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Brief paper

Attainability of the minimum data rate for stabilization of linear systems via logarithmic quantization☆

Keyou You[a], Weizhou Su[b], Minyue Fu[c], Lihua Xie[a,*]

[a] School of Electrical and Electronic Engineering, Nanyang Technological University, 639798, Singapore
[b] School of Automation Science and Engineering, South China University of Technology, Guangzhou, China
[c] Department of Control Science and Engineering, Zhejiang University, Hangzhou, China

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ABSTRACT

This paper investigates the attainability of the minimum average data rate for stabilization of linear systems via logarithmic quantization. It is shown that a finite-level logarithmic quantizer suffices to approach the well-known minimum average data rate for stabilizing an unstable linear discrete-time system under two basic network configurations. In particular, we derive explicit finite-level logarithmic quantizers and the corresponding controllers to approach the minimum average data rate.

1. Introduction

There has been a lot of interest in quantized feedback control in recent years due to the emergence of networked control systems. The idea of modeling the quantization error as an additive white Gaussian noise began to be challenged in the new environment where only very coarse information is allowed to propagate through the network due to limited network bandwidth or for the purpose of energy saving, e.g., in wireless sensor networks.

The change of view on quantization can be traced back to the paper (Delchamps, 1990) where the author treated quantization as partial information of the quantized entity rather than its approximation, and demonstrated the significance of the historical values of the quantizer output. Since then, various methods for studying quantization effects on control and estimation have been developed.

The research on quantized feedback control can be categorized depending on whether the quantizer is static or dynamic. A static quantizer is a memoryless nonlinear function while a dynamic quantizer uses memory and is more complicated and potentially more powerful. Following Delchamps (1990), Brockett and Liberzon (2000) studied a dynamic finite-level uniform quantizer for stabilization and pointed out that there exist a dynamic adjustment policy for quantizer sensitivity and a quantized state feedback controller to asymptotically stabilize an unstable linear system. This raised a fundamental question: how much information needs to be communicated between the quantizer and the controller in order to stabilize an unstable linear system? Various authors have addressed this problem under different scenarios, e.g., Baillieul (2002), Nair and Evans (2004), Tatikonda and Mitter (2004) and Wong and Brockett (1999), and the appealing data rate theorem states that the minimum average data rate required for stabilization is given by the following inequality:

$$R > \sum_{|\lambda(A)|\geq 1} \log_2 |\lambda(A)| \triangleq R_{\min}, \tag{1}$$

where $\lambda(A)$ denotes an eigenvalue of the open-loop system matrix A. To achieve the minimum data rate, a dynamic quantizer is needed.

Perhaps, one of the most interesting static quantizers is the so-called logarithmic quantizer introduced by Elia and Mitter (2001) and Fu and Xie (2005), which gives the coarsest quantization density for quadratic stabilization of an unstable single input linear system. However, the use of static logarithmic quantizers

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* Corresponding author. Tel.: +65 67904524; fax: +65 67920415.
E-mail addresses: youk0001@ntu.edu.sg (K. You), wzhsu@scut.edu.cn (W. Su), minyue.fu@newcastle.edu.au (M. Fu), elhxie@ntu.edu.sg (L. Xie).

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requires an infinite data rate. This problem is resolved by Fu and Xie (2009), which shows that an unstable linear system can be stabilized using a fixed-rate finite-level logarithmic quantizer with a dynamic scaling. Though the study of logarithmic quantizers constitutes a vast body of the literature, e.g., Ceragioli and De Persis (2007), Carli, Bullo, and Zampieri (2010), Elia and Mitter (2001), Fu and Xie (2005, 2009, 2010), Gao and Chen (2007), Hayakawa, Ishii, and Tsumura (2009) Liu and Elia (2004) and Tsumura, Ishii, and Hoshina (2009), it is unclear whether a logarithmic quantizer can approach the well-known minimum average data rate required for stabilizing an unstable linear system. This problem is of interest due to the better efficiency of a logarithmic quantizer in terms of data rate for performance control than a uniform quantizer. Furthermore, floating-point quantization whose relative quantization error is uniformly bounded and independent of the quantizer input (Widrow, Kollar, & Liu, 1996) may be treated as logarithmic quantization. Note that, scientific calculations are almost exclusively implemented by using floating-point roundoff and also more and more digital signal processors contain floating-point arithmetic (Widrow et al., 1996). It is practically important to account for logarithmic quantization effects.

The above motivates the study on the attainability of the minimum average data rate for stabilization of linear systems via a finite-level logarithmic quantizer. Precisely, we ask the following question: does a logarithmic quantizer require an average data rate higher than the minimum average data rate for stabilization? The main contribution of this paper shows that the answer is negative. The result is confirmed by showing that the use of a finite-level logarithmic quantizer with a variable data rate can approach the minimum average data rate. We also show that, in the absence of disturbance, asymptotic stabilization can be achieved via the same logarithmic quantizer with a simple dynamic scaling.

The rest of the paper is organized as follows. The problem of interest is formulated in Section 2. Section 3 constitutes the main part of the paper where the attainability of the minimum average data rate via logarithmic quantization is proved. Concluding remarks are drawn in Section 4. A technical lemma is given in the Appendix.

2. Problem formulation

Consider a discrete linear time-invariant unstable system

$$x_{k+1} = Ax_k + Bu_k + w_k, \quad \forall k \in \mathbb{N}, \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the measurable state, $u_k \in \mathbb{R}$ is the control input, and $w_k \in \mathbb{R}^n$ is a uniformly bounded disturbance input, i.e., $\|w_k\|_\infty \le d, \forall k \in \mathbb{N}$, where $\|\cdot\|_\infty$ is the l^∞ norm for vectors or the induced matrix norm for matrices. Without loss of generality, assume that $A \in \mathbb{R}^{n\times n}$ has two distinct real Jordan blocks, i.e., $A = \text{diag}(J_1, J_2)$, where $J_i \in \mathbb{R}^{n_i\times n_i}$ corresponds to one unstable real eigenvalue $\lambda_i \in \mathbb{R}$ or a pair of unstable complex conjugate eigenvalues $\lambda_i, \lambda_i^* \in \mathbb{C}$ and $|\lambda_1| \neq |\lambda_2|$. Moreover, (A, B) is a controllable pair.

