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Single landmark based collaborative multi-agent localization with time-varying range measurements and information sharing ☆

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ABSTRACT

This paper considers the collaborative localization problem for a team of mobile agents. The goal is to estimate the relative coordinate of each agent with respect to a stationary landmark. Each agent is supposed to be able to measure its own velocity and the distances to nearby agents as well as the change rates of the distances. Due to limited sensing capability, movements of agents and possible interference of severe environments, the topology describing the measurements and communication information flow among the agents and the landmark is usually time-varying. Under such a scenario, this paper develops a consensus-like fusion scheme together with a continuous-time estimator for the collaborative localization problem. It is proved that the fused estimate of each agent's position globally asymptotically converges to its true value if the movements of the agents satisfy a persistent excitation condition and each agent is uniformly jointly reachable from the landmark in the time-varying topology. The effectiveness of the proposed scheme is verified through simulations without and with measurement noises.

1. Introduction

Recent advances in computing, communication, and sensing, have made it feasible to envision large numbers of autonomous vehicles working cooperatively to accomplish a task ranging from defense, surveillance, environment monitoring to search and rescue. Usually, the vehicles' location information is vital for location-aware applications. Although Global Positioning System (GPS) is usually used for precise navigation as it provides absolute position information, there are situations such as sky occlusion, hardware failure and GPS jamming, which may inhibit the use of GPS [1]. On the other hand, for low-cost mobile platforms, it may be infeasible to equip with GPS. Moreover, in many situations (e.g., formation control [2]), localizing mobile agents with respect to a common landmark or a leader agent rather than in a global coordinate system is competent. These observations motivate us to study the localization problem with respect to a single landmark.

Existing works study either localizing targets of common interest or self-localization of agents. Localization algorithms can be categorized into two classes depending on whether the network is static or mobile. The first class is concerned with static networks such as sensor networks where sensor nodes keep stationary once deployed. For the self-localization problem, the goal is to determine the Euclidean positions of all nodes in the network based on the knowledge of the positions of a few anchor nodes and inter-agent measurements (e.g., distance, bearing, RSSI, etc.). In the two dimensional space, generally two or three anchor nodes are required for the group of static sensors to locate themselves [3–7]. For the problem of localizing a target of interest, a cluster of static nodes works collaboratively to estimate the location of a target (e.g., a jammer in sensor networks [8]). The second class focuses on a network of mobile agents. For the self-localization problem, mobile agents use one or several landmarks as references to locate themselves. But for the problem of localizing a target of interest, a cluster of mobile agents seeks to determine the coordinate of the target either in a global coordinate system or in their local coordinate systems. For both problems in a mobile setting, the agents utilize relative measurements (distance, bearing, or

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distance plus bearing) from their exteroceptive sensors (e.g. lasers, cameras, etc.) together with their motion information (velocity and turning rate) from interoceptive sensors such as wheel encoders, accelerometers, gyroscopes, etc. [9–16].

This paper falls into the second class and aims to solve the collaborative localization problem of a group of mobile agents with respect to a single landmark. Towards this goal, this paper develops a cooperative estimation scheme for each mobile agent to locate itself, i.e., estimating the relative coordinates of each agent with respect to a stationary landmark. Different from static networks of agents, a mobile agent is able to localize itself with only one landmark, e.g. [17–19]. As a dual problem, a single mobile agent is capable of localizing a target of interest as well [9,20,21]. However, essentially speaking, [17] still utilizes two landmarks (one real and one virtual) while [18] requires both distance and bearing measurements. Moreover, different from most of the existing works (e.g., [14,17–21]), which require to know the absolute positions of landmarks or mobile agents, this paper requires no absolute information but only local measurements. Besides, collaborative localization using multiple mobile agents provides several potential advantages over using single mobile agent, including increased localization accuracy and coverage areas, robustness, and flexibility in case of limited sensing ranges and possible measurement failures due to severe environments. Compared with the probabilistic approach for cooperative localization [15,22,23] and the discrete-time Kalman filter approach to simultaneous localization of a group of mobile robots [12,13], the focus of this paper is on the deterministic and continuous-time aspect. Unlike our earlier work [10], which focuses on how to use a team of mobile agents to localize a target of interest under a fixed topology assumption, this paper addresses the collaborative localization problem under a time-varying topology, which is more challenging.

In this paper, every agent is equipped with onboard interoceptive sensors for the measurement of its own absolute velocity and exteroceptive sensors for the measurement of distances to its nearby agents and the change rates of the distances. Every agent has a local inertial frame attached to its body and the orientation of every agent's local frame is the same as that of the landmark. Moreover, it is assumed that the nearby mobile agents can communicate with each other, but the landmark is silent. We abstract a group of mobile agents together with the landmark as a set of nodes, and then use a directed graph to indicate that an agent i has a relative measurement to another agent j and can receive information from it as well if there is a directed edge connecting from node j to node i. Due to possible measurement failures or possible movements of the agents outside of the sensing range about their neighbors, the directed graph describing the information flow relationship is time-varying. Here we assume the communication between neighboring agents is with acknowledgment receipts going backwards. In this way, an agent knows when its neighboring agents lose track of the information stream and when the information stream is recovered. But it should be noted that the acknowledgment going backward is used only to indicate the status of a communication link, while the main stream of the valuable information goes the way as indicated in the directed graph. It can have certain benefits (saving communication energy for example) when compared with bidirectional communications. Therefore, in this paper, we use the directed graph model to indicate the main stream of information exchange excluding the acknowledgments. To deal with the collaborative localization problem in such a scenario, we first propose a continuous-time estimator for each agent to estimate its relative position with respect to its neighbors by utilizing the distance and change rate measurements, the velocities of itself and its neighbors, and the displacements of neighbors during the intervals when the distance and change rate measurements are lost. Second, a consensus-like fusion scheme is developed for every agent to localize itself with respect to the landmark by fusing the estimate of its own position with respect to the landmark using the range measurements about the landmark when available and the estimates of its position with respect to its neighbors. By doing so, the agents, which are not able to directly measure the distance to the landmark, can also locate itself with the help of its neighbors.

