \\ c_4 &= \begin{bmatrix} 0 & 0 & V_4[k] & 1-V_4[k] & \cdots & 1-V_4[k] \end{bmatrix}^T \end{aligned}$$

The total juice sent to downstream tanks, J_s can be written as follows:

$$J_s[k] = J_1[k] + J_2[k]V_1[k] \quad (6)$$

$J_1[k]$ is the measured juice output of the first mill. $J_2[k]V_1[k]$ is the portion of the juice output of the second mill which is sent to downstream process. $V_1[k]$ is the first boot valve and $J_2[k]$ is estimated from Eqn. 1. Valve position, V_i of each boot is available online. Although the boot valve is a nonlinear element, its nonlinearity is well defined. Time varying transport lags, T_i are found from the variable surface speeds of the rolls, fixed speeds of the carriers, and related distances. They appear as parameters in our model, but their calculations will be omitted in this paper.

2.2 Fibre subsystem

The fibre subsystem defines the mass flow of fibre at the boots along the milling train. Those mass flows define the target maceration for proper crushing. The fibre rate is calculated from the first mill juice and the cane input which are the only flow measurements available online. This is then delayed using the necessary transport lags to calculate the fibre rate at each boot:

$$\begin{aligned} mf_1(t) &= (mi(t-D_{01}) - \delta_{j1}J_1(t-D_{02}))\, f_{bg1} \\ mf_i(t) &= mf_{i-1}(t-D_{i-1}) \text{ for } i = 2,3,4 \end{aligned} \quad (7)$$

where δ_{j1} and f_{bg1} are the assumed values of the density of the first mill juice and the fibre ratio of the bagasse, mi is the mass flow of input cane, J_1 is the first mill juice flow, D_{01} is the transport lag from the point where the mass flow of input cane is measured to the first boot and D_{02} is the transport lag from the flow measurement of the first mill juice to the first boot. The other delay, D_i, is the transport lag from the i^{th} boot to the $(i+1)^{th}$ boot. The variable transport lags used by the fibre subsystem are calculated as in the juice subsystem.

2.3 Tank subsystem

The dynamics of the downstream tanks; 1^{st} and 2^{nd} mixed juice and ESJ tanks, provides a mechanism to calculate the upper bound placed on the nett juice output of the milling train, see Fig. 1. Because of their closely coupled operation and the lack of measurements of intermediary flows, they can be lumped into a fictitious dynamic tank. The volume of this tank, Vol_{DTNK} is the total volume of the tanks. If the radius and height of this fictitious dynamic tank are r_{DTNK} and h_{DTNK} respectively, the nett juice flow into the tank subsystem, $J_s(t) - f_{ESJ}(t)$, determines the level of the juice in the dynamic tank:

$$\frac{dh_{DTNK}(t)}{dt} = \frac{1}{\pi r_{DTNK}^2}(J_s(t) - f_{ESJ}(t)) \quad (8)$$

Above $J_s(t)$ is the total juice entering the tank subsystem as calculated from Eqn. 6 and $f_{ESJ}(t)$ is the flow out of the ESJ tank (i.e. the flow exiting the tank subsystem). This continuous time first order system can be written in discrete time state space form with a sampling time T_s as:

$$\begin{aligned} x[k+1] &= x[k] + \frac{T_s}{\pi r_{DTNK}{}^2} u[k] \\ y[k] &= x[k] \end{aligned} \tag{9}$$

In the discrete representation above, the state, $x[k]$ is taken as the deviation of the tank level off the setpoint.

3 Maceration control

Extraction increases with increased amounts of maceration. However there is a desired ratio, k_{desi}, between the added maceration and the fibre content of the bagasse for each boot i, beyond which the additional sugar extraction is negligible. This ratio must be preserved at each boot. Hence the target maceration flow into each boot is determined by the fibre flow, $mf_i[k]$, at that boot as calculated by the fibre subsystem, see Eqn. 7. Whilst meeting the target maceration flows into the boots, the total juice flow, J_s, out of the milling train must be appropriate in order to maintain the tank levels within their safety limits. As represented by the juice model, see Eqn. 1, other than the first mill, the juice output of each mill and the maceration flow into each boot can be formulated as a function of added water flow, related transport lags and boot valves. Accordingly the only component of J_s that can be controlled by the added water is the flow of the second mill juice, see Eqn. 6. The target flow of the second mill juice, $J2_{des}$ is calculated by a controller based on the tank subsystem. The target values for the output of the discrete juice subsystem, y_t, as represented by Eqns. 4 and 5, can be written as:

$$y_t^T = [k_{des1} mf_1[k] \ \cdots \ k_{des4} mf_4[k] \ \ J2_{des}[k]] \tag{10}$$

4 Model predictive control

Model predictive control, MPC, has been used in a large number of industrial applications and is frequently considered to be the first option for the control of processes with hard constraints. Since our final aim is the control of a real plant by digital systems, the discussion here is restricted to the discrete time case.