Remark 1. There is no loss of generality to focus on the system of the form (2). In fact, consider a system as follows:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + w_k, \\ y_k = Cx_k + v_k, \end{cases} \quad \forall k \in \mathbb{N}, \tag{3}$$

where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}^l$ is the control input, $y_k \in \mathbb{R}^p$ is the output, $w_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^p$ are bounded additive disturbances. (A, B) and (C, A) are stabilizable and detectable pairs, respectively, and $\text{rank}(B) = l \le n$.

Assume that all the eigenvalues of A lie outside or on the unit circle. Otherwise, the matrix A can be transformed to a block diagonal form $\text{diag}\{A_s, A_u\}$ by a coordinate transformation, where A_s and A_u respectively correspond to the stable and unstable (including marginally unstable) subspaces. State variables associated with the stable block A_s will converge to a bounded region for any bounded control sequence. Thus, without loss of generality, we assume that A has all eigenvalues lie outside or on the unit circle and (A, B, C) are controllable and observable.

Then, a deadbeat observer (Chen, 1984) can be constructed to estimate the state of the system. The estimation error will be uniformly bounded after n steps and independent of the initial state. Hence, it is sensible to focus on the state feedback case.

By applying the Wonham decomposition to (3) (Chen, 1984), one can convert the multiple inputs system to l single input ones. More specifically, there is a nonsingular real matrix $T \in \mathbb{R}^{n\times n}$ such that $\bar{A} = T^{-1}AT$ and $\bar{B} = T^{-1}B$ take the form: $\bar{A} = \begin{bmatrix} A_1 & A_{12} & \cdots & A_{1l} \\ 0 & A_2 & \cdots & A_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_l \end{bmatrix}, \bar{B} = \begin{bmatrix} B_1 & B_{12} & \cdots & B_{1l} \\ 0 & B_2 & \cdots & B_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_l \end{bmatrix}$, where (A_i, B_i) with $A_i \in \mathbb{R}^{n_i\times n_i}$ and $B_i \in \mathbb{R}^{n_i}$, $i \in \{1, \ldots, l\}$, is a controllable pair and $\sum_{i=1}^{l} n_i = n$. For illustration and brevity, let $l = 2$ and assume that the state feedback system is already given by $x_{k+1} = \bar{A}x_k + \bar{B}u_k + w_k$. By partitioning the state $x_k \triangleq [(x_k^1)', (x_k^2)']'$ in conformity with the upper triangular form of $\bar{A}$, two single input subsystems are written as

$$x_{k+1}^1 = A_1x_k^1 + B_1u_k^1 + A_{12}x_k^2 + B_{12}u_k^2 + w_k^1; \tag{4}$$

$$x_{k+1}^2 = A_2x_k^2 + B_2u_k^2 + w_k^2. \tag{5}$$

If x_k^2 is stabilized with a communication data rate greater than $\sum_{\lambda(A_2)} \log_2 |\lambda(A_2)|$, then $\|x_k^2\|_\infty$ will be uniformly bounded and can be treated as a bounded disturbance input to subsystem (4), which can be stabilized similarly with a data rate greater than $\sum_{\lambda(A_1)} \log_2 |\lambda(A_1)|$. As in Nair and Evans (2004, Section 3), it is convenient to put A_2 into real Jordan canonical form so as to decouple its unstable dynamical modes. Consequently, it is sufficient to focus on the system of form (2) since the extension of system (2) to the case with more than two Jordan blocks can be easily carried out.

We now recall logarithmic quantizers.

Definition 1 (*Fu & Xie, 2005*)**.** A quantizer is called a *logarithmic* quantizer if it has the form:

$$Q_\infty(v) = \begin{cases} u^{(i)}, & \text{if } \dfrac{1}{1+\delta}u^{(i)} < v \le \dfrac{1}{1-\delta}u^{(i)}, \; v > 0; \\ 0, & \text{if } v = 0; \\ -Q_\infty(-v), & \text{if } v < 0 \end{cases} \tag{6}$$

where $u^{(i)}$, $i \in \mathbb{N}$, are from the set:

$$\mathcal{U} = \{\pm u^{(i)} : u^{(i)} = \rho^i u^{(0)}, i = \pm 1, \pm 2, \ldots\} \cup \{\pm u^{(0)}\} \cup \{0\},$$
$$0 < \rho < 1, \; u^{(0)} > 0, \tag{7}$$

ρ represents the quantizer *density* and $\delta = \frac{1-\rho}{1+\rho}$.

However, the logarithmic quantizer in (6) has an infinite number of quantization levels and needs an infinite number of bits to represent the quantizer output. Define a $(2N+2)$-level logarithmic quantizer with density $\rho \in (0, 1)$ as follows:

$$Q_N(v) = \begin{cases} \rho^i(1-\delta), & \text{if } \rho^{i+1} < v \le \rho^i, \; 0 \le i \le N-1; \\ 0, & \text{if } 0 \le v \le \rho^{N-1}; \\ -Q_N(-v), & \text{if } -1 \le v < 0, \\ 1, & \text{otherwise.} \end{cases} \tag{8}$$

In the above, we have chosen $u^{(0)} = \frac{2\rho}{1+\rho}$ in (6) and for any $v \notin [-1, 1]$, the alarm level 1 is used to indicate the overflow of the

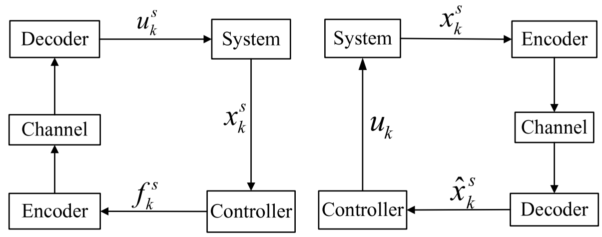

Fig. 1. Configuration I (left) vs. Configuration II (right).