We summarize the major contributions of this work as follows: (1) Different from most existing works that require the knowledge of the absolute positions of some landmarks or agents, this paper develops a collaborative localization scheme to estimate the relative coordinates of a group of mobile agents with respect to a single landmark utilizing only local measurements and limited local information exchange between nearby agents. Thus, the localization scheme is fully distributed. (2) This paper addresses collaborative localization in a very general setting that the information flow graph among the neighboring agents is unidirectional and time-varying to reflect the practical concerns of neighbor changes and measurement failures over time. However, asymptotic convergence of the proposed collaborative localization scheme is still ensured if the information flow graph has the property of being sufficiently connected over time. (3) It is proved that with the collaborative localization scheme proposed in this paper, each agent can have an uninterrupted estimation of its relative coordinate with respect to the landmark even when it does not have any relative measurements about the landmark or its neighbors. (4) The collaborative localization scheme proposed in this paper works not only in the two dimensional space but also in the three dimensional space.

2. Preliminaries and problem formulation

In this section, we first introduce basic notions of graphs, which will be used later. Then the collaborative localization problem is formulated.

2.1. Preliminaries

Let $\mathbb{R}^{n\times n}$ be the set of $n \times n$ real matrices. The superscript T represents the transpose of a real matrix. I_p represents the identity matrix of dimension p. Matrices with nonnegative off-diagonal elements are referred to as Metzler matrices [24]. The matrix inequality $A > (\geq)\ B$ means that $A - B$ is positive (semi-) definite. $A \otimes B$ denotes the Kronecker product of matrices A and B. For a vector x, $\|x\|$ denotes its 2-norm. For a finite set $\mathcal{S}$, $|\mathcal{S}|$ denotes the cardinality of $\mathcal{S}$.

A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a non-empty finite set $\mathcal{V} = \{v_1, \ldots, v_n\}$ of elements called nodes and a finite set $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ of ordered pairs of distinct nodes called edges. A walk in a graph $\mathcal{G}$ is an alternating sequence $\mathcal{W} : v_1e_1v_2e_2\cdots v_{k-1}e_{k-1}v_k$ of nodes v_i and edges e_i such that $e_i = (v_i, v_{i+1})$ for every $i = 1, 2, \ldots, k-1$. We call $\mathcal{W}$ a walk from v_1 to v_k. Let $\mathcal{R} \subset \mathcal{V}$ be a subset of nodes in $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. A node $v \in \mathcal{V} - \mathcal{R}$ is said to be reachable from $\mathcal{R}$ if there exists a walk from a node in $\mathcal{R}$ to v.

The set of neighbors of node i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}, j \neq i\}$. The Laplacian matrix $L_{\mathcal{G}} = [l_{ij}] \in \mathbb{R}^{n\times n}$ of $\mathcal{G}$ is defined as $l_{ii} = |\mathcal{N}_i|$, $l_{ij} = -1$ if $j \in \mathcal{N}_i$ and $l_{ij} = 0$ otherwise.

When the edge set in a directed graph changes over time, we call it a time-varying graph, denoted as $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$. For a time-varying graph $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$, a node v is said to be *uniformly jointly reachable* from $\mathcal{R} \subset \mathcal{V}$ if there exists $T > 0$ such that for all t, v is reachable from $\mathcal{R}$ in the union graph $\mathcal{G}([t, t+T])$, whose edge set is the union of the edge set of $\mathcal{G}(t)$ over the time interval $[t, t+T]$. An example is given in Fig. 1, for which v_3 is uniformly jointly reachable from v_1, since we can take $T = 2$ and then for any t the union graph $\mathcal{G}([t, t+T]) = \mathcal{G}_1 \cup \mathcal{G}_2$.

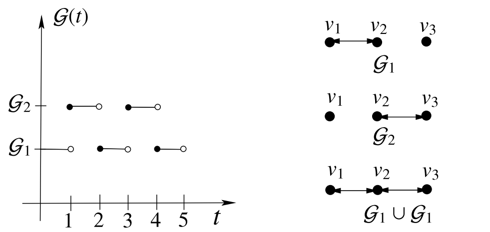


Fig. 1. Graph $\mathcal{G}(t)$ that periodically switches between $\mathcal{G}_1$ and $\mathcal{G}_2$.

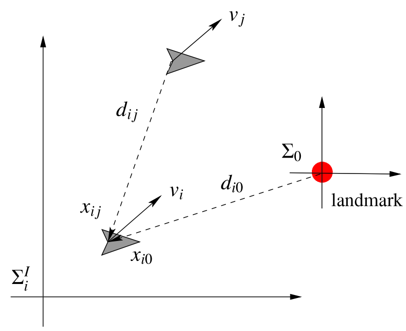


Fig. 2. Local frames and relative states.

2.2. Problem formulation

We address the collaborative localization problem in this paper. Consider a team of N mobile agents (labeled consecutively from 1 to N) working collaboratively to localize themselves in a plane or in three dimensional space. Suppose there is a stationary landmark labeled 0. The goal for each agent is to estimate its coordinates in the landmark's local frame Σ_0.

Suppose each agent i is able to access its own velocity v_i in its own inertial frame Σ_i^I. The orientations of reference frames Σ_i^I, $i = 1, \ldots, N$, are the same as that of Σ_0. We note that this can be satisfied if the agents carry compasses. Moreover, we assume that each agent i is equipped with an onboard sensor such that it can have the range measurement $d_{ij}(t)$ and the change rate $\dot{d}_{ij}(t)$ of its neighbor agent j and/or the range measurement $d_{i0}(t)$ and the change rate $\dot{d}_{i0}(t)$ of the landmark. An illustration is depicted in Fig. 2 and it follows that

$$d_{ij}(t) = d_{ji}(t) = \|x_{ij}(t)\|$$

and

$$\dot{x}_{ij}(t) = v_{ij}(t), \tag{1}$$

where $x_{ij}(t)$ and $v_{ij}(t) = v_i(t) - v_j(t)$, $j = 0, 1, \ldots, N$, are the relative coordinates and relative velocity of agent i in agent j's or the landmark's local frame respectively.