4.1 Linear model predictive control

A key feature of MPC is its ability to handle hard constraints on the outputs and inputs. Based on the current state of the process, previous input, and target state and input values, the MPC algorithm calculates a set of input values which are intended to bring the process to the target state in finite time. In reality only the very first calculated input is applied to the plant. With that input and the new process state, another set of input values are calculated in the next cycle. Since an input profile over a finite time into the future is calculated, the dynamics of the process must be embedded with a model of the process into the MPC algorithm. If the model of the process is chosen to be linear with linear constraints then the algorithm reduces to linear model predictive control.

4.2 Linear MPC algoritm

The MPC algorithm used in this paper was proposed by Kenneth R. Muske and James Rawlings, [5]. We outline the method below for completeness. The system subject to linear MPC is assumed to have the following description:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \end{aligned} \tag{11}$$

The control is then based on the minimisation of the following infinite horizon quadratic objective function at time k.

$$\min_{u^N} \sum_{j=0}^{\infty} \left((y_{k+j} - y_t)^T Q (y_{k+j} - y_t) \right.$$
$$\left. + (u_{k+j} - u_t)^T R (u_{k+j} - u_t) + \Delta u_{k+j}{}^T S \Delta u_{k+j} \right) \tag{12}$$

subject to constraints:

$$\begin{aligned} u_{min} \le u_{k+j} \le u_{max}, \ & j = 0, 1, \cdots, N-1 \\ y_{min} \le y_{k+j} \le y_{max}, \ & j = j_1, j_1+1, \cdots, j_2 \\ \Delta u_{min} \le \Delta u_{k+j} \le \Delta u_{max}, \ & j = 0, 1, \cdots, N \end{aligned} \tag{13}$$

Here j_1 and j_2 provide a mechanism for relaxing the output constraints to circumvent possible temporary infeasibilities. Since input constraints are defined by the limits of the actuators, relaxing the outputs is the only option. If infeasibility is unavoidable, the output constraints are relaxed until the time, j_1. The constraints are then applied between j_1 and j_2. j_2 is the earliest possible time to guarantee the satisfaction of the output constraints thereafter. In Eqn. 12, Q is a symmetric positive semidefinite penalty matrix on the outputs. R is a symmetric positive definite penalty matrix on the inputs. S is a symmetric positive semidefinite penalty matrix on the rate of change of inputs in which $\Delta u_{k+j} = u_{k+j} - u_{k+j-1}$ is the change of input vector at time j. y_t is the target output. Then u_t and x_t are the target input and state vectors, respectively, which hold the output of the system in Eqn. 11 at the value y_t with the minimum offset [5]. The solution of the quadratic programme, u^N contains N future control moves as shown below:

$$u^N = [\ u_k \ \ u_{k+1} \ \ \cdots \ \ u_{k+N-1} \ ]^T \tag{14}$$

After time $k+N-1$, the input is assumed to be constant at u_t. Recall that only u_k is applied to the plant. With y_{k+j} computed from Eqn. 11:

$$y_{k+j} = C(A^j x_k + \sum_{i=0}^{j-1} A^{j-1-i} B u_{k+i}) + D u_{k+j} \quad (15)$$

and by redefining system states and inputs as:

$$\bar{x}_k = x_k - x_t, \quad \bar{u}_k = u_k - u_t \quad (16)$$

the objective function of Eqn. 12 can be rewritten in terms of the input, $\bar{u}^N$ only:

$$\min_{\bar{u}^N} \bar{u}^{N^T} H \bar{u}^N + 2\bar{u}^{N^T} (G\bar{x}_k - F\bar{u}_{k-1}) \quad (17)$$

The cost matrices H, G and F are functions of matrices of the system representation and MPC parameters:

$$\begin{aligned} H &= \mathcal{H}(A, B, C, D, Q, R, S, N) \\ G &= \mathcal{G}(A, B, C, D, Q, N), \quad F = \mathcal{F}(S, N) \end{aligned} \quad (18)$$

As the objective function, the constraints (see Eqn. 13) must also be rewritten in input profile:

$$\alpha\, \bar{u}^N \leq \beta \quad (19)$$

where α and β are functions of:

$$\begin{aligned} \beta &= \beta(A, B, C, D, N, j_1, j_2, constraints, u_{k-1}, u_t) \\ \alpha &= \alpha(A, B, C, D, N, j_1, j_2) \end{aligned} \quad (20)$$

However the constraints given in Eqn. 20, depend on the system representation (A, B, C, D). Hence they are vulnerable to model errors. Online solution of quadratic programs also poses a problem. For systems driven by single input, a quick remedy for these problems is to define the finite horizon N rather short. Then the unconstrained global minimum can be taken as the solutions of the related quadratic programs. The constraints are then applied by simply saturating the controller at the limits, u_{max} and u_{min}, [2]. The unconstrained global minimum of Eqn. 17 is given as:

$$u^* = -\mathcal{H}^{-1}(G\bar{x}_k - F\bar{u}_{k-1}) \quad (21)$$

5 MPC design for maceration

We can now use the linear discrete time state space model for the maceration process as in Section 2 to apply the MPC algorithm. The proposed design consists of two cascaded linear MPCs: *Juice MPC* to regulate the added water and *Tank MPC* to estimate the target second mill juice, $J2_{des}$ for the Juice MPC. Juice MPC uses the representation given for the juice subsystem in Eqns. 4 and 5. The target output vector for the juice MPC is given in Eqn. 10. Output of the juice MPC is the optimal added water flow over the defined finite horizon. Only the first value in the profile is applied to the plant. Tank MPC is configured with the tank model as calculated in Eqn. 9. Target output for the tank MPC is zero, i.e., the tank level should be the steady state value achieved with zero nett flow. Output of the tank MPC is the optimal profile of the nett flow into the tank subsystem over the related finite horizon. The target nett flow is the first value of the profile. The target juice output of the mill, Js_{des} is found by the addition of the calculated target nett flow and the measured outflow, $fesj$. The target juice output of the second mill, $J2_{des}$ is then found from Eqn. 6. Both juice and tank subsystems are single input. Accordingly the finite horizons for both systems can be taken short (e.g. 2 samples) and the solution to all quadratic programs can be taken as unconstrained minimums. The solutions are saturated at the related u_{max} and u_{min} values as explained in Section 4. The main difference between the Juice MPC and Tank MPC is the fact that the juice subsystem, (see Eqns. 4 and 5) is stable where the dynamic tank subsystem, Eqn. 9 is unstable. Eventually the calculation of cost matrices, H and G (see Eqn. 18), are slightly different for the tank subsystem [5]. The design has been simulated for variable fibre rate where all the other process variables are kept constant. The variable fibre rate was simulated by varying the first mill juice, see Fig. 2.

6 Discussion of results

For the purpose of presentation, the maceration control of the third boot together with the MPC controlled water flow is summarised in Fig. 2. While the changing fibre rate is closely tracked, the dynamic tank level is also kept at the target value, see Fig. 3. The nett flows for the tank subsystem are shown in Fig. 4.

7 Conclusions

Maceration control of a sugar cane crushing plant has been examined. The maceration process involves three subsystems: juice circuit, fibre flow and storage tanks. All three subsystems have been modeled. Based on the models a cascaded linear model predictive control scheme has been proposed, which is expected to provide a better integration of the related subsystems, hence improving sugar extraction.

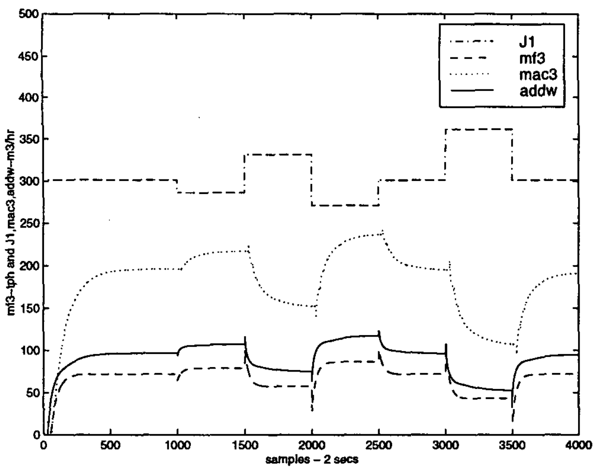

Figure 2: Maceration control of boot no 3

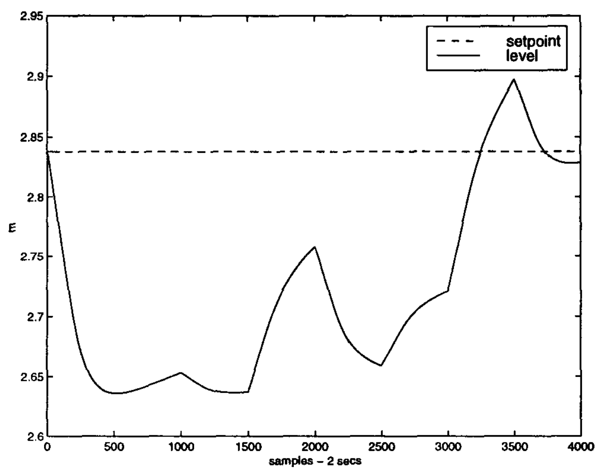

Figure 3: Dynamic tank level

8 Acknowledgements

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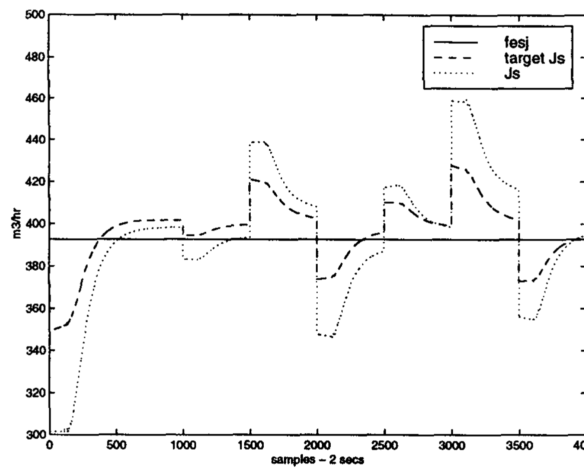

Figure 4: Flows for the dynamic tank

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