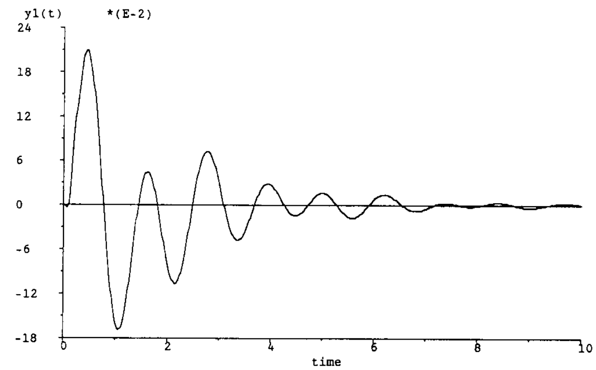

Fig. 2. The deflection of the center of the beam with the LQG controller $K_{\beta 3}$.

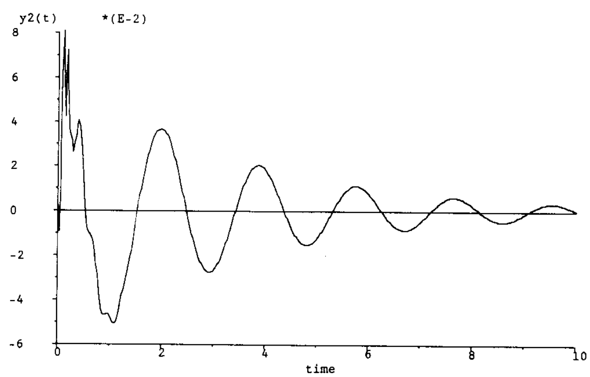

Fig. 3. The inclination at the center of the beam with the LQG controller $K_{\beta 3}$.

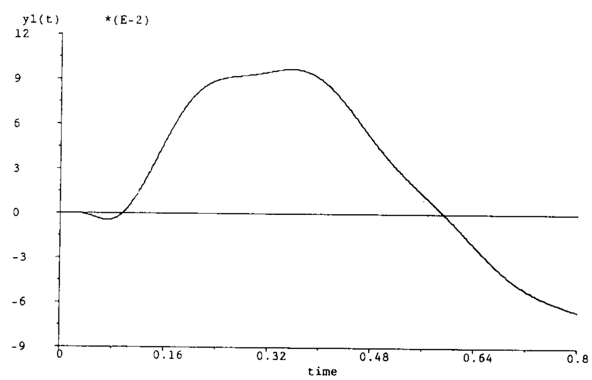

Fig. 4. The deflection of the center of the beam with the LQG controller $K_{\beta 4}$.

achievable robustness margin for a system, which is easily calculated from its unstable part, is a useful index in controller design; in particular, it tells you how well you must approximate your original system to avoid spillover effects. By designing a maximally robust controller for the approximation, one can minimize the chance of spillover. So as well as providing qualitative insights into the spillover problem, we have also given a design procedure which guarantees *a priori* no spillover. In fact, this note is a particular interpretation of the results of Curtain and Glover [12], but since the spillover problem has become such an issue in the control of flexible systems, we felt it worthwhile to elaborate on this aspect. We feel that our treatment of the example in Section IV illustrates the advantages of our interpretation of the spillover problem.

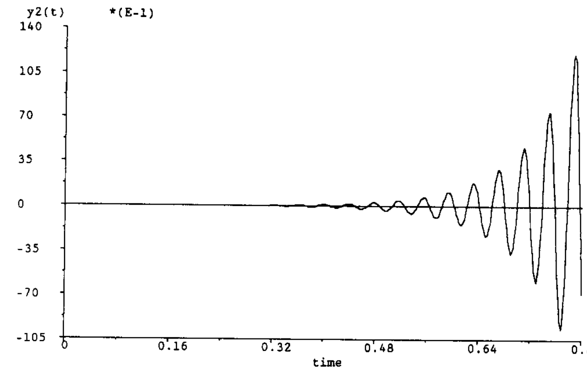

Fig. 5. The inclination at the center of the beam with the LQG controller $K_{\beta 4}$.