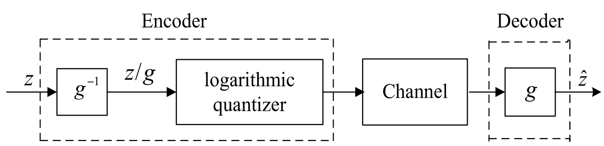

Fig. 2. Encoder/decoder pair for a digital channel: $g > 0$ is a scaling factor.

quantizer. Thus, the number of bits to represent each quantizer output is $\lceil \log_2(2N+2) \rceil$, where $\lceil \cdot \rceil$ is the standard ceiling function, i.e., $\lceil x \rceil = \min\{l \in \mathbb{Z} | l \geq x\}$.

Two basic network configurations shown in Fig. 1 are to be studied. *Configuration* I refers to the scenario where the downlink channel has limited bandwidth while in *Configuration* II, the uplink channel has limited bandwidth. Thus, the output of the controller in *Configuration* I, which is a scalar for system (2), is to be quantized. In *Configuration* II, the vector state measurement is quantized. As in Tatikonda and Mitter (2004), the encoder/decoder pair for the limited data rate communication is described in Fig. 2. The first stage of the encoding process consists of designing a scaling factor g^{-1} such that the quantizer input z/g is within the quantization range. The output of the finite-level logarithmic quantizer $Q_N(z/g)$, which takes values from the set $\{\pm\rho^i(1-\delta) : i = 0, \ldots, N-1\} \cup \{0, 1\}$, is encoded into a binary sequence and transmitted via a limited data rate communication channel. The decoder receives the binary sequence and decodes it as $Q_N(z/g)$ since we neglect transmission errors of the channel. The quantizer output is then scaled back by g, i.e., $\hat{z} = gQ_N(z/g)$ to recover z if $Q_N(z/g) \neq 1$. In *Configuration* I, z and $\hat{z}$ respectively correspond to f_k^s and u_k^s. While in *Configuration* II, z is a vector and corresponds to x_k^s, which is the state of the system after down sampling. Thus, the quantizer in Fig. 2 is a product quantizer and consists of n finite-level logarithmic quantizers. The above notations will be defined in what follows. Note that, there is no separate channel to communicate the gain value g. The main task is to jointly design the scaling factor g, the finite-level logarithmic quantizer and the corresponding control law to approach the minimum average data rate of the channel, for stabilizing the unstable system in (2). We mention that an earlier attempt has been made on scalar systems under *Configuration* I in Fu, Xie, and Su (2008).

Remark 2. The two configurations differ in the way that *Configuration* II quantizes the state first and use the quantized state to construct the control signal whereas in *Configuration* I, the control signal is constructed using the unquantized state and then quantized by a finite-level logarithmic quantizer. From the information preservation point of view, *Configuration* II appears to generate worse control actions because quantization (or information loss) happens earlier. However, what we show in the paper is that for the purpose of stabilization, the two configurations require the same minimum average data rate, if variable rate logarithmic quantization is used.

Remark 3. The two configurations have been widely adopted in the literature. For example, Elia and Mitter (2001), Fu and Xie (2005) and Tsumura et al. (2009) focus on *Configuration* I while Brockett and Liberzon (2000), Nair and Evans (2004) and Tatikonda and Mitter (2004) are restricted to *Configuration* II. The differences in the present paper are that the quantizer in the encoder of Fig. 2 is limited to a finite-level logarithmic quantizer and we aim to approach the minimum average data rate of the channel for stabilizing system (2).

3. Attainability of the minimum average data rate via logarithmic quantization

In this section, we shall design finite-level logarithmic quantizers and the corresponding control laws to approach the minimum average data rate for stabilizing the unstable system (2) under *Configurations* I and II, respectively.

3.1. Stabilization using quantized control feedback

Theorem 4. *Consider system* (2) *and network* Configuration I *of* Fig. 1, *stabilization can be achieved based on quantized control feedback with a finite-level logarithmic quantizer if and only if the average data rate R of the channel exceeds $R_{\min}$, i.e., $R > n_1 \log_2 |\lambda_1| + n_2 \log_2 |\lambda_2|$.*

Before giving the proof, the controller and the quantizer are first proposed. Note that $|\lambda_1| \neq |\lambda_2|$, define the subset $\mathcal{L}(A) \subset \mathbb{N}$ by

$$\mathcal{L}(A) = \begin{cases} \mathbb{N}, & \text{if } \lambda_1, \lambda_2 \in \mathbb{R}, \\ \{i \in \mathbb{N} | \lambda_1^i \neq (\lambda_1^*)^i\}, & \text{if } \lambda_1 \in \mathbb{C}, \lambda_2 \in \mathbb{R}; \\ \{i \in \mathbb{N} | \lambda_2^i \neq (\lambda_2^*)^i\}, & \text{if } \lambda_1 \in \mathbb{R}, \lambda_2 \in \mathbb{C}; \\ \{i \in \mathbb{N} | \lambda_j^i \neq (\lambda_j^*)^i, j = 1, 2\}, & \text{otherwise.} \end{cases}$$

Obviously, $\mathcal{L}(A)$ has infinitely many elements. Since (A, B) is a controllable pair, it is readily verified that $(A^m, A^{m-1}B)$ is a controllable pair if $m \in \mathcal{L}(A)$. By applying the control input $u_{mk+t} = 0$, if $1 \leq t \leq m-1$, the down-sampled system of (2) with a down-sampling factor m is expressed as

$$x_{m(k+1)} = A^m x_{mk} + A^{m-1} B u_{mk} + d_k, \tag{9}$$

where $d_k = \sum_{t=0}^{m-1} A^{m-1-t} w_{mk+t}$. Due to the controllability of $(A^m, A^{m-1}B)$, (9) can be transformed into a controllable canonical form, i.e., there exists a nonsingular real matrix $P \in \mathbb{R}^{n\times n}$ that transforms (9) into the controllable canonical form:

$$x_{k+1}^s = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_n & -\alpha_{n-1} & -\alpha_{n-2} & \cdots & -\alpha_1 \end{bmatrix} x_k^s + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u_k^s + w_k^s. \tag{10}$$

Here, we define $x_k^s \triangleq P x_{mk}$, $u_k^s \triangleq u_{mk}$ and $w_k^s \triangleq P d_k$.