The objective is to develop an estimator such that each agent can estimate its relative coordinates $x_{i0}(t)$ in the landmark's frame Σ_0 and locate itself.

If each agent as well as the landmark is regarded as a node, then we can use a directed graph $\mathcal{G}$ to describe the coupling topology of N agents and the landmark. If agent i ($i = 1, \ldots, N$) can measure $d_{ij}(t)$ and $\dot{d}_{ij}(t)$, and it can receive local information from agent j ($j = 1, \ldots, N, j \neq i$), there is an edge from node j to node i in $\mathcal{G}(t)$. The information includes agent j's velocity $v_j(t)$, agent j's estimated coordinates and agent j's displacement during the interval when the measurements and communication are lost. Moreover, if agent i can measure $d_{i0}(t)$ and $\dot{d}_{i0}(t)$, then there is an edge from node 0 to node i. We note that the landmark is silent, that is, it does not take any measurement or communicate with any agent. Note that $\mathcal{G}$ may change over time as the measurements and communication may be unreliable, so we denote $\mathcal{G}$ by $\mathcal{G}(t)$ to indicate that it is time-varying. For each agent i, we denote the set of its neighbors by $\mathcal{N}_i(t)$. Since $\mathcal{G}(t)$ shows how information flows among the agents and the landmark, we call $\mathcal{G}(t)$ the *information flow graph*. Denote the corresponding Laplacian matrix of $\mathcal{G}(t)$ by $L_{\mathcal{G}}(t)$.

Remark 1. In practice, the change rate $\dot{d}_{ij}(t)$ may be obtained approximately by applying appropriate differentiation or low-pass filters, e.g., the linear time-varying differentiator [25] and the extended state observer (ESO) [26].

Remark 2. In this paper, though the main stream goes the way as indicated by a directed graph, we assume that the acknowledgment signal can go backward, which is only used to indicate whether a link is on or off. That is, by allowing this, agent i knows when its out-neighbor j loses track of the information stream and when the communication is resumed. However, we do not need agent j to send its measurement and estimate information to agent i for the reason of energy saving.

Throughout this paper, we suppose the following assumptions.

Assumption 1. The velocity $v_i(t)$ of each agent i, $i = 1, \ldots, N$, is continuously differentiable and bounded.

Assumption 2. For any i and $j \in \mathcal{N}_i$, $d_{ij}(t) : \mathbb{R}_{>0} \mapsto \mathbb{R}$ is continuously differentiable and bounded.

Assumption 1 ensures that the motions of the agents are driven by finite forces. The continuous differentiability of $d_{ij}(t)$ in Assumption 2 can be inferred from Assumption 1.

3. Consensus-based collaborative localization algorithm

In this section, we develop a distributed algorithm for the collaborative localization problem of multi-agent systems based on a single landmark. Firstly, we address the problem of how to estimate the relative coordinates of an agent in its neighbor's local frame. By this method, if the landmark is a neighbor of an agent, i.e., the agent can have the relative measurements with respect to the landmark, the agent can then estimate its own coordinates in the landmark's frame directly. We term this as a *direct estimate*. Instead, if the landmark is not a neighbor of an agent, then the agent is not able to estimate its own coordinates with respect to the landmark using the relative measurements between the landmark and itself. In this case, if the agent can localize themselves in its neighbors' local frames and if the coordinates of its neighbors with respect to the landmark are known, then the agent is still capable of determining its own coordinates in the landmark's frame. We term this as an *indirect estimate*. However, there may exist multiple neighbors that can help the agent localize itself with respect to the landmark in this way. Such multiple estimates should be combined in order not to rely on a sole neighbor. Moreover, even in the case that the direct estimate is available, the indirect estimates can also be combined to improve the estimation accuracy and convergence rate. Therefore, in the second part of this section, we design a consensus-based cooperative fusion scheme for each agent to fuse the direct estimate and all available indirect estimates.

3.1. Localization subject to measurements and communication loss

As we know, agents may lose its measurements and communication due to severe environments or sensor failures. In this subsection, we assume that agent i ($i = 1, 2, \ldots, N$) is able to communicate with agent j and access to the range measurement $d_{ij}(t)$ and the change rate $\dot{d}_{ij}(t)$ during some time intervals. We seek to design a direct estimator to estimate the relative coordinates $x_{ij}(t)$ of agent i in agent j's local frame.

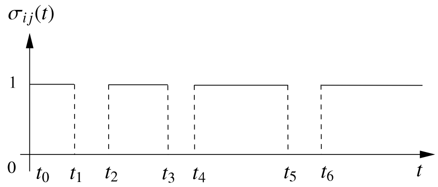


Fig. 3. An illustration of the indicator function $\sigma_{ij}(t)$.

Since the local relative measurements and communication may be unreliable, without loss of generality, we assume that agent i is able to obtain the local relative measurement $d_{ij}(t)$ and $\dot{d}_{ij}(t)$ at time $t \in [t_0, t_1) \cup [t_2, t_3) \cup \ldots$ and lose the measurements at time $t \in [t_1, t_2) \cup [t_3, t_4) \cup \ldots$. To be more specific, we define an indicator function $\sigma_{ij}(t)$ to indicate the status, i.e., $\sigma_{ij}(t) = 1$ if $d_{ij}(t)$ and $\dot{d}_{ij}(t)$ are available to agent i at time t and $\sigma_{ij}(t) = 0$ otherwise. Then, as illustrated in Fig. 3, it follows that

$$\sigma_{ij}(t) = \begin{cases} 1, & t \in [t_{2k}, t_{2k+1}), \\ 0, & t \in [t_{2k+1}, t_{2k+2}), \end{cases} \qquad k = 0, 1, \ldots$$

Taking the derivative of both sides of $d_{ij}^2(t) = \|x_{ij}(t)\|^2$ with respect to time and considering (1), one obtains