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Stability of a Polytope of Matrices: Counterexamples

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***Abstract*—The problem of robust stability leads to a considerable body of research on the stability of a polytope of polynomials and matrices. Since Kharitonov's seminal result on interval polynomials, there have**

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been significant breakthroughs for the stability of a polytype of polynomials. However, for a polytope of matrices, the stability problem is far from completely resolved. In this paper, we provide counterexamples for three conjectures which are directly motivated by the results in the polynomial case. These counterexamples illustrate the fundamental differences between the polynomial stability problem and the matrix stability problem.

I. Introduction and Formulation

The modeling of physical systems is a process which inherently depends on making various approximations. In this paper, we focus on linear systems and inaccuracies in the model which are attributable to uncertain parameters. The uncertainties lead to perturbations in the coefficients of the characteristic polynomial and subsequently jeopardize the stability of the system.

If a state space approach is considered for modeling, the uncertainties lead to perturbations of the elements of the various matrices relating the state variables, the inputs, and the outputs of the system. In this paper, it is assumed that these uncertain parameters are only known within given bounds, and within this framework, the robust stability problem centers on whether stability is preserved for all admissible variations of the uncertain parameters. To motivate the mathematical formulation of the problem discussed here, consider the state equation

$$\dot{x}(t)=A(q)x(t); \qquad q \in Q \tag{1.1}$$

where $x(t) \in \boldsymbol{R}^n$ and q is a vector of uncertain parameters varying in the prescribed set Q. Notice that if $A(\cdot)$ depends (affine) linearly on q, then we can write

$$A(q)=A_0+\sum_{i=1}^{m} A_i q_i \tag{1.2}$$

where q_i is the ith component of q,

$$A_0=A(0),$$
$$A_i=A(q_i e_i),$$

and e_i represents a unit vector in the ith coordinate direction. Furthermore, if an *a priori* bound

$$|q_i| \le \bar{q}_i; \qquad \bar{q}_i \ge 0 \tag{1.3}$$

is available for the components of q, then it is easily shown that the set of possible $A(q)$ matrices $\{A(q): q \in Q\}$ is a polytope in $\boldsymbol{R}^{n \times n}$. This leads us to study the following problem: Given $n \times n$ matrices $M_1, M_2, \cdots, M_m$, let

$$\mathbb{M}=\left\{M_\lambda=\sum_{i=1}^{m} \lambda_i M_i : \lambda_i \ge 0,\ i=1,2,\cdots,m;\ \sum_{i=1}^{m} \lambda_i=1\right\}.$$

Determine if all matrices $M_\lambda \in \mathbb{M}$ have all their eigenvalues in the strict left-half plane. This being the case, we call $\mathbb{M}$ a (strictly) *Hurwitz polytope* of matrices.

An important special case of this problem is obtained by placing additional restrictions on the matrices M_i. To motivate this special case, consider again state equation (1.1) and form the characteristic polynomial

$$\Delta(s, q)=\det(sI-A(q)).$$

If $A(\cdot)$ depends linearly on q, the coefficient $a_i(q)$ of $\Delta(s, q)$ is, in general, a multidimensional polynomial in the q_i. In some special cases, however, $a_i(q)$ turns out to depend (affine) linearly on the q_i. For example, this occurs when the q_i only enter into a single row or a single column of $A(q)$; e.g., consider the companion canonical form. For such special cases, with componentwise bounds on the q_i, the set of possible polynomials $\{\Delta(s, q): q \in Q\}$ is a polytope in the space of nth-order polynomials. This motivates the following problem: Given nth-order polynomials $p_1(s), p_2(s), \cdots, p_m(s)$, let

$$\mathbb{P}=\left\{p_\lambda(s)=\sum_{i=1}^{m} \lambda_i p_i(s) : \lambda_i \ge 0,\ i=1,2,\cdots,m;\ \sum_{i=1}^{m} \lambda_i=1\right\}.$$

Determine if all polynomials $p_\lambda(s) \in \mathbb{P}$ have all their zeros in the strict left half plane. This being the case, we call $\mathbb{P}$ a (strictly) *Hurwitz polytope* of polynomials.

