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Optimal PMU Placement for Power System State Estimation with Random Communication Packet Losses

Xin Tai, Damián Marelli, Eduardo Rohr and Minyue Fu

***Abstract*— Phasor measurement units (PMUs) become important to state estimation for power systems by providing globally synchronized measurements of real-time phasors of voltage and currents with a high sampling rate. However the large quantities of measurement data produced by PMUs brings a serious burden to the communication system, which aggravates communication constraints such as the packet loss rate. In this paper, a novel optimization criterion for choosing PMU placements is proposed, considering random communication packet losses. Based on this criterion, a simplified optimal solution searching algorithm is given. Finally numerical simulations are given to test the validity of this algorithm. The dependence of the optimal PMU placement solution on the packet loss rate is indicated as well.**

I. INTRODUCTION

State estimation in power systems is traditionally done using static estimation methods, i.e., ignoring previous measurements. This is due to the low sampling rate of traditional SCADA measurements. The new phasor measurement unit (PMU) devices permit obtaining synchronous measurements (via global synchronous time stamps) at a much higher sampling rate [1]. Hence, when using these measurements, dynamic estimation, i.e., using a dynamic model of the power system and a Kalman filter, yields a better performance [2], [3].

A number of papers have recently studied the problem of choosing the optimal placement of PMUs. Generally speaking, these methods aim at minimizing the number of PMUs under the constraint that the whole system is *topologically observable*. Roughly speaking, topological observability requires the state of the system at any time instant k to be uniquely computable directly from noise-free measurements obtained at time k. The optimal placement can be searched using simulated annealing optimization [4], [5] or integer linear programming [6], [7], [8]. Also some papers study the optimal PMU placement issue of making the system topologically observable considering communication constraints [9], [10], [11]. In general, these algorithms provide a set of feasible solutions which guarantee topological observability using minimum number of PMUs. Hence a criterion is needed to choose among these solutions. For the case of dynamic estimation, the authors of [12] proposed to use the steady state estimation error covariance of the Kalman filter as a criterion to clear the ambiguity left from the initial optimization problem.

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The methods described above implicitly assumes that the communication channel is ideal. When random packed drops occur, the solutions provided by these results are not necessarily optimal. Hence, a different selection criterion needs to be chosen to find the optimal PMU placement. Notice that packet loss is a particularly serious issue for wireless communication in noisy environment where the loss rate can often reach up to 35%.

In this paper we propose a method for optimally choosing the placement of PMUs, when measurements are subject to random communication packet losses. As in previous works, we also require the solution to guarantee topological observability. However, due to random packet drops, the state estimation error covariance becomes stochastic. Hence, a natural extension of the criterion used for the ideal channel case is to choose the optimal location so that the asymptotic value of the expected value of the norm of this covariance is minimized. A difficulty in doing so is that, to the best of our knowledge, there is no exact expression to compute this asymptotic value. To go around this, we derive a sequence of upper and lower bounds on it, which become monotonically tight. Using these bounding sequences, we derive an algorithm to find the optimal solution. The proposed algorithm is sequential: At each step it chooses a set of tighter bounds; it then eliminates, from the set of candidate solutions, those which are no longer candidate to become the optimal solution, and it stops when the set of candidates has only one solution left.

We present numerical experiments based on the IEEE 9-bus test system. Our experiments show that the proposed method is able to find the optimal PMU locations in a relatively small number of iterations. We also show that the optimal solution depends on the given packet loss rate.

II. PROBLEM FORMULATION

A. System Modeling

The dynamic behavior of a power system is described by the following model

$$x_{k+1} = Ax_k + B\bar{x} + \omega_k, \tag{1}$$

where x_k is the state vector at sample time k, whose entries are the complex phasors of all bus voltages. The matrix B is given by $B = I - A$, where I is the identity matrix, so that

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$\lim_{k\to\infty} E(x_k) = \bar{x}$ ($E(\cdot)$ denotes expectation). The noise ω_k is a white Gaussian vector random process with zero mean and covariance matrix Σ_ω. We also assume that the initial state x_0 has Gaussian distribution with mean $\bar{x}$ and covariance Σ_{x_0}.

