Proceedings of the 2011 4th International Symposium on
Advanced Control of Industrial Processes, Thousand
Islands Lake, Hangzhou, P.R. China, May 23-26, 2011

Power System Dynamic State Estimation with Random Communication Packets Loss

Xin Tai, Damián Marelli and Minyue Fu

Abstract— **In this paper power system dynamic state estimation problem is studied considering random communication packet losses. Two sorts of stochastic processes, i.i.d. process and Markov process, are respectively utilized to model two different communication packet losses cases. The first case only considers packet losses rate, and the second case includes both of packet losses rate and recovery rate. The degradation of the performance of state estimation caused by communication packet losses is analyzed on IEEE 14 buses test system, and numerical results are given.**

I. INTRODUCTION

State estimation is one of the essential elements of the online security analysis function in modern power system energy management centers by providing a real time database of the system state variables. After Schweppe and Wildes [1]introduced the concept of state estimation in the field of power system in the early 1970s, many methods have been proposed to estimate the state variables of the power system[2]-[5]. Especially, with the power system growing larger and more complex, dynamic state estimation methods [6]-[9] draw more and more attentions, which can save a large number of computing resources compared with the traditional static state estimation methods.

Signal communication technology forms the backbone of state estimation by providing the database of real time raw measurement. In most of the papers, which talked about power system state estimation problem, the effects of network communication were rarely focused on. In traditional power system state estimation methods the failure of communication usually is addressed as normal bad data. In this paper the state estimation performance degradation due to signal communication constraint will be studied. When communication constraint problem is studied, the following several properties usually are considered, time delay, packets loss, bandwidth constraint and quantization error. Su studied the random time delay problem in his paper [7], and a state estimation method was proposed for power system.

In this paper we just focus on the random packet loss behavior of communication network. There are two sorts of packet loss behaviors, in the first case, only packet loss rate is considered, and in the second case, both of packet loss rate and recovery rate are entertained. The details will be given in next section.

Xin Tai and Damián Marelli are with School of Electrical Engineering and Computer Science, University of Newcastle, NSW 2308, Australia Xin.Tai@uon.edu.au; Damian.Marelli@newcastle.edu.au

Minyue Fu is with Department of Control Science and Engineering, Zhejiang University, China and School of Electrical Engineering and Computer Science, University of Newcastle, NSW 2308, Australia Minyue.Fu@newcastle.edu.au

II. PROBLEM FORMULATION

We first take the standard IEEE 14 bus test system, shown in Fig. 1, as an example to generally represent the network structure of power systems. Based on the three assumptions of the power system dynamic modeling in previous section, the following dynamic network model is used to describe the general power system:

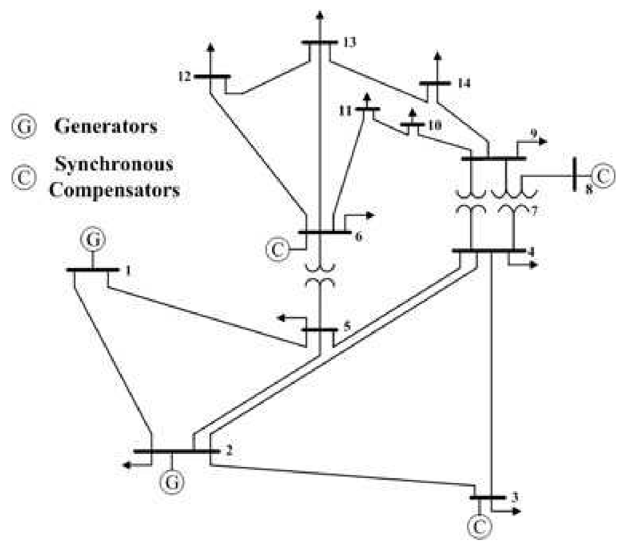

Fig. 1. IEEE 14 bus test system

The following group dynamic equations (1) are used to simply model the power system and signal communication network.

$$\begin{aligned} X_{k+1} &= AX_k + B\bar{X} + \omega_k \\ Y_k &= h(X_k) + \nu_k \\ Z_k &= \Upsilon_k Y_k \end{aligned} \quad (1)$$

where k is the time sample. X_k is the state vector comprised by the phase angles and magnitudes of the bus voltages,

$$X_k = [\theta_2(k)\,\theta_3(k)\cdots\theta_N(k)\,V_1(k)\,V_2(k)\cdots V_N(k)]^T$$