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[d_{ij}^2(t)\right] = 2v_{ij}^{\mathrm{T}}(t)x_{ij}(t),$$

i.e.,

$$d_{ij}(t)\dot{d}_{ij}(t) = v_{ij}^{\mathrm{T}}(t)x_{ij}(t).$$

Denote by $\hat{x}_{ij}(t)$ the estimate of $x_{ij}(t)$. When $j \in \mathcal{N}_i(t)$, agent i seeks to estimate its coordinate by using the local relative measurements $d_{ij}(t)$, $\dot{d}_{ij}(t)$, and the motion information of itself and its neighbors. When $j \notin \mathcal{N}_i(t)$, i.e., agent i loses its measurements and the communication with agent j, agent i maintains its previous estimate. Once the measurements and communication are recovered, agent i uses its own inertial navigation information and the inertial navigation information received from its neighbors to eliminate the estimation error accumulated during the period when the measurements and communication are lost. Following this idea, we develop the following estimator

$$\begin{cases} \dot{\hat{x}}_{ij}(t) = v_{ij}(t) + v_{ij}(t)\left[d_{ij}(t)\dot{d}_{ij}(t) - v_{ij}^{\mathrm{T}}(t)\hat{x}_{ij}(t)\right], \\ \quad t \in [t_{2k}, t_{2k+1}) \\ \dot{\hat{x}}_{ij}(t) = 0, \quad t \in [t_{2k+1}, t_{2k+2}) \\ \hat{x}_{ij}(t_{2k+2}) = \hat{x}_{ij}(t_{2k+1}) + s_i - s_j, \end{cases}$$

where $s_i = \int_{t_{2k+1}}^{t_{2k+2}} v_i(\tau)\mathrm{d}\tau$ and $s_j = \int_{t_{2k+1}}^{t_{2k+2}} v_j(\tau)\mathrm{d}\tau$ are the displacements of agent i and agent j during the time interval $[t_{2k+1}, t_{2k+2}]$, respectively. According to Remark 2, we note that here s_i and s_j are calculated by agent i and agent j respectively since only respective velocities are required. Once the measurements and communication are recovered at time $t = t_{2k+2}$, agent j sends its displacement s_j to agent i.

We denote the estimation error by $\tilde{x}_{ij}(t) = \hat{x}_{ij}(t) - x_{ij}(t)$ and obtain the error dynamics as follows

$$\begin{cases} \dot{\tilde{x}}_{ij}(t) = -v_{ij}(t)v_{ij}^{\mathrm{T}}(t)\tilde{x}_{ij}(t), & t \in [t_{2k}, t_{2k+1}) \\ \dot{\tilde{x}}_{ij}(t) = -v_{ij}(t), & t \in [t_{2k+1}, t_{2k+2}) \\ \tilde{x}_{ij}(t_{2k+2}) = \tilde{x}_{ij}(t_{2k+1}). \end{cases} \tag{2}$$

Before discussing the convergence property of the error system (2), we first introduce the notion of persistent excitation.

Lemma 1 ([27]). *Let $V(\cdot) : \mathbb{R}_+ \to \mathbb{R}^{n\times r}$ be regulated (i.e., one-sided limits exist for all $t \in \mathbb{R}_+$). Then*

$$\dot{x} = -VV^{\mathrm{T}}x$$

is exponentially asymptotically stable if and only if for some positive δ, α_1 and α_2, and for all $s \in \mathbb{R}_+$

$$\alpha_1 I \le \int_s^{s+\delta} V(t)V^{\mathrm{T}}(t)\mathrm{d}t \le \alpha_2 I. \tag{3}$$

Remark 3 ([27]). In the context of this lemma, we require that although $\|x(t)\|$ decays exponentially fast, it decays no faster than exponentially. This means that there exist positive $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ such that $\gamma_1 e^{-\gamma_2(t-s)} \le \|\Phi(t,s)\| \le \gamma_3 e^{-\gamma_4(t-s)}$ for all $t \ge s \ge 0$, where $\Phi(\cdot,\cdot)$ is the transition matrix associated with $-V(\cdot)V(\cdot)^{\mathrm{T}}$.

We note that (3) is well known as the persistent exciting (p.e.) condition. To focus our attention on the motions of the agents during the period when the local relative measurements are available, we define

$$\bar{v}_{ij}(t) = \sigma_{ij}(t)v_{ij}(t).$$

Then we provide the following result regarding the convergence of the error system (2).

Theorem 1. *There exist $\lambda > 0$ and $c > 0$ such that $\|\sigma_{ij}(t)\tilde{x}_{ij}(t)\| \le ce^{-\lambda t}$ for all $t \ge 0$ if there exist $\lambda_1 > 0$, $\lambda_2 > 0$ and $T > 0$ such that for all $t \ge 0$,*

$$\lambda_1 I \le \int_t^{t+T} \bar{v}_{ij}(\tau)\bar{v}_{ij}^{\mathrm{T}}(\tau)\mathrm{d}\tau \le \lambda_2 I. \tag{4}$$

Proof. We first consider a system related to (2):

$$\dot{\varepsilon}(t) = -\bar{v}_{ij}(t)\bar{v}_{ij}^{\mathrm{T}}(t)\varepsilon(t). \tag{5}$$

It can be observed from (2) and (5) that if $\tilde{x}_{ij}(t_{2k}) = \varepsilon(t_{2k})$, then $\tilde{x}_{ij}(t) = \varepsilon(t)$ for all $t \in [t_{2k}, t_{2k+1})$. Also $\varepsilon(t)$ remains unchanged for $t \in [t_{2k+1}, t_{2k+2})$ while $\tilde{x}_{ij}(t)$ changes with time. However, for $t = t_{2k+2}$, $\tilde{x}_{ij}(t_{2k+2}) = \tilde{x}_{ij}(t_{2k+1})$ according to (2). Therefore, it can be concluded that $\tilde{x}_{ij}(t) = \varepsilon(t)$ for all $t \in [t_{2k}, t_{2k+1})$ as long as $\tilde{x}_{ij}(0) = \varepsilon(0)$.

Moreover, according to the definition of $\sigma_{ij}(t)$, it follows that given any k, $\sigma_{ij}(t)\tilde{x}_{ij}(t) = \tilde{x}_{ij}(t)$ for $t \in [t_{2k}, t_{2k+1})$ and $\sigma_{ij}(t)\tilde{x}_{ij}(t) = 0$ for $t \in [t_{2k+1}, t_{2k+2})$. To understand the relationship, schematic evolution curves of $\|\tilde{x}_{ij}(t)\|$, $\|\varepsilon(t)\|$, and $\|\sigma_{ij}(t)\tilde{x}_{ij}(t)\|$ are depicted in Fig. 4.

