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Robust Stabilization of Non-Minimum Phase Linear Time Invariant Systems

Huaizhong Li   Minyue Fu

Department of Electrical and Computer Engineering
The University of Newcastle
NSW 2308, Australia
Email: eelhz@tesla.newcastle.edu.au, eemf@cc.newcastle.edu.au

Abstract

This paper considers the problem of robust stabilization of non-minimum phase linear time-invariant single-input single-output uncertain systems. Under certain assumptions, we show that a non-minimum phase LTI uncertain systems with known order and known and invariant relative degree can be controlled to achieve robust stability by using a linear periodic output feedback controller. Our result generalizes similar results for LTI minimum phase uncertain systems.

1. Introduction

It has been widely acknowledged that minimum phase systems have great advantages over their non-minimum phase counterparts. In general, an LTI minimum phase stable controller can be applied to a LTI minimum phase SISO uncertain plant to achieve robust stability alone [1] or robust stability with specified robust performance simultaneously [2, 3]. An LTI minimum phase stable controller can even be applied to a linear time-varying minimum phase SISO uncertain plant to guarantee both the robust stability and the robust model following performance [4]. However, since open-loop zeros are not affected in the LTI cases by LTI feedback, the results mentioned above are not directly applicable to non-minimum phase SISO LTI uncertain plants.

This paper generalizes the robust stabilization results in [1, 2, 3] to non-minimum phase SISO LTI plants.

2. Problem Statement

Consider a SISO LTI uncertain plant

$$D(p,\theta)y(t) = N(p,\theta)u(t) \tag{1}$$

where $p = \frac{d}{dt}$ and

$$N(p,\theta) = \sum_{i=0}^{m} b_{m-i}(\theta)p^i, \quad D(p,\theta) = \sum_{i=0}^{n} a_{n-i}(\theta)p^i$$

and θ is an uncertain parameter vector belonging to a set $\Theta \in \mathbb{R}^q$, $D(p,\theta)$ and $N(p,\theta)$ are both real monic polynomials and they are possibly unstable.

We introduce the following assumptions:

Assumptions:

1: The plant order n is known;
2. The uncertainty vector θ belongs to a known compact set Θ;
3: The relative degree $r = n - m \geq 1$ of the plant is known and invariant;
4. The coefficients of $N(p,\theta)$ and $D(p,\theta)$ are continues for all $\theta \in \Theta$;
5: $a_i(\theta) \in [\underline{a}_i,\ \bar{a}_i]$, $i = 1,\cdots,n$; $b_i(\theta) \in [\underline{b}_i,\ \bar{b}_i]$, $i = 1,\cdots,m$, where $\underline{a}_i$, $\bar{a}_i$, $\underline{b}_i$ and $\bar{b}_i$ are known *a priori*;
6. $N(p,\theta)$ and $D(p,\theta)$ are coprime for all $\theta \in \Theta$.

Following Barmish and Wei [1], Bellman *et al.* [5] and Lee *et al.* [6], we show that a linear periodic output feedback controller of order n can be designed to make the averaged open loop system minimum phase. Further, the averaged system can be made robustly stable by tuning the parameters of the linear periodic controller.

Due to space limitation, we briefly describe our main result in the next section. The details can be found in [8].

3. Main Result

Lemma 1: [5] Consider the following fast periodic systems of the form:

$$\dot{x} = (A + \frac{\alpha}{\epsilon}B(\frac{t}{\epsilon}))x \tag{2}$$

Let

$$R = \lim_{T\to\infty} \frac{1}{T}\int_0^T \Phi^{-1}(\tau,0)A\Phi(\tau,0)d\tau \tag{3}$$

where $\Phi(\tau,0)$ is the state transition matrix of

$$\frac{dx}{d\tau} = \alpha B(\tau)x, \qquad \tau = \frac{t}{\epsilon} \tag{4}$$

Assume that $\Phi(\tau,0)$ is bounded for all $\tau \in (-\infty,\infty)$. Then there exists ϵ_0, such that for any $0 < \epsilon < \epsilon_0$, the system is asymptotically stable if R is a Hurwitz matrix, i.e., the system is asymptotically stable if its averaged system is asymptotically stable.