It is clear from (10) that if we can stabilize the last element of the vector state x_k^s, denoted by $x_k^s(n)$, then x_k^s is stabilized, which further implies the stabilization of (2) due to $m < \infty$. Thus, a deadbeat controller is proposed whose output is then quantized by a finite-level logarithmic quantizer and applied to the down-sampled system. Specifically, the quantized control input to the down-sampled system is given by

$$\begin{cases} u_k^s = gQ_N(f_k^s/g); \\ f_k^s = \begin{cases} 0, & \text{if } k < n; \\ \sum_{j=0}^{n-1} \alpha_{j+1} x_{k-j}^s(n), & \text{if } k \geq n, \end{cases} \end{cases} \tag{11}$$

where the quantization level parameter N and scaling factor $g > 0$ are to be designed.

Denote $|A| = |\lambda_1|^{n_1}|\lambda_2|^{n_2}$, it follows that there exists an $\alpha_0 > 0$ such that $|\alpha_k| \leq \alpha_0|A|^m, \forall k \in \{1, \dots, n\}$ since $|\lambda_j| \geq 1, \forall j \in \{1, 2\}$.

Proof of Theorem 4. The necessity part has been well established in Nair and Evans (2004) and Tatikonda and Mitter (2004). Only the sufficiency needs to be elaborated.

First, note that given any $R > \log_2 |A|$, there exists an $\alpha > 1$ satisfying $R \geq \log_2(\alpha|A|)$. Based on Lemma 9 in the Appendix and by choosing $\beta_1 = 0$, $\beta_2 = n\alpha_0$, it is possible to select a pair of $m \in \mathcal{L}(A)$ and $N > 0$ such that $\forall \epsilon > 0$,

$$\log_2\left[1 + \frac{2\log_2(n\alpha_0|A|^m)}{\log_2 \frac{n\alpha_0|A|^m+\epsilon+1}{n\alpha_0|A|^m+\epsilon-1}}\right] < \log_2(2N+2) \leq m\log_2\alpha + \log_2|A|^m - 1. \quad (12)$$

The quantizer level parameter N is determined by (12) and the number of bits required to represent each quantizer output is $\lceil \log_2(2N+2)\rceil$. The quantizer works as follows. At time k, the quantizer first detects the overflow of $x_k^s(n)$ and then proceeds to detect the overflow of f_k^s/g. Precisely, if $|x_k^s(n)| > \triangle$ is detected, it generates the alarm level 1 and in this case there is no need to further check f_k^s/g. Here the parameters g and $\triangle$ are to be determined later. Otherwise, it continues to check f_k^s/g. If $|f_k^s/g| > 1$ is detected, the quantizer generates the alarm level 1. Thus, the alarm level 1 will be generated if either $|x_k^s(n)| > \triangle$ or $|f_k^s/g| > 1$.

It is verified from (12) that the average data rate of this protocol satisfies

$$\frac{\lceil \log_2(2N+2)\rceil}{m} \leq \log_2(\alpha|A|) \leq R.$$

Since R is any given number greater than $\log_2|A|$, the average data rate of the proposed quantizer can be made arbitrarily close to $\log_2|A|$. Thus, what remains to be proved is the stability.

Mathematical induction arguments are adopted to show that $\limsup_{k\to\infty} |x_k^s(n)| < \infty$ for any given initial condition.

First, assume $|x_k^s(n)| \leq \triangle, \forall k \in \{0, \dots, n-1\}$, which will be relaxed later. Then, for any $k \geq n$, assume that $|x_j^s(n)| \leq \triangle, \forall j \leq k$, it is obvious that $\forall j \in \{n, n+1, \dots, k\}$, $|f_j^s| = |\sum_{t=0}^{n-1}\alpha_{t+1}x_{j-t}^{(n)}| \leq n\alpha_0|A|^m\triangle \triangleq g$. Since $|f_j^s/g| \leq 1$ and $|x_j^s(n)| \leq \triangle, \forall j \leq k$, no alarm level 1 occurs before time k. From (10), there exist vectors $c_j \in \mathbb{R}^n, j \in \{0, \dots, n-1\}$, such that $s_k^s = \sum_{j=0}^{n-1} c_j^T w_{k-j}^s$ and the down-sampled system is expressed by

$$x_{k+1}^s(n) = -\sum_{j=0}^{n-1}\alpha_{j+1}x_{k-j}^s(n) + u_k^s + s_k^s, \quad k \geq n. \quad (13)$$

Moreover, $|s_k^s| \leq \sum_{j=0}^{n-1}\|c_j^T\|_\infty\|w_{k-j}^s\|_\infty \triangleq \tilde{s}, \forall k \in \mathbb{N}$. Choose the quantizer density parameters $\delta = \frac{1}{n\alpha_0|A|^m+\epsilon}$, $\rho = \frac{1-\delta}{1+\delta}$ and $\triangle > 0$ to satisfy that

$$\triangle > \max\left\{\frac{\tilde{s}}{1-(n\alpha_0|A|^m)^2\rho^{2N+1}}, \frac{\tilde{s}}{1-n\alpha_0|A|^m\delta}\right\}. \quad (14)$$

In light of (12), it is easy to verify that

$$\begin{cases}(n\alpha_0|A|^m)^2\rho^{2N+1} < 1\\ n\alpha_0|A|^m\delta < 1.\end{cases} \quad (15)$$