For the system (5), we know by Lemma 1 that the zero solution of (5) is exponentially asymptotically stable if and only if there exist $\lambda_1 > 0$, $\lambda_2 > 2$ and $T > 0$ such that (4) is satisfied for all $t \ge 0$. Therefore, according to the relationship of $\|\varepsilon(t)\|$ and $\|\sigma_{ij}(t)\tilde{x}_{ij}(t)\|$, the conclusion follows. □

Next we come to understand the p.e. condition (4). The upper bound of (4) holds obviously because of Assumption 1. We will focus on the lower bound of (4). First we recall the notion of *linearly independent functions.*

Definition 1 ([28]). The n functions $f_1(t), f_2(t), \ldots, f_n(t)$ are linearly dependent if, for some $c_1, c_2, \ldots, c_n \in \mathbb{R}$ not all zero,

$$\sum_{i=1}^{n} c_i f_i(t) = 0$$

for all t in some interval $\mathcal{I}$. Otherwise, they are said to be linearly independent.

Consider in two dimensions and let $\bar{v}_{ij}(t) = [\bar{v}_{ij_x}(t),\ \bar{v}_{ij_y}(t)]^{\mathrm{T}}$. Then

$$\bar{v}_{ij}(t)\bar{v}_{ij}^{\mathrm{T}}(t) = \begin{bmatrix} \bar{v}_{ij_x}^2(t) & \bar{v}_{ij_x}(t)v_{ij_y}(t) \\ \bar{v}_{ij_x}(t)v_{ij_y}(t) & \bar{v}_{ij_y}^2(t) \end{bmatrix}.$$

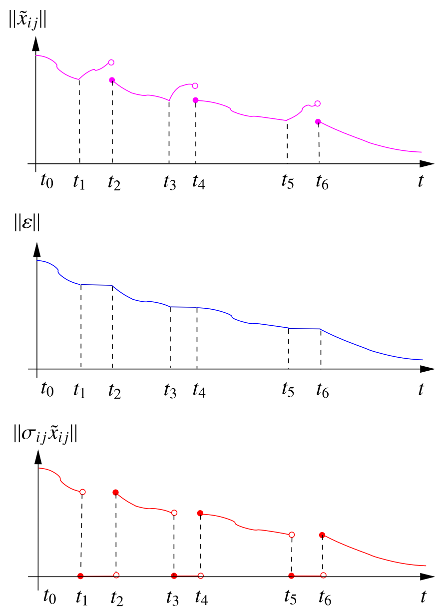


Fig. 4. Schematic evolution of $\|\tilde{x}_{ij}(t)\|$, $\|\varepsilon(t)\|$, and $\|\sigma_{ij}(t)\tilde{x}_{ij}(t)\|$.

Define

$$P(t) = \int_t^{t+T} \bar{v}_{ij}(\tau)\bar{v}_{ij}^{\mathrm{T}}(\tau)\mathrm{d}\tau = \begin{bmatrix} \int_t^{t+T} \bar{v}_{ij_x}^2(\tau)\mathrm{d}\tau & \int_t^{t+T} \bar{v}_{ij_x}(\tau)\bar{v}_{ij_y}(\tau)\mathrm{d}\tau \\ \int_t^{t+T} \bar{v}_{ij_x}(\tau)\bar{v}_{ij_y}(\tau)\mathrm{d}\tau & \int_t^{t+T} \bar{v}_{ij_y}^2(\tau)\mathrm{d}\tau \end{bmatrix}.$$

By the Cauchy–Bunyakovsky inequality [29], it follows that

$$\int_t^{t+T} \bar{v}_{ij_x}^2(\tau)\mathrm{d}\tau \int_t^{t+T} \bar{v}_{ij_y}^2(\tau)\mathrm{d}\tau \geq \left(\int_t^{t+T} \bar{v}_{ij_x}(\tau)\bar{v}_{ij_y}(\tau)\mathrm{d}\tau\right)^2$$

holds for any given $T \geq 0$. The Cauchy–Bunyakovsky inequality becomes an equality if and only if $\bar{v}_{ij_x}(t)$ and $\bar{v}_{ij_y}(t)$ are linearly dependent in the time interval $[t, t+T]$. Hence, to satisfy the lower bound of (3), $P(t)$ must be positive definite, which requires that there exists $T > 0$ such that for all $t \geq 0$, the two components $\bar{v}_{ij_x}(t)$ and $\bar{v}_{ij_y}(t)$ of the relative velocity to be linearly independent in the time interval $[t, t+T]$.

Remark 4. Similarly, in three dimensions, denote

$$\bar{v}_{ij}(t) = [\bar{v}_{ij_x}(t),\ \bar{v}_{ij_y}(t),\ \bar{v}_{ij_z}(t)]^{\mathrm{T}}$$

and it would require that there exists $T > 0$ such that for all $t \geq 0$, $\bar{v}_{ij_x}(t)$, $\bar{v}_{ij_y}(t)$ and $\bar{v}_{ij_z}(t)$ to be pairwisely linearly independent in the time interval $[t, t+T]$.

3.2. Collaborative multi-agent localization

In the preceding subsection, we assume that the local relative measurements ($d_{ij}(t)$ and $\dot{d}_{ij}(t)$) are unreliable and are only available to agent i from time to time. Then an estimator is designed for the agent to locate itself. If the landmark is a neighbor of agent i, then agent i can have a direct estimate $\hat{x}_{i0}(t)$ of its coordinates in the landmark's frame. However, agents may lose its local relative measurements due to severe environments or temporary sensor failure, or even worse, some agents may not have relative measurements with respect to the landmark all the time because the landmark is out of its sensing range. In such cases, collaboration from neighbor agents is necessary to help the agents locate themselves. In this subsection, we develop a collaborative estimator for each agent i to estimate its relative coordinate with respect to the landmark though it may not have a direct measurement about the landmark or may have measurement failures over times.