where $\theta_i(k)$ and $V_i(k)$ are the voltage phase angle and magnitude of bus i at time k. However in the following sections θ_i and V_i will be used instead for simplicity. N is the number of buses. Assuming the initial value of state X_0 is a white Gaussian noise with mean value $\bar{X}$ and covariance matrix Σ_{X_0}. Matrix A represents how fast the transitions between states are, which is assumed to be stable. Matrix B is associated with the trend behavior of the state trajectory. Let $B = I - A$, where I is the identity matrix, then $E(X_k)_{k\to\infty} = \bar{X}$. Therefore $\bar{X}$ is called the expected steady state. Vector ω_k represents modeling uncertainties, which is a white Gaussian noise with zero mean and covariance matrix Σ_ω. $Y_k$is the raw measurement vector. v_k is measurement noise, which is also a white Gaussian noise with zero mean and covariance matrix Σ_v. Assuming X_0, ω_k and v_k are uncorrelated.

h is the load_flow function vector for the current power system network configuration, including the following measurements.

- Real and reactive power injection at bus i:

$$\begin{array}{l} P_i = V_i \sum\limits_{j\in\aleph} V_j \left(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}\right) \\ Q_i = V_i \sum\limits_{j\in\aleph} V_j \left(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}\right) \end{array}$$

where $G_{ij} + jB_{ij}$ is the ijth element of the complex bus admittance matrix. $\aleph_i$ is the set of bus numbers that are directly connected to bus i.

- Real and reactive power flow from bus i to bus j:

$$\begin{array}{l} P_{ij} = V_i^2\left(g_{si} + g_{ij}\right) - V_iV_j\left(g_{ij}\cos\theta_{ij} + b_{ij}\sin\theta_{ij}\right) \\ Q_{ij} = -V_i^2\left(b_{si} + b_{ij}\right) - V_iV_j\left(g_{ij}\sin\theta_{ij} - b_{ij}\cos\theta_{ij}\right) \end{array}$$

where $g_{ij} + jb_{ij}$ is the admittance of the series branch connecting buses i and j. $g_{si} + jb_{si}$ is the admittance of the shunt branch connected at bus i.

- Voltage magnitude at bus i: V_i.

The above measurements are all accessable but not necessary. Generally there are m measurements and n state variables, $m > n = 2N - 1$.

Z_k is the received measurement vector after transmission from the raw measurement vector Y_k via a communication network. Random matrix $\Upsilon_k = diag\left[\begin{array}{cccc}\gamma_k^1, & \gamma_k^2, & \cdots, & \gamma_k^m\end{array}\right]$ is used to indicate whether the measurement of the corresponding dimension is correctly received, $\gamma_k^i = 1$ or $0, i = 1, 2, \cdots, m$ indicates measurement received or lost respectively. Note that when $\gamma_k^i = 0$, the corresponding dimension measurement will be canceled, and the whole dimension of measurement will be reduced to $m - l$ at time k, l is the number of 0 in Υ_k.

In this paper, we study two different cases of communication packet loss. In the first case, only packet loss rate p is considered. Furthermore assuming packet received independently for different sample time point, $\mathrm{cov}\left(\gamma_k^i\gamma_s^i\right) = 0$, $k \neq s$. I.i.d. process is used here to model this kind of packet loss behavior, like follows: Υ_k yields i.i.d. with

$$\begin{array}{l} p = \Pr(\gamma_k^i = 0) \\ i = 1, 2, \cdots, m \\ k = 0, 1, \cdots \end{array}$$

In the second case, both of packet loss rate p and recovery rate q are considered. Packet on current sample time point is received or lost only depend on the state of previous time point. Υ_k is assumed to yield one order Markov distribution. Transmission probability matrix is

$$\left[\begin{array}{cc} 1-p & p \\ q & 1-q \end{array}\right]$$

where

$$\begin{array}{l} p = \Pr\left(\gamma_{k+1}^i = 0 \,\middle|\, \gamma_k^i = 1\right) \\ q = \Pr\left(\gamma_{k+1}^i = 1 \,\middle|\, \gamma_k^i = 0\right) \\ i = 1, 2, \cdots, m \\ k = 0, 1, \cdots \end{array}$$

In both of the above two cases, each dimension of measurements of the same sample time point is assumed to be transmitted via communication network independently with the others, $\mathrm{cov}\left(\gamma_k^i\gamma_k^j\right) = 0, i \neq j$.

III. DYNAMIC STATE ESTIMATION

In this paper extended Kalman filter (EKF) is used to estimate the state variables. Note that MAP estimation method is also valid for this system, and actually EKF is a particular case of MAP method with only one iterative step on each time point. Furthermore both of the results of the two methods are very similar, but the calculation speed of EKF is much faster than MAP, and mathematical analysis is also simpler for EKF. Therefore, EKF is preferred here.