Typically, a PMU is able to measure not only the voltage phasor of the bus where it is installed, but also the current phasors of all lines connecting to this bus. Hence, there is a linear relationship between PMU measurements and the system state variables. Thus, the measurement equation can be written as

$$y_k = C_l x_k + \upsilon_k, \tag{2}$$

where

$$C_l = \begin{bmatrix} \mathrm{Re}(H_l) & -\mathrm{Im}(H_l) \\ \mathrm{Im}(H_l) & \mathrm{Re}(H_l) \end{bmatrix}, H_l = \begin{bmatrix} I \\ Y_l \end{bmatrix}$$

and y_k being the PMU measurement vector, which includes bus voltage and branch current phasors. The matrix C_l depends on the chosen PMU placement, hence we use the subscript $_l$ to denote different PMUs placements. Also, Y_l is the branch admittance matrix. υ_k is the measurement noise, assumed to be white and Gaussian with zero mean and covariance matrix Σ_υ. We further assume that x_0, ω_k and υ_k are uncorrelated.

The packet loss behavior of the system communication channel is described by

$$z_k = \Gamma_k y_k \tag{3}$$

where z_k is the measurement vector received by the estimator. The random matrix $\Gamma_k = diag\left[\begin{array}{cccc} \gamma_k^1, & \gamma_k^2, & \cdots, & \gamma_k^m \end{array}\right]$ models the packet drops on each channel (i.e., $\gamma_k^i = 1$ when a packed is received and 0 otherwise). We assume that the binary random variables γ_k^i are i.i.d., with $p_i = \Pr(\gamma_k^i = 0)$ being the packet drop rate of channel i.

B. Topological Observability

As mentioned above, our proposed method searches the optimal PMU placement from a set of candidates guaranteeing topological observability. In this paper we use the definition given in [4]. That is, *a power system is topologically observable if each node is either directly measured, or connected to a measured node through at least one branch with a metered or calculated current phasor.* Mathematically, topological observability means that the matrix C_l has full column rank.

C. State Estimation

State estimation of the system (1)-(3) is done using a Kalman filter [13]. This is given by the following recursions

$$\begin{aligned} &\hat{x}_{k|k} = \hat{x}_{k|k-1} + G_k\left[z_k - \Gamma_k C_l \hat{x}_{k|k-1}\right] \\ &\hat{x}_{k+1|k} = A\hat{x}_{k|k} + B\bar{x} \\ &G_k = \Sigma_{k|k-1} C_l' \Gamma_k' \left(\Gamma_k C_l \Sigma_{k|k-1} C_l' \Gamma_k' + \Gamma_k \Sigma_\upsilon \Gamma_k'\right)^{-1} \\ &\Sigma_{k|k} = \Sigma_{k|k-1} - G_k \Gamma_k C_l \Sigma_{k|k-1} \\ &\Sigma_{k+1|k} = A\Sigma_{k|k} A' + \Sigma_\omega \end{aligned} \tag{4}$$

which are initialized by $\Sigma_{0|-1} = \Sigma_{x_0}$ and $\hat{x}_{0|-1} = \bar{x}$.

Equation (4), shows that the estimation covariance $\Sigma_{k|k}$ is a function of the PMU placement l as well as the packet losses matrix Γ_k. Hence, $\Sigma_{k|k}$ is a random matrix sequence which depends on the PMU locations.