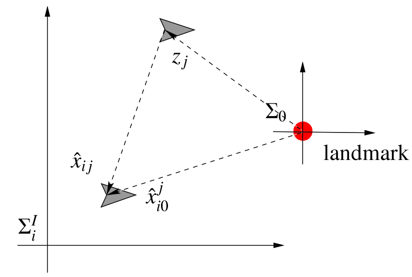


Fig. 5. Indirect estimation of the relative coordinates of agent i.

Denote by $z_i(t)$ the fused estimate of each agent i's relative coordinates with respect to the landmark by combining all the available measurements and information from its neighbors. Then, as illustrated in Fig. 5, if agent i is able to estimate its relative coordinates $x_{ij}(t)$ in its neighbor agent j's local frame and agent j is able to communicate its own fused estimate $z_j(t)$ to agent i, then agent i can obtain an indirect estimate $\hat{x}_{i0}^j(t)$ of its coordinates with respect to the landmark by

$$\hat{x}_{i0}^j(t) = \hat{x}_{ij}(t) + z_j(t). \tag{6}$$

Furthermore, since the landmark is silent, agents can imagine the landmark as a dummy agent broadcasting zero position estimate $z_0(t) \equiv 0$. Let

$$\hat{x}_{i0}^0(t) = \hat{x}_{i0}(t) + z_0(t). \tag{7}$$

We develop a consensus-like estimation fusion scheme for agent i to update its fused estimate, i.e.,

$$\dot{z}_i(t) = v_i(t) + \sum_{j\in\mathcal{N}_i(t)} \left[\hat{x}_{i0}^j(t) - z_i(t)\right]. \tag{8}$$

That is to say, each agent i updates its fused estimate with (8) no matter whether the agent can directly have the relative measurements about the landmark or not.

Remark 5. Regarding (8), the physical meaning is that when agent i does not have any neighbors, it just locates itself using the inertial navigation information, and when agent i recovers the local relative measurements to its neighbors and receives information from its neighbors, it makes use of this information to reduce the error accumulated with only inertial navigation.

In summary, the estimator executed on each agent i is as follows:

$$\begin{cases} \dot{\hat{x}}_{ij} = v_{ij} + v_{ij}(d_{ij}\dot{d}_{ij} - v_{ij}^{\mathrm{T}}\hat{x}_{ij}), & t \in [t_{2k}, t_{2k+1}), j \in \mathcal{N}_i(t) \\ \dot{\hat{x}}_{ij} = 0, & t \in [t_{2k+1}, t_{2k+2}), j \notin \mathcal{N}_i(t) \\ \hat{x}_{ij}(t_{2k+2}) = \hat{x}_{ij}(t_{2k+1}) + s_i - s_j, & j \in \mathcal{N}_i(t) \\ \dot{z}_i(t) = v_i(t) + \sum_{j\in\mathcal{N}_i(t)} \left[\hat{x}_{i0}^j(t) - z_i(t)\right], \end{cases} \tag{9}$$

where $\hat{x}_{i0}^j(t)$ is defined in (6) and (7). Regarding the convergence of (9), we have the following result.

Theorem 2. *Suppose the p.e. condition (4) holds for every pair (i, j) that occurs infinite times in $\mathcal{G}(t)$. If every agent i is uniformly jointly reachable from the landmark (node 0) in $\mathcal{G}(t)$, then every agent's fused estimate $z_i(t)$ in (9) asymptotically converges to the true relative coordinates $x_{i0}(t)$ of agent i.*

The proof of Theorem 2 uses the input-to-state stability theory [30].

Proof of Theorem 2. For $i = 1, \ldots, N$, we let $y_i(t) = z_i(t) - x_{i0}(t)$. Then (8) can be transformed to

$$\begin{aligned}\dot{y}_i(t) &= \sum_{j\in\mathcal{N}_i(t)} \left[y_j(t) - y_i(t)\right] + \sum_{j\in\mathcal{N}_i(t)} \tilde{x}_{ij}(t) \\ &= \sum_{j\in\mathcal{N}_i(t)} \left[y_j(t) - y_i(t)\right] + u_i(t), \qquad (10)\end{aligned}$$

where $y_0(t) \equiv \mathbf{0}$ and $u_i(t) = \sum_{j\in\mathcal{N}_i(t)} \tilde{x}_{ij}(t)$. This is a typical consensus system, in which $y_i(t)$'s ($i = 1, \ldots, N$) are the individual states of N agents, y_0 can be treated as the state of a leader agent, and u_i can be thought as an external input signal.

Since $y_0(t) \equiv \mathbf{0}$, we add an augmented equation $\dot{y}_0(t) = \mathbf{0}$ with $y_0(0) = \mathbf{0}$. Denote $y(t) = [y_0^{\mathrm{T}}(t)\ y_1^{\mathrm{T}}(t)\ \cdots\ y_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, $u_0(t) = \mathbf{0}$, $u = [u_0^{\mathrm{T}}(t)\ u_1^{\mathrm{T}}(t) \cdots u_N^{\mathrm{T}}(t)]^{\mathrm{T}}$. Then (10) can be written in a matrix form by

$$\dot{y}(t) = -\left[L_{\mathcal{G}}(t) \otimes I_p\right] y(t) + u(t), \qquad (11)$$

where $p = 2$ or 3 depending on the dimension of the ambient space and $L_{\mathcal{G}}(t)$ is the Laplacian matrix of $\mathcal{G}(t)$. We first consider the unforced system

$$\dot{y}(t) = -\left[L_{\mathcal{G}}(t) \otimes I_p\right] y(t). \qquad (12)$$

It can be verified that $-L_{\mathcal{G}}(t)$ is Metzler with zero row sums. Since any agent i is uniformly jointly reachable from node 0, then there exist an interval length $T > 0$ and a threshold δ such that for all $t \in \mathbb{R}$ the δ-digraph (see [31]) associated to

$$-\int_t^{t+T} L_{\mathcal{G}}(s)\mathrm{d}s$$

has the property that all nodes are reachable from node 0. Thus, according to Theorem 1 in [31], all components of any solution of (12) uniformly exponentially converge to a common value as $t \to \infty$. As $y_0(t) \equiv \mathbf{0}$, we conclude that all $y_i(t) \to \mathbf{0}$ as $t \to \infty$, $i = 1, \ldots, N$, which means that the unforced system (12) is exponentially stable.