Considering the communication packets loss behaviors described in previous section, the following modified EKF is operated on the system (1).

$$\begin{array}{l} \hat{X}_{k|k} = \hat{X}_{k|k-1} + G_k\left[Z_k - \Upsilon_k h\left(\hat{X}_{k|k-1}\right)\right] \\ \hat{X}_{k+1|k} = A\hat{X}_{k|k} + B\bar{X} \\ G_k = \Sigma_{k|k-1}H_k'\Upsilon_k'\left(\Upsilon_k H_k \Sigma_{k|k-1} H_k'\Upsilon_k' + \Upsilon_k\Sigma_v\Upsilon_k'\right)^{-1} \\ \Sigma_{k|k} = \Sigma_{k|k-1} - G_k\Upsilon_k H_k \Sigma_{k|k-1} \\ \Sigma_{k+1|k} = A\Sigma_{k|k}A' + \Sigma_\omega \end{array} \tag{2}$$

Initialization is provided by $\Sigma_{0|-1} = \Sigma_{X_0}$, $\hat{X}_{0|-1} = \bar{X}$. $H_k = \left.\frac{\partial h(X_k)}{\partial X_k}\right|_{X_k = \hat{X}_{k|k-1}}$ is the Jacobian matrix of $h(X_k)$.

From the EKF (2), we can see that the state estimation error covariance is function of the random packet loss parameter Υ_k. The numerical test in next section analyzes the effect of random packet loss to the state estimation performance.

IV. NUMERICAL SIMULATION

The performance of the EKF in (2) is tested via the generated simulation measurement data in this section. In this simulation the trace of state estimation error covariance is used as criterion to evaluate the performance. There are two methods of calculating the trace of estimation error covariance considered, the first one is

$$\Xi_k = trace\left(\frac{1}{d}\sum_{j=1}^{d}\left(X_k^j - \hat{X}_k^j\right)\left(X_k^j - \hat{X}_k^j\right)'\right) \tag{3}$$

which is called the trace of practical state estimation error covariance. The second one is

$$\Psi_k = trace\left(\frac{1}{d}\sum_{j=1}^{d}\Sigma_{k|k}^j\right) \tag{4}$$

which is called the trace of theoretical state estimation error covariance. Where d is the number of independent realizations of simulated results used to calculate the average values, and j indicates different realizations. Note that the two kinds of error covariances will equal to each other when $d \to \infty$. Here we let $d = 100$.

A	$diag(0.98,\cdots,0.98)$
B	$diag(0.02,\cdots,0.02)$
Σ_W	$diag(0.01^2,\cdots,0.01^2)$
Σ_V	$diag(0.02^2,\cdots,0.02^2)$
Σ_{X_0}	$diag(0.025,\cdots,0.025)$

TABLE I
SIMULATION VALUES OF PARAMETERS

Bus	Voltage magnitude	Phase angle
1	1.060	0.0
2	1.045	-4.98
3	1.010	-12.72
4	1.019	-10.33
5	1.020	-8.78
6	1.070	-14.22
7	1.062	-13.37
8	1.090	-13.36
9	1.056	-14.94
10	1.051	-15.10
11	1.057	-14.79
12	1.055	-15.07
13	1.050	-15.16
14	1.036	-16.04

TABLE II
SIMULATION VALUE OF $\bar{X}$

A. Simulation Data Generation

In practical case the value of the parameters A and B as well as the noise covariance Σ_W and Σ_V should be estimated online. In this paper, we just fix these parameters on some given value shown in Table I. The value of $\bar{X}$ is given in IEEE 14_bus test system, which is shown in Table II.

Using the first two equations in (1) and the values in Table I and Table II, we generated d sets of simulation measurement. Fig. 2 and Fig. 3 show only one realization of them.