III. OPTIMIZATION CRITERION FOR CHOOSING PMU PLACEMENTS

The natural criterion for choosing the PMU placement is to minimize the asymptotic expected covariance norm $\lim_{k\to\infty} E(||\Sigma_{k|k}||)^1$, where $||\Sigma_{k|k}||$ denotes the largest eigenvalue of $\Sigma_{k|k}$. A difficulty in doing so is that there is no exact expression for computing this limit. However, it turns out that we can still find the optimal locations by using instead a sequence of lower and upper bounds for this value. We derive these bounds below. To simplify the notation we use $\Sigma_k = \Sigma_{k|k}$.

The next lemma introduces the starting point for obtaining the desired bounds.

Lemma 1: Let

$$\begin{aligned} \underline{\Sigma} &= \operatorname*{sol}_{\Sigma}\{\Sigma = A\Sigma A' + \Sigma_\omega - A\Sigma C'(C\Sigma C'^{-1} C\Sigma A'\} \\ \overline{\Sigma} &= \operatorname*{sol}_{\Sigma}\{\Sigma = A\Sigma A' + \Sigma_\omega\}. \end{aligned}$$

Then, if $\Sigma_0 \geq \underline{\Sigma}$, for all k,

$$\underline{\Sigma} \leq \Sigma_k \leq \overline{\Sigma}. \tag{5}$$

Proof: Notice that $\underline{\Sigma}$ is the steady state solution for the standard Kalman filtering problem when no packet drop occurs, and $\overline{\Sigma}$ is the same solution obtained when all the measurement are lost. ■

The bounds in Lemma 1 are too loose to be used for our optimization problem. Below we explain how they can be refined to make them arbitrarily tight, at the expense of increased computational effort.

Let G^N describe the measurements received from time $k-N$ to $k-1$, i.e.,

$$G_k^N = \{\Gamma_{k-N+1},\ \Gamma_{k-N+2},\ \ldots,\ \Gamma_{k-1}\}. \tag{6}$$

Also, we let S_m^N, $m = 1,\ldots,2^{Ng}$ denote all 2^{Ng} possible arrival patterns in a time interval of length N. Let $\Pr(S_m^N)$ be the probability that $S_m^N = G_k^N$, i.e., that the sequence S_m^N was observed from times $k-N$ to $k-1$. Also, let $\phi(\cdot,\cdot)$ be the function describing the evolution of the error covariance according to a given sequence, i.e., $\Sigma_k = \phi(\Sigma_{k-N}, G_k^N)$. Then, we define

$$\underline{C}_N = \sum_{m=1}^{2^{Ng}} \Pr(S_m^N)||\phi(\underline{\Sigma}, S_m^N)|| \tag{7}$$

$$\overline{C}_N = \sum_{m=1}^{2^{Ng}} \Pr(S_m^N)||\phi(\overline{\Sigma}, S_m^N)||. \tag{8}$$

The next lemma shows that $\underline{C}_N$ is monotonically increasing and $\overline{C}_N$ is monotonically decreasing.

[1] Conditions for the existence of this limit are given in [14, §2.2]

Lemma 2:

$$\underline{C}_{N+1} < \underline{C}_N \tag{9}$$
$$\overline{C}_{N+1} > \overline{C}_N \tag{10}$$

Proof: We will only show the monotonicity of the lower bound. That of the upper bound follows from the same argument. Consider the following partition of the sequence S_j^{N+1}:

$$S_j^{N+1} = \{S_n^1, S_m^N\}. \tag{11}$$

Then,

$$\begin{aligned} \underline{C}_{N+1} &= \sum_{m=1}^{2^{Ng}} \sum_{n=1}^{2^g} \Pr(S_m^N)\Pr(S_n^1)||\phi\left(\underline{\Sigma},\{S_n^1,S_m^N\}\right)|| && (12)\\ &= \sum_{m=1}^{2^{Ng}} \Pr(S_m^N) \sum_{n=1}^{2^g} \Pr(S_n^1)||\phi\left(\phi(\underline{\Sigma},S_n^1),S_m^N\right)|| && (13)\\ &\geq \sum_{m=1}^{2^{Ng}} \Pr(S_m^N)||\phi\left(\underline{\Sigma},S_m^N\right)|| && (14)\\ &= \underline{C}_N && (15) \end{aligned}$$