Now let us look at the system (11). Since $u_i(t) = \sum_{j\in\mathcal{N}_i(t)} \tilde{x}_{ij}(t) = \sum_{j=0}^{N} \sigma_{ij}(t)\tilde{x}_{ij}(t)$ and $\sigma_{ij}(t)\tilde{x}_{ij}(t)$ tends to zero followed from Theorem 1, we come to the conclusion that $\lim_{t\to\infty} u(t) = \mathbf{0}$. Denote $f(t, y, u) = -\left(L_{\mathcal{G}}(t) \otimes I_p\right) y + u$. Then it can be checked that $f(t, y, u)$ is globally Lipschitz in (y, u), uniformly in t. So it follows that (11) is input-to-state stable, i.e., for all $t \geq t_0$,

$$\|y(t)\| \leq \beta(\|y(t_0)\|, t - t_0) + \gamma(\sup_{t_0\leq\tau\leq t} \|u(\tau)\|).$$

For any given $\varepsilon > 0$, choose $\mu > 0$ such that $\gamma(\mu) \leq \varepsilon/2$. Since $\lim_{t\to\infty} u(t) = 0$, it follows that there exists $t_1 > 0$ such that $\|u(t)\| \leq \mu$ when $t \geq t_1$. Now, since $y(t)$ is bounded, suppose the bound of $y(t)$ is ρ. Then, it follows that

$$\begin{aligned}\|y(t)\| &\leq \beta(\|y(t_1)\|, t - t_1) + \gamma(\mu) \\ &\leq \beta(\rho, t - t_1) + \varepsilon/2 \qquad (13)\end{aligned}$$

for $t \geq t_1$. Since $\beta(\rho, t - t_1) \to 0$ as $t \to \infty$, there exists $t_2 > 0$ such that $\beta(\rho, t) \leq \varepsilon/2$, $t \geq t_2$. Thus, it follows from (13) that $\|y(t)\| < \varepsilon$, $t > \mathcal{T}$, where $\mathcal{T} = \max(t_1, t_2)$, which implies that

$$\lim_{t\to\infty} \|y(t)\| = 0,$$

i.e., $y(t)$ converges to $\mathbf{0}$ as $t \to \infty$. □

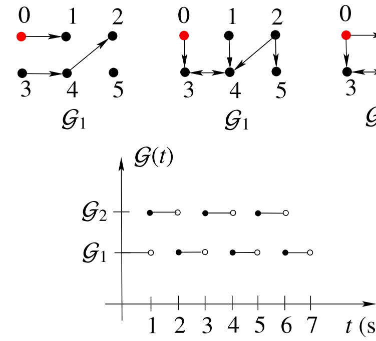


Fig. 6. The periodic switching graph $\mathcal{G}(t)$ that switches between two different topologies $\mathcal{G}_1$ and $\mathcal{G}_2$ for the first simulation.

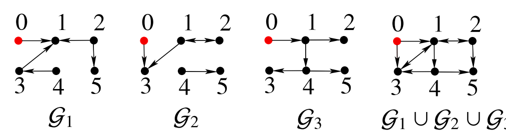


Fig. 7. The three graphs $\mathcal{G}_1$, $\mathcal{G}_2$ and $\mathcal{G}_3$, among which $\mathcal{G}(t)$ randomly switches, and the union of the three graphs used in the second simulation.

Remark 6. It can be inferred from Theorem 2 that once the estimator (9) converges, agent i can have an uninterrupted precise estimation of its coordinate even when it does not have any measurements about its neighbor agents or about the landmark.

4. Simulation

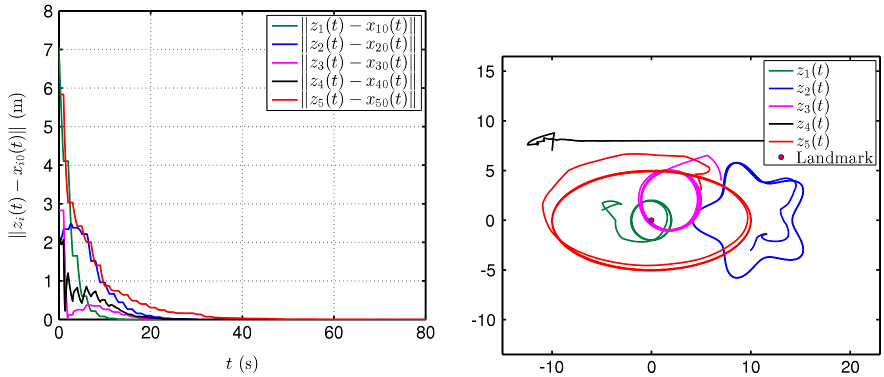
In this section, we carry out simulations of five mobile agents achieving collaborative localization in the two dimensional space. Just for demonstration of our proposed estimation scheme, we set the landmark at the origin and let the five agents be governed by the following dynamics:

$$\dot{x}_1(t) = \begin{bmatrix} -2\sin t \\ 2\cos t \end{bmatrix}, \qquad \dot{x}_2(t) = \begin{bmatrix} \cos\frac{t}{5} - \cos t \sin\frac{t}{5} \\ \sin\frac{t}{5} + \cos t \cos\frac{t}{5} \end{bmatrix},$$

$$\dot{x}_3(t) = \begin{bmatrix} -\sin\frac{t}{3} \\ \cos\frac{t}{3} \end{bmatrix}, \qquad \dot{x}_4(t) = \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix}, \qquad \dot{x}_5(t) = \begin{bmatrix} -\frac{10}{8}\sin\frac{t}{8} \\ \frac{5}{8}\cos\frac{t}{8} \end{bmatrix}.$$