The discussion above sets the stage for the recent surge in the literature dealing with the stability of polytopes of polynomials and matrices; e.g., see [1]-[9] and their bibliographies. It was Kharitonov's paper [1] that paved the way for the recent literature aimed at constructing *computationally tractable* methods to check the stability of a polytope of polynomials. Although Kharitonov's Theorem is restricted to the special case of interval polynomials (a hyper-rectangle in the space of polynomials), it is important largely because of the simplicity of its use. Kharitonov's Theorem states that the stability of four specially constructed extreme polynomials is both necessary and sufficient for the stability of the entire hyper-rectangle. The motivation for further research in this area stems from the fact that interval polynomials are to be associated with the uncertainties which are independent. For the case of linearly dependent uncertainties, the resulting polytope of polynomials is no longer a hyper-rectangle. Hence, the desire to obtain similar results to Kharitonov's for the problem of a general polytope of polynomials leads to a considerable body of research. A significant result in this new line of research is given in a paper by Bartlett, Hollot, and Lin [8]. These authors show that the strict stability of the exposed edges of a polytope of polynomials is both necessary and sufficient for the stability of the entire polytope. The significance of this result is that checking the stability of an edge involves a convex combination of two nth-order polynomials. Hence, stability of an edge can be verified by varying only one parameter and this task can be carried out using the classical root locus method, or even more simply, using the result given by Bialas [13]; see also [5]. It is shown that the convex combination of two nth-polynomials $p_0(s)$ and $p_1(s)$ is strictly stable if and only if $p_0(s)$ is strictly stable and $H_0^{-1}H_1$ has no eigenvalues in $(-\infty, 0]$ where H_i is the so-called Hurwitz testing matrix of the polynomial $p_i(s)$.

The more general stability problem for a polytope of matrices is still unresolved. Existing literature deals with special cases and/or sufficient conditions. For example, in [9], interval matrices ($\mathbb{M}$ is a hyper-rectangle) are considered and the strong assumption of symmetry is imposed. In [5], a complete solution to the problem is given but only for the case $m = 2$; see also [3] and [10] for sufficient conditions. To date, general results for the stability of an arbitrary polytope of matrices (or even the special case of interval matrices) have not been published. Given the lack of results for the matrix case, one is tempted to argue that the stability problem for a polytope of matrices can be solved using known results for a polytope of polynomials. In fact, a result quite similar to Kharitonov's was published for interval matrices [11], but was later found to be false [12]. Our objective in this paper is to provide counterexamples to the most "tempting" conjectures in the matrix case. These conjectures are motivated by the recent literature and it is interesting to note that these conjectures are true for 2×2 matrices.

Conjecture 1 (Checking the Edges): In view of the "Edge Theorem" for polynomials [8], a conjecture is made for the matrix case. Namely, $\mathbb{M}$ is strictly Hurwitz if and only if the edges of $\mathbb{M}$ are strictly Hurwitz; i.e., strict stability of $\lambda M_i + (1 - \lambda)M_j$ for all $i, j \in \{1, 2, \cdots, m\}$ and all $\lambda \in [0, 1]$ is a necesary and sufficient condition for strict stability of $\mathbb{M}$.

Conjecture 2 (Checking Edges of a Hyper-Rectangle): Given the failure of the first conjecture, the obvious question to ask is whether the conjecture holds if the hypothesis is strengthened so that $\mathbb{M}$ is a hyper-rectangle rather than an arbitrary polytope; i.e., we consider the case of interval matrices.

Conjecture 3 (Mapping into Polynomials): Consider the case of interval matrices ($\mathbb{M}$ is a hyper-rectangle) and form the set of characteristic polynomials.

$$\mathbb{P}_{\mathbb{M}}=\{p(s) : p(s)=\det(sI-M_\lambda) \text{ for some } M_\lambda \in \mathbb{M}\}.$$

Now, it is conjectured that $\mathbb{M}$ is strictly Hurwitz if and only if the convex hull, conv $\mathbb{P}_{\mathbb{M}}$ is strictly Hurwitz.