Inserting the quantized control in (11) into system (13) results in

$$\begin{aligned}|x_{k+1}^s(n)| &\leq \begin{cases}|f_k^s| + \tilde{s}, & \text{if } |f_k^s/g| \leq \rho^N\\ \delta|f_k^s| + \tilde{s}, & \text{if } \rho^N < |f_k^s/g| \leq 1\end{cases}\\ &\leq \begin{cases}n\alpha_0|A|^m\rho^N\triangle + \tilde{s}, & \text{if } |f_k^s/g| \leq \rho^N\\ n\alpha_0|A|^m\delta\triangle + \tilde{s}, & \text{if } \rho^N < |f_k^s/g| \leq 1\end{cases}\\ &\leq \triangle \text{ due to the selection of } \triangle \text{ in (14) and (15).}\end{aligned}$$

Inductively, $|x_k^s(n)| \leq \triangle, \forall k \in \mathbb{N}$. Next, assume that the assumption that $|x_j^s(n)| \leq \triangle, \forall j \in \{0, \dots, n-1\}$ is violated, which can be detected by the decoder via the alarm level. Denote the *first* time of receiving the alarm level 1 by k. Choose a scaling factor

$$\gamma = \sum_{j=1}^{n}\alpha_j + 1 \quad (16)$$

to dynamically update the scaling factor. Specifically, set $\triangle_k = \triangle$ and update the scaling factor as follows:

$$\triangle_{k+j+1} = \begin{cases}\gamma\,\triangle_{k+j}, & \text{alarm level 1 occurs,}\\ \triangle_{k+j}, & \text{otherwise,}\end{cases}$$

which is simultaneously processed on both sides of the channel. Set $u_{k+j}^s = 0$, the increasing speed of $\triangle_{k+j}$ is thus faster than that of $x_{k+j}^s(n)$ by (10) and (16). $\triangle_{k+j}$ will eventually capture $x_{k+j}^s(n)$ or $\lim_{j\to\infty}\frac{x_{k+j}^s(n)}{\triangle_{k+j}} = 0$. Let the scaling factor be $g_{k+j} = n\alpha_0|A|^m\triangle_{k+j}$, it follows that $\lim_{j\to\infty} f_{k+j}^s/g_{k+j} = 0$, implying that there exists a finite $k_0 \geq n-1$ such that the signals received within the time period $\{k+k_0-n+1, \dots, k+k_0\}$ do not give rise to the alarm level 1, which suggests that $|x_{k+j}^s(n)| \leq \triangle_{k+k_0}, \forall j \in \{k_0-n+1, \dots, k_0\}$. Then, repeating the above proof as the bounded case at time $k+k_0+n-1$ yields that $|x_{k+j}^s(n)| \leq \triangle_{k+k_0}, \forall j \geq k_0+n-1$.

Finally, it follows that $\limsup_{k\to\infty}|x_k^s(n)| < \infty$, which eventually leads to $\limsup_{k\to\infty}\|x_k\|_\infty < \infty$. □

The following corollary gives the corresponding result for asymptotic stabilization, i.e., $\lim_{k\to\infty}\|x_k\|_\infty = 0$.

Corollary 5. *Consider system (2) with $w_k = 0$ and having network* Configuration I *of Fig. 1; asymptotic stabilization can be achieved via a quantized control feedback with a finite-level logarithmic quantizer if and only if the average data rate R of the channel is strictly greater than $R_{\min}$, i.e., $R > n_1\log_2|\lambda_1| + n_2\log_2|\lambda_2|$.*

Proof. Similarly, only the sufficiency part needs to be established. Define a scaling factor

$$\eta \triangleq \max\{(n\alpha_0|A|^m)^2\rho^{2N+1}, n\alpha_0|A|^m\delta\}, \quad (17)$$

which is strictly less than one by (15), i.e., $\eta < 1$. Let $\triangle_{k+1} = \eta\,\triangle_k$ with an *arbitrary* $\triangle_0 > 0$, which is assumed to be agreed by both the quantizer and the decoder. Assume that $|x_k^s(n)| \leq \triangle_0, \forall k \in \{0, \dots, n-1\}$, the control input and quantizer are given in Theorem 4 with $\triangle$ replaced by $\triangle_k$. Then, it is straightforward that $|x_{k+1}^s(n)| \leq \eta\,\triangle_k = \triangle_{k+1}$. Thus, $x_k^s(n)$ can be driven exponentially to zero since $\lim_{k\to\infty}|x_k^s(n)| \leq \triangle_0\lim_{j\to\infty}\eta^j = 0$. Due to $m < \infty$, it follows that $\lim_{k\to\infty}\|x_k\|_\infty = 0$. The removal of the boundedness assumption for the initial state is similar to what we have done in Theorem 4. □

3.2. Stabilization using quantized state feedback

We proceed to validate the attainability of the minimum average data rate under *Configuration* II via logarithmic quantization where the control design solely relies on the quantized state. Intuitively, this might require a larger average data rate since the quantized state contains less information than its unquantized version. However, the result of this subsection shows that the logarithmic quantizer can still approach the minimum average data rate.

Theorem 6. *Consider system (2) and network* Configuration II *of Fig. 1, stabilization can be achieved based on the quantized state feedback with a finite-level logarithmic quantizer if and only if the average data rate R of the channel exceeds $R_{\min}$, i.e., $R > n_1\log_2|\lambda_1| + n_2\log_2|\lambda_2|$.*