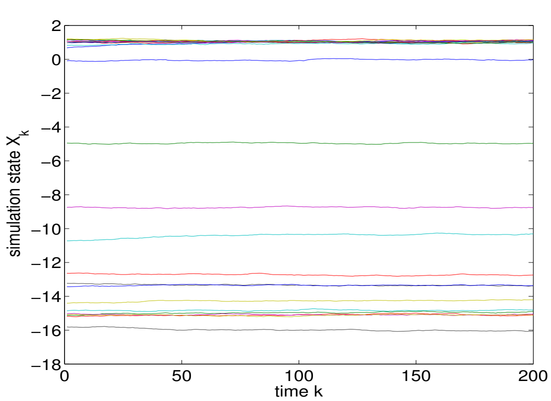

Fig. 2. one realization of simulation state variables

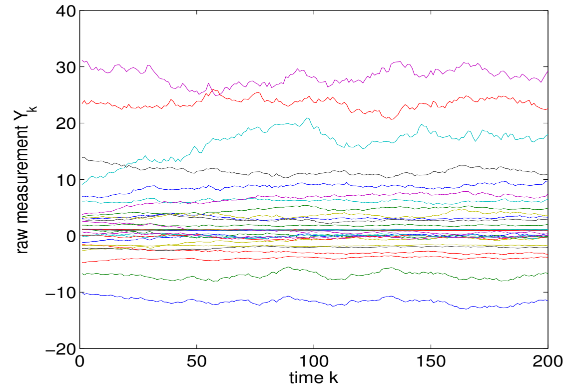

Fig. 3. one realization of simulation measurement

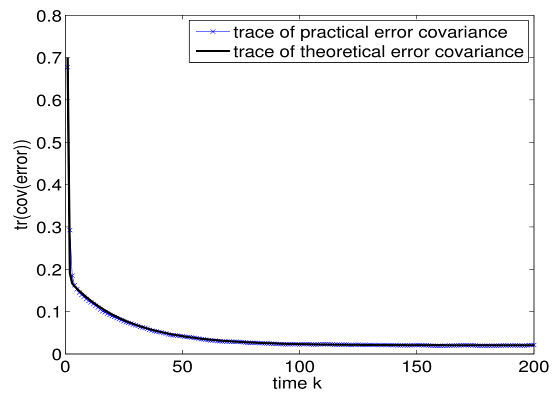

Fig. 4. $p=0.1$, EKF state estimation result

B. Only Packet Loss Rate p with i.i.d. Distribution

Let Υ_k yields i.i.d. with $\Pr(\gamma_k^i=0)=p$, $i=0,1,\cdots,m$. When the packet loss rate p is fixed, the state estimation error covariance will decreases with time going to some steady value. Fig. 4 shows the result when $p=0.1$. Otherwise, when time k is fixed, especially let k equal to some steady time point in Fig. 4, $k=200$, the relationship between packet loss rate p and state estimation error covariance can be represented clearly. From Fig. 4, it's noted that the trace of the estimation error covariance increases monotonically with the packet loss rate increases, especially in the last 10% of p, changing speed increases quickly. However, when p is small, the effect of the packet loss is not very distinct. In both Fig. 4 and Fig. 5, the two kinds of trace of error covariances are close to each other, but the so_called trace of practical error covariance fluctuates more roughly.

C. Packet Loss Rate p and Recovery Rate q with Markov Distribution

When both of packet loss rate p and recovery rate q are considered, the one order Markov process is used to model the packet loss behavior, which is introduced in Section II. Fig. 6 shows the EKF estimation process when $p=0.1, q=0.9$, from which we can see that the state estimation error

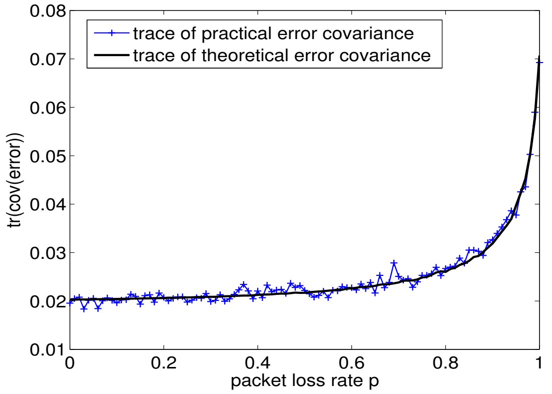

Fig. 5. i.i.d. packet loss rate, $p = 0 : 0.01 : 1$

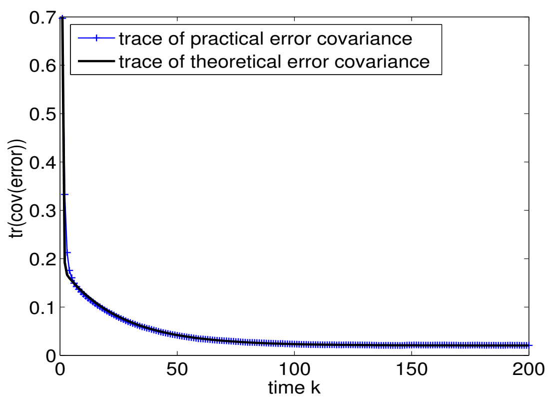

Fig. 6. $p = 0.1, q = 0.9$, EKF state estimation result

covariance decreases to some steady value. Fig. 7 and Fig. 8 show the effects of packet loss rate p and recovery rate q to the state estimation performance. Fig. 7 shows the so_called practical case and Fig. 8 shows the so_called theoretical case. From both of Fig. and Fig., it's noted that the trace of the estimation error covariance increases monotonically with packet loss rate p increasing or recovery rate q decreasing.