where the inequality in (14) follows since

$$\phi(\underline{\Sigma},S_n^1) \geq \underline{\Sigma}. \tag{16}$$

■

Now, it follows from [14, Th. 2.4] that the bounds $\underline{C}_N$ and $\overline{C}_N$ approach the asymptotic expected norm of the error covariance as $N \to \infty$, i.e.,

$$\lim_{N\to\infty} \underline{C}_N = \lim_{k\to\infty} E(||\Sigma_k||) \tag{17}$$
$$\lim_{N\to\infty} \overline{C}_N = \lim_{k\to\infty} E(||\Sigma_k||). \tag{18}$$

Hence, from Lemma 2, we have that

$$\underline{C}_N \leq E(||\Sigma_k||) \leq \overline{C}_N, \tag{19}$$

i.e., $\underline{C}_N$ and $\overline{C}_N$ are bounds on $\lim_{k\to\infty} E(||\Sigma_k||)$, which become monotonically tight on the limit.

IV. OPTIMAL PMU PLACEMENT ALGORITHM

In this section we use the bounds $\underline{C}_N$ and $\overline{C}_N$ to propose an algorithm for finding the optimal PMU placement in the sense of minimizing $\lim_{k\to\infty} E(||\Sigma_{k|k}||)$. In principle, this could be done by choosing N large enough that both bounds are "sufficiently close". However, in a large power system, this approach is numerically unaffordable. To avoid this, we propose an alternative algorithm. The main idea is to proceed sequentially, for $N = 1, 2, \cdots$. For each N, the bounds $\underline{C}_N$ and $\overline{C}_N$ are computed for all candidate PMU placements. Then, all placements whose lower bound is greater than the smallest upper bound of all candidate placements, are eliminated from the set of candidates, before continuing to the next step. The steps proceed until only one candidate is left. To reduce the numerical complexity, the initial set of candidates is formed by those PMU placements which result in systems being topologically observable. This can be the done using any available method [7].

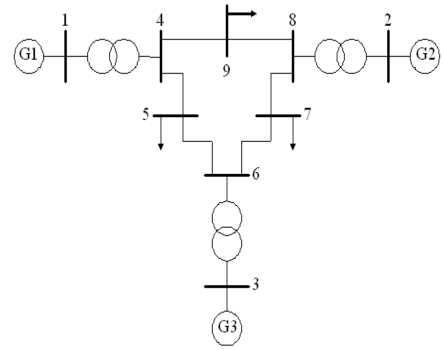

Fig. 1. IEEE 9 bus system

We summarize the proposed algorithm below.

(a) Set $N = 1$, and obtain the initial set of candidate solutions. To this end we use the integer linear programming algorithm proposed in [7].

(b) Compute the upper bound $\overline{C}_N$ for all candidate solution and let $\overline{\overline{C}}_N$ be the smallest among them.

(c) For each candidate solution compute the lower bound $\underline{C}_N$, and if this value is greater than $\overline{\overline{C}}_N$, remove this candidate from the set of candidates.

(d) If the set of candidates has only one solution, stop the iterations. Otherwise, put $N = N + 1$ and go to (b).

Because, for each candidate, the lower bound and upper bound converge as $N \to \infty$, we know that the algorithm above will terminate at some finite N. As we will show in simulations, this value is typical small.

V. SIMULATION RESULTS

To test the proposed method we use the IEEE 9-bus test system, shown in Fig. 1. The associated dynamical system model is summarized in Table I. To obtain the initial set of candidate solutions, we use the linear integer programming method in [7], which gives all PMU placements leading to topologically observable systems, using the minimum number of PMUs. The resulting set has four solutions, each of them using three PMUs. The installation buses of these four solutions are $\{1,6,8\}$, $\{2,4,6\}$, $\{3,4,8\}$ and $\{4,6,8\}$, respectively. We point out that the solution $\{4,6,8\}$ has two extra current measurements, in comparison with the other three solutions. This makes the comparison somehow unfair, since this solution consumes more communication resources than the others. Hence, we cancel two current measurements from the fourth solution, so that all solutions have the same number of measurements.