In this case, two scalar logarithmic quantizers with appropriately chosen parameters are designed and applied to the down-sampled state x_k^s of (2), where the down-sampling factor $m \geq 2n$ is to be determined later, more precisely, indexing the scalar components of the state of (2) by an additional superscript $h \in \{1, \dots, n\}$. At time $t = mk + h - 1$, the hth element of x_k^s will be quantized by $Q_{N_1}(x_k^s(h)/\triangle)$ if $h \leq n_1$ and $Q_{N_2}(x_k^s(h)/\triangle)$ otherwise, where the quantization level parameters N_i and $\triangle$ are determined by the available data rate. Neglecting the transmission time implies that the quantized x_k^s can reach the controller before time $mk+n$. Since $m \geq 2n$, the control law within one cycle $\{mk, \dots, m(k+1)-1\}$ can be proposed as follows:

$$\begin{cases} \begin{bmatrix} u_{mk+m-1} \\ \vdots \\ u_{mk+m-n} \end{bmatrix} = -\triangle \mathcal{C}^T(\mathcal{C}\mathcal{C}^T)^{-1}A^m Q\left(\dfrac{x_k^s}{\triangle}\right), \\ u_{mk+t} = 0, \quad \forall t \in \{0, \dots, m-n-1\}, \end{cases} \tag{18}$$

where the controllability matrix $\mathcal{C}$ is defined as $\mathcal{C} \triangleq [B, AB, \dots, A^{n-1}B]$ and the product quantizer $Q(\cdot)$ is composed by

$$Q(\cdot) = [\underbrace{Q_{N_1}(\cdot), \dots, Q_{N_1}(\cdot)}_{n_1}, \underbrace{Q_{N_2}(\cdot), \dots Q_{N_2}(\cdot)}_{n_2}]^T.$$

Lemma 7. *There is a positive ζ such that for any $m \in \mathbb{N}$,*

$$\|J_i^m\|_\infty \leq \zeta\sqrt{n_i}m^{n_i-1}|\lambda_i|^m, \quad \forall i \in \{1, 2\}. \tag{19}$$

Proof. It is known that there exists a $\zeta > 0$, independent of J_i, n_i, and m_i, such that $\|J_i\| \leq \zeta m_i^{n_i-1}|\lambda_i|^m$, where $\|\cdot\|$ is the spectral norm induced from the Euclidean norm (Nair & Evans, 2004). Together with the fact that $\|J_i\|_\infty \leq \sqrt{n_i}\|J_i\|$, (19) is immediately inferred. □

Proof of Theorem 6. As in the case of *Configuration* I, only the sufficiency part requires to be proved. Given any $R > n_1 \log_2|\lambda_1| + n_2\log_2|\lambda_2|$, there exists an $\alpha > 1$ satisfying $R = R_1 + R_2$ and $R_i \geq n_i\log_2(\alpha|\lambda_i|)$, $\forall i \in \{1, 2\}$. In view of Lemma 9 in the Appendix, for any $\epsilon > 0$, we can choose a pair of integers $m \geq 2n$ and N_i satisfying:

$$\log_2\left[1 + \frac{2\log_2 \zeta\sqrt{n_i}m^{n_i-1}|\lambda_i|^m}{\log_2 \frac{\zeta\sqrt{n_i}m^{n_i-1}|\lambda_i|^m+\epsilon+1}{\zeta\sqrt{n_i}m^{n_i-1}|\lambda_i|^m+\epsilon-1}}\right] < \log_2(2N_i+2)$$
$$\leq m\log_2\alpha + \log_2|\lambda_i|^m - 1, \quad \forall i \in \{1, 2\}. \tag{20}$$

The quantization level parameter N_i is selected based on (20). The average data rate of this protocol is computed by $\frac{n_1\lceil\log_2(2N_1+2)\rceil + n_2\lceil\log_2(2N_2+2)\rceil}{m} \leq \log_2(\alpha|A|) \leq R$, implying that the minimum average data rate can be approached by the above protocol. Also, the quantizer density parameters for $Q_{N_i}(\cdot)$ are chosen by $\delta_i = \frac{1}{\zeta\sqrt{n_i}m^{n_i-1}|\lambda_i|^m+\epsilon}$ and $\rho_i = \frac{1-\delta_i}{1+\delta_i} = \frac{\zeta\sqrt{n_i}m^{n_i-1}|\lambda_i|^m+\epsilon-1}{\zeta\sqrt{n_i}m^{n_i-1}|\lambda_i|^m+\epsilon+1}$, which gives that

$$\begin{cases} (\zeta\sqrt{n_i}m^{n_i-1})^2\rho_i^{2N_i+1} < 1, \\ \delta_i\zeta\sqrt{n_i}m^{n_i-1} < 1. \end{cases} \tag{21}$$

Define the uniform upper bound of d_k in (9) by $D \triangleq d\sum_{t=0}^{m-1}\|A\|_\infty^{m-1-t}$, then $\|d_k\|_\infty \leq D$, $\forall k \in \mathbb{N}$. Similarly, the initial state x_0 is assumed to be bounded by $\triangle > 0$, where $\triangle$ is selected to satisfy

$$\triangle \geq \max_{i\in\{1,2\}}\left\{\frac{D}{1-(\zeta\sqrt{n_i}m^{n_i-1})^2\rho_i^{2N_i+1}}, \frac{D}{1-\delta_i\zeta\sqrt{n_i}m^{n_i-1}}\right\}. \tag{22}$$

Inserting the control law in (18) into (2) yields that

$$\begin{aligned} x_{k+1}^s &= A^m x_k^s + \sum_{t=0}^{m-1}A^{m-1-t}(Bu_{mk+t} + w_{mk+t}) \\ &= A^m x_k^s + \sum_{t=m-n}^{m-1}A^{m-1-t}Bu_{mk+t} + d_k \\ &= A^m\left[x_k^s - \triangle Q\left(\frac{x_k^s}{\triangle}\right)\right] + d_k. \end{aligned} \tag{23}$$