Remark: The motivation for this conjecture comes from the fact that conv $\mathbb{P}_{\mathbb{M}}$ is easily shown to be the polytope whose extreme points are generated by computing the characteristic polynomials associated with the extreme points of $\mathbb{M}$. Hence, the objective is to reduce the matrix problem to one involving a polytope of polynomials for which there are strong results.

II. The First Counterexample: Checking the Edges of a Polytope

This counterexample is generated by taking

$$M_1=\begin{bmatrix}-1.0 & 0 & 1.0\\ 0 & -1.0 & 0\\ -1.0 & 0 & 0.1\end{bmatrix};$$

$$M_2=\begin{bmatrix}-1.0 & 0 & 0\\ 0 & -1.0 & 1.0\\ 0 & -1.0 & 0.1\end{bmatrix};$$

$$M_3=\begin{bmatrix}-1.0 & 0 & -1.0\\ 0 & -1.0 & -1.0\\ 1.0 & 1.0 & 0.1\end{bmatrix}.$$

Now, we check the edges of the polytope $\mathbb{M}$ obtained by taking the convex hull of M_1, M_2, and M_3. Indeed, the convex combination of M_1 and M_2 is strictly Hurwitz since for any $\lambda \in [0, 1]$

$$\det\,[sI-(\lambda M_1+(1-\lambda)M_2)]=(s+1)(s^2+0.9s+(\lambda^2+(1-\lambda)^2-0.1))$$

is strictly Hurwitz (notice that the coefficients of the second factor are always positive). Also, the convex combination of M_1 and M_3 is strictly Hurwitz since for any $\lambda \in [0, 1]$

$$\det\,[sI-(\lambda M_1+(1-\lambda)M_3)]$$
$$=(s+1)(s^2+0.9s+((1-\lambda)^2+(1-2\lambda)^2-0.1))$$

is strictly Hurwitz. Finally, the convex combination of M_2 and M_3 is also strictly Hurwitz since for any $\lambda \in [0, 1]$

$$\det\,[sI-(\lambda M_2+(1-\lambda)M_3)]$$
$$=(s+1)(s^2+0.9s+((1-\lambda)^2+(1-2\lambda)^2-0.1))$$

is also strictly Hurwitz, Hence, the edges of the polytope are strictly Hurwitz. However, note that the matrix

$$\frac{1}{3}M_1+\frac{1}{3}M_2+\frac{1}{3}M_3=\begin{bmatrix}-1.0 & 0 & 0\\ 0 & -1.0 & 0\\ 0 & 0 & 0.1\end{bmatrix}$$

is unstable!

III. The Second Counterexample: Checking Edges of a Hyper-Rectangle

In view of the previous counterexample, we strengthen the hypothesis from "polytope of matrices" to "hyper-rectangle of matrices." As in Section II, we show here that the sufficiency condition in the conjecture above is false. Indeed, consider the interval matrix described by

$$M=\begin{bmatrix}m_{11} & -12.06 & -0.06 & 0\\ -0.25 & -0.03 & 1.00 & 0.5\\ 0.25 & -4.0 & -1.03 & 0\\ 0 & 0.5 & 0 & m_{44}\end{bmatrix}$$

where

$$-1.5\le m_{11}\le -0.5;$$
$$-4.0\le m_{44}\ge -1.0.$$

Writing

$$m_{11}=-0.5-q_1;$$
$$m_{44}=-1.0-q_2$$

with $q_1 \in [0, 1]$ and $q_2 \in [0, 3]$, the characteristic polynomial is computed to be

$$\begin{aligned}\Delta(s, q_1, q_2)=&s^4+(2.56+q_1+q_2)s^3\\ &+(2.871+2.06q_1+1.561q_2+q_1q_2)s^2\\ &+(3.164+4.841q_1+1.56q_2+1.06q_1q_2)s\\ &+(1.853+3.773q_1+1.985q_2+4.032q_1q_2).\end{aligned}$$