We assume that a packet loss affects all the measurements of a PMU. In other words, when a packet loss occurs, all the measurements of the corresponding PMU will be lost. Hence, there are 2^3 packet loss patterns at each sample time.

To illustrate the convergence of the bounds $\underline{C}_N$ and $\overline{C}_N$, we show in Figs. 2 and 3 their values for different values of N,

A	$diag(0.8, 0.8, 0.95, 0.8, 0.95, 0.95, 0.8, 0.95, 0.8)$
B	$diag(0.2, 0.2, 0.05, 0.2, 0.05, 0.05, 0.2, 0.05, 0.2)$
Σ_ω	$diag(0.1^2, \cdots, 0.1^2)$
Σ_υ	$diag(0.1^2, \cdots, 0.1^2)$
Σ_{x_0}	$diag(0.1^2, \cdots, 0.1^2)$

TABLE I

SIMULATION VALUES OF PARAMETERS

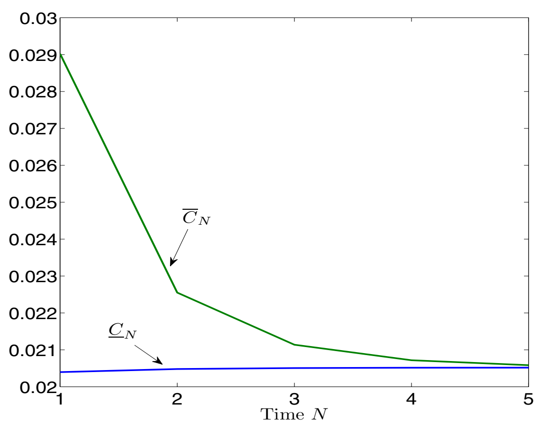

Fig. 2. Bounds of $\lim_{k\to\infty} E(\|\Sigma_k\|)$, PMUs located in buses $\{4,6,8\}$, $p_i = 0.05$.

corresponding to the fourth solution $\{4,6,8\}$. The packet loss rates are 0.05 and 0.35 respectively. We can see how both bounds converge monotonically to the same limit value and that different packet loss rates lead to different convergence rates.

The bounds of the four candidate solutions, for packet loss rates of 0.05 and 0.35, are shown in Figs. 4 and 5, respectively. From the two figures, we get the following observations: 1) The optimal PMU placements for both cases can be found after 4 iterations. Even if the gap between the upper bound and lower bound at sample time 4 shown in Fig. 2 (or Fig. 3) is still visible, the optimal solution can already be identified. 2) The two different packet loss rates lead to two different optimal solutions. When the packet loss rate has a lower value, $p_i = 0.05$, the third candidate solution $\{3,4,8\}$ has the best state estimation performance. However when p_i increases to 0.35, the second candidate solution $\{2,4,6\}$ becomes the optimal placement. In addition, we find that when $p_i = 0$, the optimal placement is $\{3,4,8\}$ (the lower/upper bound evolutions are not shown). These simulations indicate the dependence of the optimal PMU placement on the given communication packet loss rate. In other words, we point out that the steady Kalman filter error covariance used in [12] is only a particular case with $p_i = 0$. The candidate solutions should be re-evaluated when the communication environment changes.