V. CONCLUSIONS

The relationship between state estimation error covariance of power system and random communication packet loss is studied via numerical simulation based on IEEE 14 bus test system. This test method is also valid for the other power systems. This simulation result will be very useful for further study of state estimation of power system with communication constraints. Furthermore all of these monotonicity can be approved mathematically.

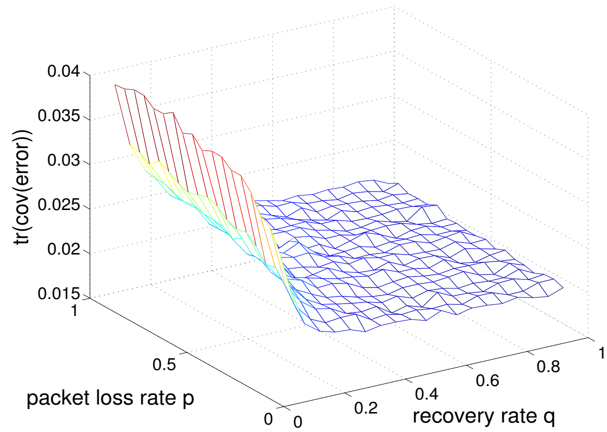

Fig. 7. practical error covariance, Markov distribution packet loss rate $p = 0.05 : 0.05 : 0.95$ and recovery rate $q = 0.05 : 0.05 : 0.95$

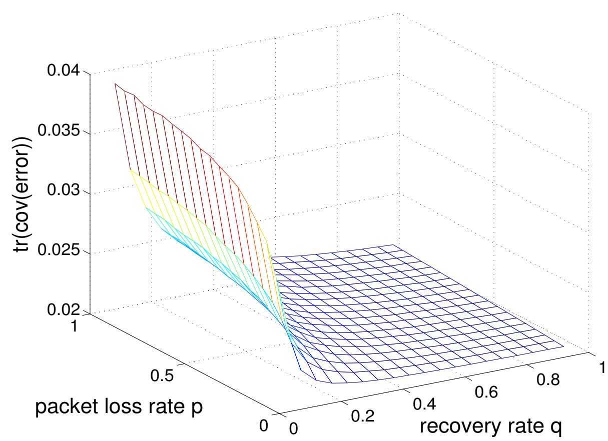

Fig. 8. theoretical error covariance, Markov distribution packet loss rate $p = 0.05 : 0.05 : 0.95$ and recovery rate $q = 0.05 : 0.05 : 0.95$

REFERENCES

[1] F. C. Schweppe and J. Wildes, "Power System Static-State Estimation, Part I,II and III", *IEEE Trans. Pourer App. Syst.*, vol. PAS-89, pp.120-135, Jan. 1970.

[2] F. F. Wu, "Power System State Estimation: A Survey ", *Elect. Power & Energy Syst.*, vol. 12, no. 1, pp.80-87, Jan. 1990.

[3] M. B. Do Coutto Filho, A. M. Leite da Silva, and D. M. Falcão, "Bibliography on Power System State Estimation (1968-1989)", *IEEE Trans. power Syst.*, vol. 5, no. 3, pp.950-961, Aug. 1990.

[4] o. Alsac, N. Vempati, B. Stott, and A. Monticelli, "Generalized State Estimation", *IEEE Trans. power Syst.*, vol. 13, no. 3, pp.1069-1075, Aug. 1998.

[5] A. Abur and A. G. Expósito, *Power Systen State Estimation: Theory and Implementation*, New York: Marcel Decker, 2004.

[6] K. Nishiya, J. Hasegawa and T. Koike, "Dynamic State Estimation including Anomaly Detection and Identification for Power Systems ", *IEEE PROC*, vol. 129, Pt. C, no. 5, Sep. 1982.

[7] C. Su and C. L, "Interconnected Network State Estimation Using Randomly Delay Measurements", *IEEE Trans. power Syst.*, vol. 16, , no. 4, pp. 870-878, Nov. 2001.

[8] Shivakumar, N.R. and Jain, A., "A Review of Power System Dynamic State Estimation Techniques", *Power System Technology and IEEE Power India Conference, 2008. POWERCON 2008. Joint International Conference on*, pp. 1-6, Oct. 2008.

[9] M. B. D. C. Filho and J. C. S. de Souza, "Forecasting-aided State Estimation-Part I and II", *IEEE Trans. power Syst.*, vol. 24, , no. 4, pp. 1667-1685, Nov. 2009.