Next, we investigate the stability of the four edges of this hyper-rectangle; i.e., we consider the following four cases:

Case 1: $q_1 = 0$, $q_2 \in [0, 3]$. We obtain

$$\begin{aligned}\Delta(s, 0, q_2)=&s^4+(2.56+q_2)s^3+(2.871+1.56q_2)s^2\\ &+(3.164+1.561q_2)s+(1.853+1.985q_2).\end{aligned}$$

Case 2: $q_1 \in [0, 1]$, $q_2 = 0$. We obtain

$$\begin{aligned}\Delta(s, q_1, 0)=&s^4+(2.56+q_1)s^3+(2.871+2.06q_1)s^2\\ &+(3.164+4.841q_1)s+(1.853+3.773q_1).\end{aligned}$$

Case 3: $q_1 = 1$, $q_2 \in [0, 3]$. We obtain

$$\begin{aligned}\Delta(s, 1, q_2)=&s^4+(3.56+q_2)s^3+(4.931+2.56q_2)s^2\\ &+(8.005+2.621q_2)s+(5.626+6.017q_2).\end{aligned}$$

Case 4: $q_1 \in [0, 1]$, $q_2 = 3$. We obtain

$$\begin{aligned}\Delta(s, q_1, 3)=&s^4+(5.56+q_1)s^3+(7.551+5.06q_1)s^2\\ &+(7.847+8.021q_1)s+(7.808+15.869q_1).\end{aligned}$$

Now, for each of these four cases, we construct the Routh table parametrically in q_1 and q_2. By varying q_1 and q_2 within their bounds, it is easy to verify that there are no sign changes in the first column and hence all four edges are strictly Hurwitz. However, the interior point obtained by setting $q_1 = 0.5$ and $q_2 = 1.0$ leads to the characteristic polynomial

$$\begin{aligned}\Delta(s, 0.5, 1.0)&=s^4+4.06s^3+5.961s^2+7.676s+7.741\\ &=(s+2.2389)(s+1.8263)(s-0.0026+j1.376)\\ &\quad\cdot(s-0.0026-j1.376)\end{aligned}$$

which is unstable

IV. The Third Counterexample: Mapping into Polynomials

Let $\mathbb{M}$ be a hyper-rectangle in $R^{n\times n}$ associated with an interval matrix having entries

$$m_{ij}^-\le m_{ij}\le m_{ij}^+;\qquad i, j=1, 2, \cdots, n$$

where m_{ij}^- and m_{ij}^+ are prescribed bounds for the ijth element of the matrix M. Let $M_1, M_2, \cdots, M_m$ be the vertexes of $\mathbb{M}$ obtained by considering only the extreme values of the m_{ij}, and consider the characteristic polynomials $p_1(s), p_2(s), \cdots, p_m(s)$ associated with these extreme points. Now, as indicated in the remark in Section I,

$$\text{conv}\ \mathbb{P}_{\mathbb{M}}=\left\{p_\lambda(s)=\sum_{i=1}^{m}\lambda_i p_i(s) : \lambda_i\ge 0,\ i=1, 2, \cdots, m;\ \sum_{i=1}^{m}\lambda_i=1\right\}.$$

We now provide an example to show that $\mathbb{M}$ might be strictly Hurwitz but conv $\mathbb{P}_{\mathbb{M}}$ might contain unstable polynomials. Indeed, consider the interval matrix described by

$$M=\begin{bmatrix}m_{11} & 0 & 0\\ 0 & m_{22} & m_{23}\\ 0 & m_{32} & m_{33}\end{bmatrix}$$

where

$$-2.4780 \leq m_{11} \leq -1.4471;$$
$$-0.0518 \leq m_{22} \leq -0.0194;$$
$$2.000 \leq m_{23} \leq 3.4370;$$
$$m_{32} = -0.7115;$$
$$-0.0026 \leq m_{33} \leq -0.0012.$$