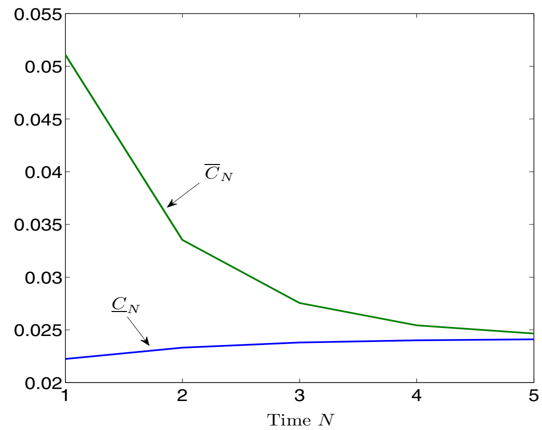

Fig. 3. Bounds of $\lim_{k\to\infty} E(\|\Sigma_k\|)$, PMUs located in buses $\{4,6,8\}$, $p_i = 0.35$.

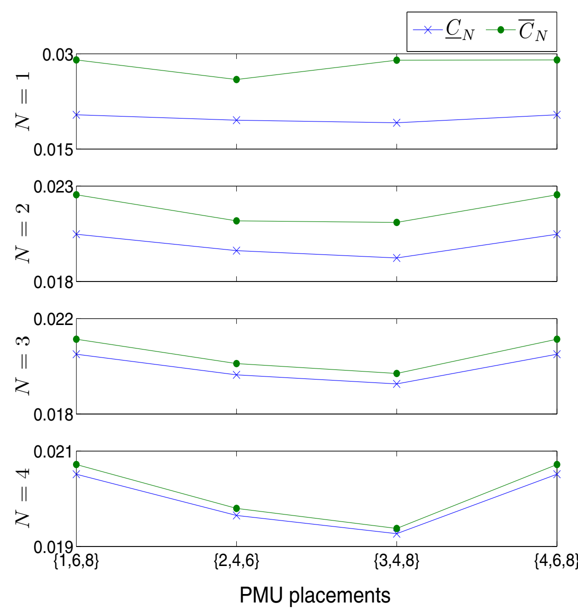

Fig. 4. Bounds of $\lim_{k\to\infty} E(\|\Sigma_k\|)$, $p_i = 0.05$.

VI. CONCLUSIONS

We have proposed a method for determining the optimal PMU placement in power systems, in the presence of communication packet losses. The proposed method chooses the locations from a set of candidate PMU placements, each of them guaranteeing topological observability. Then the optimal PMU placements is chosen to minimize the asymptotic expected value of the norm of the state estimation error covariance. To reduce the computational complexity, we propose a sequential algorithm which uses a sequence of lower and upper bounds for the error covariance, which are monotonically tight. Numerical experiments show that the proposed method is able to find the optimal locations with a relatively small amount of computation.

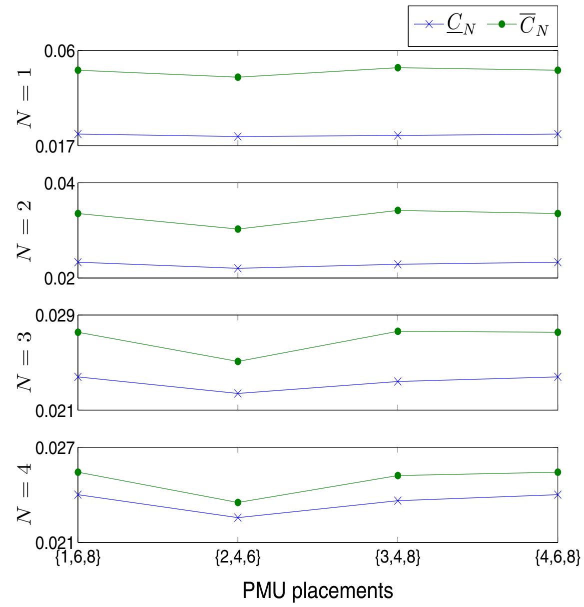

Fig. 5. Bounds of $\lim_{k\to\infty} E(\|\Sigma_k\|)$, $p_i = 0.35$.

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