Assuming that $\|x_k^s\|_\infty \leq \triangle$, there is no alarm level 1 for the scaled state $x_k^s(h)$ by the scaling factor $\triangle$. Denote by $(x_k^s)^{(1)}$ the state vector consisting of the first n_1 elements of x_k^s while $(x_k^s)^{(2)}$ is the state vector by collecting the remaining elements of x_k^s. Similar notations will be given for $Q^{(i)}$ and $d_k^{(i)}$, $i \in \{1, 2\}$. Consider system (23), we obtain:

$$\begin{aligned} \|(x_{k+1}^s)^{(i)}\|_\infty &= \|J_i^m\left[(x_k^s)^{(i)} - \triangle Q^{(i)}\left(\frac{(x_k^s)^{(i)}}{\triangle}\right)\right] + d_k^{(i)}\|_\infty \\ &\leq \|J_i^m\|_\infty\left\|(x_k^s)^{(i)} - \triangle Q^{(i)}\left(\frac{(x_k^s)^{(i)}}{\triangle}\right)\right\|_\infty + D \\ &\leq \begin{cases} \|J_i^m\|_\infty\|(x_k^s)^{(i)}\|_\infty + D, \\ \quad \text{if } \|(x_k^s)^{(i)}/\triangle\|_\infty \leq \rho_i^{N_i-1} \\ \|J_i^m\|_\infty\delta_i\|(x_k^s)^{(i)}\|_\infty + D, \\ \quad \text{if } \rho_i^{N_i-1} < \|(x_k^s)^{(i)}/\triangle\|_\infty \leq 1 \end{cases} \\ &\leq \begin{cases} \zeta\sqrt{n_i}m^{n_i-1}\rho_i^{N_i-1}\triangle + D, \\ \quad \text{if } \|(x_k^s)^{(i)}/\triangle\|_\infty \leq \rho_i^{N_i-1} \\ \zeta\sqrt{n_i}m^{n_i-1}\delta_i\triangle + D, \\ \quad \text{if } \rho_i^{N_i-1} < \|(x_k^s)^{(i)}/\triangle\|_\infty \leq 1. \end{cases} \\ &\leq \triangle \quad \text{by (21) and (22).} \end{aligned}$$

Inductively, $\|x_k^s\|_\infty \leq \triangle, \forall k \in \mathbb{N}$. Since $m < \infty$, it follows that $\limsup_{k\to\infty}\|x_k\|_\infty < \infty$.

The removal of the boundedness assumption of $\|x_0^s\|_\infty \leq \triangle$ is similar to the case of *Configuration* I and is omitted. □

Remark 8. It is worth mentioning that the parallel result in Corollary 5 can be given under *Configuration* II. Furthermore, the attainability of the logarithmic quantization can also be established for the case where quantization appears in both the state measurement and in the control signal.

4. Conclusion

We have addressed the attainability of the logarithmic quantizer in the sense of approaching the minimum average data rate for stabilizing an unstable discrete-time linear system. For any average data rate greater than the minimum rate, given by the data rate theorem, a finite-level logarithmic quantizer and a controller were constructed to stabilize the system under two different network configurations with different schemes of quantizer bits assignment. It should be noted that since our main concern is the attainability of the minimum average data rate by logarithmic quantization, our proposed control law and quantizer may produce a poor transient response. The study of performance control via finite-level logarithmic quantization will be our future work.

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Appendix. A technical lemma

Define $\kappa(m,\lambda)=(\beta_1 m^{n-1}+\beta_2)|\lambda|^m$, $\forall n\geq 1$, $\beta_1,\beta_2\geq 0$ and $\beta_1+\beta_2>0$, we have the following result.

Lemma 9. *$\forall\alpha>1$, $\forall\epsilon>0$ and $|\lambda|\geq 1$, there exist positive integers m and N such that*

$$\log_2\left[1+\frac{2\log_2\kappa(m,\lambda)}{\log_2\frac{\kappa(m,\lambda)+\epsilon+1}{\kappa(m,\lambda)+\epsilon-1}}\right]<\log_2(2N+2)\leq m\log_2\alpha+\log_2|\lambda|^m-1. \tag{24}$$

Proof. It is trivial if $|\lambda|=1$ and $\beta_1=0$. Assume $|\lambda|>1$ or $\beta_1>0$, then $\kappa(m,\lambda)\to\infty$ as $m\to\infty$. Jointly with the fact that $\lim_{x\to\infty}(1+\frac{1}{x})^x=e$ yields

$$\log_2\frac{\kappa(m,\lambda)+\epsilon+1}{\kappa(m,\lambda)+\epsilon-1}\cong 2\kappa^{-1}(m,\lambda)\log_2 e, \tag{25}$$

if m is sufficiently large. Next, two cases are discussed.

Case 1: $\beta_1>0$, $\beta_2\geq 0$ and $|\lambda|\geq 1$.

Selecting a large $m\geq 1$ such that $\ln\kappa(m,\lambda)\geq 1$, we have

$$\begin{aligned}&\left(1+\kappa(m,\lambda)\ln\kappa(m,\lambda)\right)^{1/m}\\&\quad=\left(1+(\beta_1 m^{n-1}+\beta_2)|\lambda|^m\ln\kappa(m,\lambda)\right)^{1/m}\\&\quad\geq|\lambda|\beta_1^{1/m}(m^{1/m})^{n-1}(\ln\kappa(m,\lambda))^{1/m}\\&\quad\geq|\lambda|\beta_1^{1/m}\to|\lambda|\quad\text{as } m\to\infty\end{aligned}$$

due to that $\lim_{m\to\infty}x^{1/m}=1$, $\forall x>0$. On the other hand, choosing a large m such that $\kappa(m,\lambda)\ln\kappa(m,\lambda)\geq 1$, $\beta_1 m^n\geq\beta_2$ and $2\beta_1 m^n\geq\ln(2\beta_1 m^n)$ gives the following inequalities:

$$\begin{aligned}&\left(1+\kappa(m,\lambda)\ln\kappa(m,\lambda)\right)^{1/m}\leq\left(2\kappa(m,\lambda)\ln\kappa(m,\lambda)\right)^{1/m}\\&\quad\leq|\lambda|(4\beta_1)^{1/m}(m^{1/m})^n[m\ln|\lambda|+\ln(2\beta_1 m^n)]^{1/m}\\&\quad\leq|\lambda|(4\beta_1)^{1/m}(m^{1/m})^n[m^n\ln|\lambda|+2\beta_1 m^n]^{1/m}\\&\quad=|\lambda|[4\beta_1(\ln|\lambda|+2\beta_1)]^{1/m}(m^{1/m})^{2n}\\&\quad\to|\lambda|\quad\text{as } m\to\infty,\end{aligned}$$

due to that $\lim_{m\to\infty}m^{1/m}=1$.