We first show that this hyper-rectangle is strictly Hurwitz, i.e., that every matrix $M \in \mathbb{M}$ is strictly Hurwitz. Since M is block diagonal, we can write

$$M = \begin{bmatrix} M_{11} & 0 \\ 0 & M_{22} \end{bmatrix},$$

where

$$M_{11} = m_{11};$$
$$M_{22} = \begin{bmatrix} m_{22} & m_{23} \\ m_{32} & m_{33} \end{bmatrix}.$$

Next note that the eigenvalues of M are the union of eigenvalues of M_{11} and the eigenvalues of M_{22}, and moreover, the eigenvalue of M_{11} is negative real for all admissible variations of m_{11}. Also, a straightforward calculation yields

$$\det(sI - M_{22}) = s^2 - (m_{22} + m_{33})s + m_{22}m_{33} - m_{23}m_{32}.$$

Using the given bounds on the m_{ij}, it is easy to verify that the coefficients of $\det(sI - M_{22})$ remain positive over the allowable range of variation. Hence, the hyper-rectangle $\mathbb{M}$ is strictly Hurwitz.

To show that conv $\mathbb{P}_{\mathbb{M}}$ is not strictly Hurwitz, consider the extreme matrices

$$M_1 = \begin{bmatrix} -2.478 & 0 & 0 \\ 0 & -0.0194 & 3.437 \\ 0 & -0.7115 & -0.0026 \end{bmatrix};$$
$$M_2 = \begin{bmatrix} -1.4471 & 0 & 0 \\ 0 & -0.0518 & 2.0 \\ 0 & -0.7115 & -0.0012 \end{bmatrix}$$

in $\mathbb{M}$. Now it is easily verified that the characteristic polynomials of M_1 and M_2 are given by

$$p_{M_1}(s) = \det(sI - M_1) = s^3 + 2.5s^2 + 2.5s + 6.06;$$
$$p_{M_2}(s) = \det(sI - M_2) = s^3 + 1.5s^2 + 1.5s + 2.06$$

and the convex combination

$$\frac{1}{2} p_{M_1}(s) + \frac{1}{2} p_{M_1}(s) = s^3 + 2.0s^2 + 2.0s + 4.06$$

is unstable. Hence, the polytope conv $\mathbb{P}_{\mathbb{M}}$ contains an unstable polynomial.

V. Conclusion

The three counterexamples presented in this paper indicate that some obvious lines of attack on the matrix polytope stability problem will fail.

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Asymptotic Stability for a Class of Linear Discrete Systems with Bounded Uncertainties

MAGDI S. MAHMOUD AND AHMED A. BAHNASAWI

Abstract—**The problem of stabilizing linear discrete systems with additive-type bounded uncertainties is considered. It is established that when matching conditions hold a two-part feedback control can be designed: a linear part to assign the eigenvalues within the unit circle and a nonlinear part to ensure the uniform asymptotic stability for arbitrary initial conditions and bounded admissible uncertainties.**

I. Introduction

Regulation of discrete systems with unknown bounded parameters and/or parasitic elements (henceforth termed uncertain systems) is a problem of paramount importance in computer control applications [1], [2]. Several approaches have been developed [2]-[5] to characterize the uncertainty, and subsequently, deal with different aspects of the cited problem. In [2], an overview of adaptive control techniques has been presented. Stability properties of reduced-order adaptive systems for singularly perturbed discrete plants with bounded fast parasitics has been investigated in [4]. An in-depth study of adaptive model-reference identification schemes and their performance behavior has been carried out in [5]. The present work extends the idea of [3] and examines the problem of stabilizing linear discrete systems with a class of additive-type uncertainties. This class belongs to the set of unknown but bounded parameters of the system and input matrices. When matching conditions hold, it is established that unstable discrete systems can be stabilized by a two-part feedback control: a linear part to move the eigenvalues into the unit disk and a nonlinear part to ensure the uniform asymptotic stability of the original system.

II. Problem Statement

Consider a dynamical system that has an additive type of uncertainty shown in Fig. 1 and described by the following difference equation:

$$x(k+1) = [A + \Delta A(r(k))]x(k) + [B + \Delta B(s(k))]u(k) + cv(k)$$
$$x(k_o) = x_o \qquad (1)$$

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