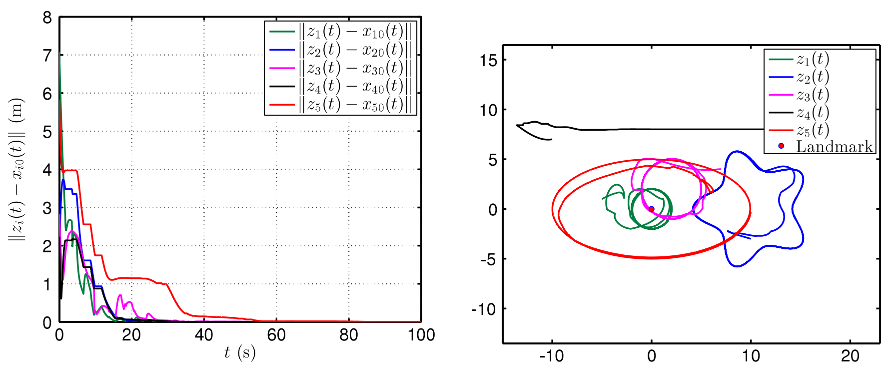
Moreover, the five agents are initially positioned at $(2, 0)$, $(10, -5)$, $(5, 2)$, $(-12, 8)$, $(10, 0)$.

In the first simulation, as depicted in Fig. 6, we let the information flow graph $\mathcal{G}(t)$ of the five agents together with the landmark be periodically switching between two graphs $\mathcal{G}_1$ and $\mathcal{G}_2$. That is, agent 1 and 3 have direct measurements about the landmark periodically while agent 2, 4 and 5 do not and they can only have indirect estimates through their neighbors. In the second simulation, we consider a more general situation. Rather than a periodically switching topology, we let the information flow graph randomly switch among three graphs $\mathcal{G}_1$, $\mathcal{G}_2$ and $\mathcal{G}_3$, as shown in Fig. 7 and also the duration for which each graph holds is random. For the first simulation, it can be checked that for the time-varying information flow graph every node is uniformly jointly reachable from the landmark. Moreover, it can be verified that the p.e. condition (4) is satisfied. For the second simulation, generally speaking, the condition that every node is uniformly jointly reachable from the landmark and the p.e. condition (4) is satisfied though it is difficult to confirm it rigorously. We adopt the estimator described in (9) to estimate the coordinates of every agent. By Theorem 2, every agent has its estimate $z_i(t)$



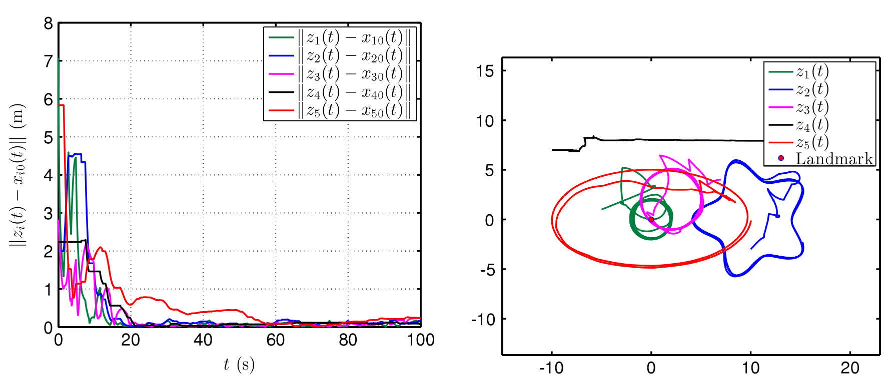
(a) $\|z_i(t) - x_{i0}(t)\|$. (b) The estimates $z_i(t)$.

Fig. 8. The first simulation.



(a) $\|z_i(t) - x_{i0}(t)\|$. (b) The estimates $z_i(t)$.

Fig. 9. The second simulation.



(a) $\|z_i(t) - x_{i0}(t)\|$. (b) The estimates $z_i(t)$.

Fig. 10. The third simulation.

converging to the true coordinates $x_{i0}(t)$ in the landmark's frame. The evolution curves of the estimation errors $\|z_i(t) - x_{i0}(t)\|$ $(i = 1, \ldots, 5)$ for the first and second simulation are shown in Figs. 8(a) and 9(a) respectively, which also validate our theoretic results. To see the estimates more visually, we plot the trajectories of the estimates in the plane, namely, $z_1(t), \ldots, z_5(t)$ (see Figs. 8(b) and 9(b)).

As we know, usually there are inevitable measurement noises. Therefore, we conduct another simulation to inspect the influence of the measurement noises on the estimation result. The setup of the third simulation is the same as that of the second one except that d_{ij}, $\dot{d}_{ij}$ and v_i used in (9) are assumed to be contaminated with white Gaussian noises. The mean and variance of the noises for d_{ij} and $\dot{d}_{ij}$ are 0 and 4 respectively. All the means of noises for v_i $(i = 1, 2, \ldots, 5)$ are 0. The covariance matrices for noises in v_i $(i = 1, 2, 3, 5)$ are all $\begin{bmatrix} 0.0025 & 0 \\ 0 & 0.0025 \end{bmatrix}$. The covariance matrix for noise in v_4 is $\begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}$. That is, the standard deviation of the noises in velocities is about 5% of the moving speed. The evolution curves of the estimation errors and the trajectories of the estimates are depicted in Figs. 10(a) and 10(b) respectively, from which we can see that (9) still works fine and the estimates of the agents' coordinates stay close to the true values.

5. Conclusion

This paper studies the collaborative localization problem for a group of mobile agents based on local relative measurements and local information exchange in two or three dimensional space. First, a continuous-time estimator is developed for an agent to estimate the relative coordinates of itself with respect to its neighbor. Second, a consensus-like fusion scheme is proposed for collaborative localization by fusing the direct estimate by itself (if there is one) and the indirect estimates from the neighboring agents. The proposed estimation scheme is fully distributed and only requires the exchange of agents' velocities, displacements during the interval when agents lose the measurements and also their estimates. Yet the estimate of every agent is globally asymptotically convergent as long as every agent is uniformly jointly reachable from the landmark in the time-varying information flow graph. There are a few interesting research problems remaining such as robustness to measurement noises, communication delays, etc.

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