Case 2: $\beta_1=0$, $\beta_2>0$ and $|\lambda|>1$.

Let $m\geq 1$, it immediately follows that

$$\begin{aligned}\left(1+\kappa(m,\lambda)\ln\kappa(m,\lambda)\right)^{1/m}&\geq|\lambda|\beta_2^{1/m}(\ln\beta_2+m\ln|\lambda|)^{1/m}\\&\geq|\lambda|\beta_2^{1/m}(\ln\beta_2+\ln|\lambda|)^{1/m}\\&\to|\lambda|\quad\text{as } m\to\infty.\end{aligned}$$

Also, for a sufficiently large m, e.g., $\kappa(m,\lambda)\ln\kappa(m,\lambda)\geq 1$ and $m\geq\ln\beta_2/\ln|\lambda|$, one can establish that

$$\begin{aligned}\left(1+\kappa(m,\lambda)\ln\kappa(m,\lambda)\right)^{1/m}&\leq(2\beta_2|\lambda|^m\ln(\beta_2|\lambda|^m))^{1/m}\\&\leq|\lambda|(4\beta_2\ln|\lambda|)^{1/m}m^{1/m}\\&\to|\lambda|\quad\text{as } m\to\infty.\end{aligned}$$

Consequently, under any situation, we derive the limit

$$\lim_{m\to\infty}\left(1+\kappa(m,\lambda)\ln\kappa(m,\lambda)\right)^{1/m}=|\lambda|.$$

In the light of (25), it is clear that

$$\lim_{m\to\infty}\left[1+\frac{2\log_2\kappa(m,\lambda)}{\log_2\frac{\kappa(m,\lambda)+\epsilon+1}{\kappa(m,\lambda)+\epsilon-1}}\right]^{\frac{1}{m}}=|\lambda|, \tag{26}$$

which further implies that for a sufficiently large m,

$$\log_2\left[1+\frac{2\log_2\kappa(m,\lambda)}{\log_2\frac{\kappa(m,\lambda)+\epsilon+1}{\kappa(m,\lambda)+\epsilon-1}}\right]\cong\log_2|\lambda|^m.$$

Since $\alpha>1$, $m\log_2\alpha\to\infty$ as $m\to\infty$, the difference between the left-hand side and the right-hand side of (24) tends to infinity if $m\to\infty$. Thus, it is always possible to select m and N to satisfy (24). □

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Keyou You was born in Jiangxi Province, China, in 1985. He received the B.S. degree in statistical science from the Sun Yat-sen (Zhongshan) University, Guangzhou, China, in 2007 and is currently pursuing the Ph.D. degree in electrical and electronic engineering at Nanyang Technological University, Singapore. From May 2010 to July 2010, he was with the ARC Center of Excellence for Complex Dynamic Systems and Control, the University of Newcastle, Australia, as a visiting scholar. He won the Guan Zhaozhi best paper award at the 29th Chinese Control Conference, Beijing, China, 2010. His current research interests include quantized estimation and control, networked control, system identification and distributed control and estimation.

Weizhou Su received the B.Eng. and M.Eng. degrees in automatic control engineering from the Southeast University, Nanjing, Jiangsu, China, in 1983 and 1986, respectively; the M.Eng. degree in electrical and electronic engineering from Nanyang Technological University, Singapore, in 1996 and the Ph.D. degree in electrical engineering from the University of Newcastle, Newcastle, NSW, Australia, in 2000. From 2000 to 2004, he held research positions in Hong Kong University of Science and Technology, Hong Kong, China, the University of Newcastle, Australia and the University of Western Sydney, Australia.

In 2004, he joined the School of Automation Science and Engineering, South China University of Technology, Guangzhou, China, where he is currently a professor. His research interests include networked control, robust and optimal control, fundamental performance limitation of feedback control, and signal processing.

Minyue Fu received his Bachelor's Degree in Electrical Engineering from the University of Science and Technology of China, Hefei, China, in 1982, and M.S. and Ph.D. degrees in Electrical Engineering from the University of Wisconsin-Madison in 1983 and 1987, respectively. From 1987 to 1989, he served as an Assistant Professor in the Department of Electrical and Computer Engineering, Wayne State University, Detroit, Michigan. He joined the Department of Electrical and Computer Engineering, the University of Newcastle, Australia, in 1989. Currently, he is a Chair Professor in Electrical Engineering. He has served as Head of School at the University of Newcastle. In addition, he was a Visiting Associate Professor at the University of Iowa in 1995–1996, and a Senior Fellow/Visiting Professor at Nanyang Technological University, Singapore, 2002, and a Visiting Professor at Tokyo University in 2003. He has held a ChangJiang Visiting Professorship at the Shandong University, China. Currently, he holds a Qian-ren Professorship at the Zhejiang University, China.

He is a Fellow of IEEE. His main research interests include control systems, signal processing and communications. He has been an Associate Editor for the IEEE Transactions on Automatic Control, Automatica and Journal of Optimization and Engineering.

Lihua Xie received his B.E. and M.E. degrees in Electrical Engineering from the Nanjing University of Science and Technology in 1983 and 1986, respectively, and his Ph.D. degree in Electrical Engineering from the University of Newcastle, Australia, in 1992. Since 1992, he has been with the School of Electrical and Electronic Engineering, the Nanyang Technological University, Singapore, where he is currently a Professor and the Director, Centre for Intelligent Machines. He held teaching appointments in the Department of Automatic Control, Nanjing University of Science and Technology from 1986 to 1989.

Dr. Xie's research interests include robust control and estimation, networked control systems, sensor networks, time-delay systems and 2-D systems. He served as an Associate Editor of IEEE Transactions on Automatic Control, Automatica, IEEE Transactions on Circuits and Systems-II, International Journal of Control, Automation and Systems, and the Conference Editorial Board, IEEE Control Systems Society, and was a member of the Editorial Board of IET Proceedings on Control Theory and Applications. Dr. Xie is also a Fellow of IEEE.