We propose the following periodic controller for (1):

$$\dot{z} = [F + \frac{1}{\epsilon}\Delta F(\frac{t}{\epsilon})]z + g_c y \tag{5}$$

$$u = (h + \frac{1}{\epsilon^r}h_r(\frac{t}{\epsilon}))z \tag{6}$$

where

$$F = \left[\begin{array}{c|c} 0 & I_{n-1} \\ \hline \multicolumn{2}{c}{f^T} \end{array}\right], \ \Delta F(\tfrac{t}{\epsilon}) = \left[\begin{array}{c|c} 0_{(n-1)} & 0 \\ \hline 0 & \alpha(\frac{t}{\epsilon}) \end{array}\right]$$

$$g_c = [0\ \cdots\ 0\ 1]^T,\ h = [h_0\ h_1\ \cdots\ h_{n-1}]$$

$$h_r(\tfrac{t}{\epsilon}) = [\beta_0(\tfrac{t}{\epsilon})\ \cdots\ \beta_{n-r-1}(\tfrac{t}{\epsilon})\ 0\ \cdots\ 0]$$

r is the plant relative degree, and f^T, h, $\alpha(t/\epsilon)$ and $\beta_i(t/\epsilon)$ are design parameters (vectors). Both $\alpha(t)$ and $\beta_i(t)$ are periodic functions, and $\alpha(t)$ can be chosen *a priori*.

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Denote an invertible Sylvester matrix $S(\theta)$ as follows:

$$S(\theta) = \left[\begin{array}{c|c} 1 \quad 0 & 1+(-1)^{r+1} \quad 0 \quad 0 \\ \hline N_1(\theta) & N_2(\theta) \end{array}\right]$$

where

$$N_1(\theta) = \begin{bmatrix} b_{n-r-1} & \cdots & b_0 & 0 & \cdots & \cdots & 0 \\ & \ddots & & \ddots & & & \\ 0 & \cdots & 0 & 1 & b_{n-r-1} & \cdots & b_0 \end{bmatrix}^T$$

$$N_2(\theta) = \begin{bmatrix} a_{n-1} & \cdots & a_r & \cdots & a_0 & 0 & \cdots \\ & \ddots & & \ddots & & & \\ 0 & \cdots & 0 & 1 & a_{n-1} & \cdots & a_0 \end{bmatrix}^T + (-1)^{r+1} \left[\begin{array}{cccccc} b_{n-r-1} & \cdots & b_0 & 0 & \cdots & 0 \\ \hline & & 0 & & & \end{array}\right]^T$$

The basic idea of controller design is as follows:

We first apply the Averaging Principle (see, e.g. [5]) to the state-space realization of plant (1) with controller (5)-(6). Then re-arrange the averaged closed-loop system to get an equivalent averaged open loop system with unit output feedback. This equivalent averaged open loop system is time-invariant and its transfer function takes the following form:

$$T_{eq-ol}(p,\theta,f^T,h,q) = \frac{g(p,\theta,h,q)}{D(p,\theta)D_c(p,f^T)} \qquad (7)$$

where $D_c(p,f^T)$ is a nth order polynomial with the design parameter f^T, $g(p,\theta,h,q)$ is a $(2n-r-1)$th order polynomial with its coefficient vector $\bar{g}(\theta) = S(\theta)[h\ q]^T$,

$$\begin{aligned} q &= [q_{n-r-1}\ q_{n-r-2}\ \cdots\ q_0] \\ q_i &= \lim_{T\to\infty} \frac{1}{T} \int_0^T e^{-\int_0^\tau \alpha(t)dt} \int_0^\tau \cdots \int_0^\tau \beta_i(t)(dt)^r d\tau \end{aligned}$$

Note that the parameters h and q can be arbitrarily chosen, implying that $\bar{g}(\theta_0)$ can be made Hurwitz where $\theta_0 \in \Theta$ denotes the nominal vector. The following assumption will guarantee that the Hurwitzness of the $\bar{g}(\theta_0)$ implies that of all $\bar{g}(\theta)$ for all $\theta \in \Theta$:

Assumption 7:
$\exists$ a constant vector $\bar{g} = [g_{2n-r-1} \cdots g_0]^T$ such that

$$J = \begin{bmatrix} 0 & \cdots & 0 & -\frac{g_0}{g_{2n-r-1}} \\ 1 & \cdots & 0 & -\frac{g_1}{g_{2n-r-1}} \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & 1 & -\frac{g_{2n-r-2}}{g_{2n-r-1}} \end{bmatrix}$$

is a stable matrix and the following inequality is satisfied:

$$\|E\|_\infty < \frac{1}{\rho\{(M \otimes I + I \otimes M)([J]^+ \otimes I + I \otimes [J]^+)^{-1}\}}$$

where

$$E = [0\ \ [N_1(\theta-\theta_0)\ N_2(\theta-\theta_0)]S^{-1}(\theta_0)\bar{g}]$$

and $M = [\frac{|E_{ij}|}{\|E\|_\infty}]$. $[\cdot]^+$ denotes a matrix obtained by replaced its elements with their absolute values, ρ is the spectral radius.

Once the transfer function (7) is minimum phase for all $\theta \in \Theta$, we then tune the design parameter vector f^T such that the equivalent averaged open loop system $T_{eq-ol}(p,\theta,f^T,h,q)$ with unit output feedback is robustly stable for all $\theta \in \Theta$(see [8]). Therefore, the averaged closed-loop system is robustly stable for all $\theta \in \Theta$. The robust stability of the closed-loop system of (1),(5),(6) is then guaranteed by Lemma 1. This leads to our main result as follows:

Theorem 1: *Under Assumptions 1-7, there exists a stable linear periodic controller (5),(6) to the given LTI uncertain plant (1) such that the closed-loop system is asymptotically stable.*

Remark 1. It is well known that periodic controllers will introduce oscillation into the system and therefore worsen the system performance or even make the system performance unacceptable (see, e.g. [9]). As observed by the authors, the same performance problem occurs in our design although our plant and controller are continuous. Basically, periodic controllers must use oscillation to obtain extra design freedom which is not feasible for their LTI counterparts. Therefore, the performance problem is generally unavoidable. However, it is usually very hard for a non-minimum phase LTI uncertain plant to achieve satisfactory performance using a LTI controller. In such cases, linear periodic controllers may be used to achieve specific design objective, for example, gain margin improvement while still maintaining reasonable performance requirement. An open problem is how to analyze the tradeoff systematically and to find possible applications for linear periodic controllers.

References

[1] B. R. Barmish and K. Wei, "Simultaneous stabilizability of single input single output systems," in *Modelling, Identification and Robust Control*, (North-Holland, Amsterdam), 1986.

[2] J. Sun, A. W. Olbrot, and M. P. Polis, "Robust stabilization and robust performance using model reference control and modelling error compensation," presented at *Inter. Workshop on Robust Contr.*, (San Antonio, Texas), March 1991.

[3] M. Fu, "Model reference robust control," presented at *Inter. Workshop on Robust Contr.*, (Tokyo, Japan), June 1991.

[4] M. Fu and H. Li, "Model following robust control of linear time-varying uncertain systems," in *Robustness of Dynamic Systems with Parameter Uncertainties*, (Birkhauser Verlag Basel), 1992.

[5] R. Bellman, J. Bentsman, and S. M. Meerkov, "Stability of fast periodic systems," *IEEE Trans. Auto. Contr.*, vol. AC-30, pp. 289–291, March 1985.

[6] S. Lee, S. M. Meerkov, and T. Runolfsson, "Vibrational feedback control: Zeros placement capabilities," *IEEE Trans. Auto. Contr.*, vol. AC-32, pp. 604–611, July 1987.

[7] R. K. Yedavalli, "A Kronecker based theory for robust root clustering of linear state space models with real parameter uncertainty" in *Proc. American Contr. Confer.*, (San Francisco, California), pp. 2755-2759, June 1993.

[8] H. Li and M. Fu, "Robust stabilization of non-minimum phase linear time invariant systems with real parametric uncertainty," Technical Report EE9433, Dept. Elect. Comp. Engg., The University of Newcastle, 1994.

[9] G. C. Goodwin and A. Feuer, "Linear periodic control: A frequency domain viewpoint", *Systems & Control Letters*, vol. 19, pp. 379-390, 1992.