



H_∞ state estimation via asynchronous filtering for descriptor Markov jump systems with packet losses



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ARTICLE INFO

Article history:

Received 23 February 2018

Revised 11 August 2018

Accepted 2 September 2018

Available online 3 September 2018

Keywords:

Descriptor Markov jump systems

Packet losses

Asynchronous filter

H_∞ state estimation

ABSTRACT

This paper is concerned with the problem of asynchronous state estimation for a class of discrete-time descriptor Markov jump systems with packet losses. Packet losses and asynchronous modes are simultaneously considered in the communication link between the plant and the estimator, where the missing probability of packet losses is governed by a binary distribution. Firstly, by constructing a stochastic Lyapunov functional, a sufficient condition is given such that the filtering error system is stochastically admissible and has a prescribed H_∞ noise attenuation performance index. Then, based on the matrix inequality decoupling technique, a linear matrix inequality (LMI)-based condition on the existence of the H_∞ asynchronous filter is presented, where the asynchronous filter can be of full-order or reduced-order. Compared with the previous methods, the proposed design method does not impose constraints on slack variables and dimensions of the designed filter, which shows less conservative results. Finally, numerical examples are given to illustrate the benefit and applicability of the new method.

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1. Introduction

Markov jump systems (MJSs) have received increased attention in the process industry in recent years, which attribute to its effective ability of describing the abrupt failures or sudden environmental changes in practical systems. MJSs have found wide applications in such areas as networked control systems, economic systems, biological systems, power systems and so on [1]. As a consequence, lots of elegant results have been presented for MJSs. To just mention a few, the problems of stability analysis and controller design for MJSs were studied in [2–4]. The authors in [5–8] discussed the issues of the filtering for MJSs, where the designed filters are mode-dependent or mode-independent. However, the above literatures cannot cope with the complex asynchronous phenomenon about modes information between the system and the filter. To deal with this challenge, the asynchronous control and filtering for MJSs were studied in [9,10]. It should be noted that all the mentioned literatures are concerned with the regular MJSs.

Descriptor systems are also called singular systems that can effectively describe a larger number of practical systems, such as biological systems, network control systems, economic systems, power systems and so on [11]. Descriptor systems are much more complicated than regular systems because the solution of descriptor systems may contain infinite modes. Thus, for descriptor systems, not only the stability need to be considered, but also the regularity (impulsive-freeness for continuous-time systems) need to be considered, which guarantees the existence and uniqueness of the solution for the systems. Note that random failures or repairs of components, sudden environmental disturbances, changing subsystem interconnections, abrupt variations may occur in the descriptor plants. We naturally model them as descriptor MJSs (DMJSs) [12]. Such models are commonly found in practical engineering systems, many analysis results and design theories for DMJSs have been developed. For the practical areas, the applications on the electrical circuit in [13], the inverted pendulum devices in [14] and the oil catalytic cracking process in [15] were concerned. On its theoretical sides, there are also many elegant results. For example, by applying two equivalent sets, necessary and sufficient conditions of dissipative control for singular MJSs were proposed in [16]. An observer-based sliding-mode controller design for singular MJSs has been synthesized in [17]. By imposing hard restrictions

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on the key freedom matrices M and N , the issue of the $l_2 - l_\infty$ filter design of DMJSs was designed in [18]. By introducing some matrix inequalities, the problem of the filter design for singular MJSSs was given in [19,20]. Authors in [21] considered the extended passive filtering problem for singular MJSSs with time-varying delays, wherein the hard restriction on the key freedom matrices G_i and the matrix inequality $-G_i X_i^{-1} G_i^T \leq X_i - G_i^T - G_i$ were used. The restrictions on freedom matrices and the matrix inequalities are used in [18–21], which gave conservative results. How to reduce the conservatism caused by these restrictions is one of the directions in this paper. Note that in the above mentioned results, the communication link has been assumed to be perfect when exchanging data among devices connected to the shared medium. However, in practice, the data may be lost while in transit through the networks due to limited bandwidth.

Networked control systems (NCSs) have many advantages and fruitful applications in a broad range of areas such as automated highway systems, remote surgery and mobile sensor networks. Different from the traditional control systems, the data may be lost in the network. Over the past decades, due to their significance both in theory and application, there has been increasing interest in the control synthesis and estimation problems for NCSs. Note that the effects of packet losses can degrade the system performances or even cause faults, many useful results on designing NCSs against the packet losses have been investigated [22–32]. For example, by viewing the packet loss as a binary switching sequence that obey the Bernoulli random binary distribution, the design of controllers for NCSs has been concerned in [28–30]. For NCSs with MJSSs plants, the finite-time energy-to-peak filtering problem has been studied in [32]. However, the aforementioned results are concerned with state-space plants (i.e. regular plants). For NCSs with singular plants, the H_∞ filtering problem was investigated in [33,34], and the fault detection issue was discussed in [35], respectively. By setting $G_{4i}^{-1} G_{3i} = I$ in free-weighting matrix G_i , the problem of fault detection filter design for discrete-time singular MJSSs with intermittent measurements has been proposed in [36], wherein the dimension of the filter is actually of full-order. By defining $X_i = \begin{bmatrix} X_{1i} & Y_i \\ X_{2i} & Y_i \end{bmatrix}$ and using the matrix inequality $-X_i \hat{P}_i^{-1} X_i^T \leq \hat{P}_i - X_i^T - X_i$, the issue of the non-fragile filter design was addressed for discrete-time singular MJSSs subject to missing measurements in [37]. By setting $J_{im} = \begin{bmatrix} J_{1im} & b_1 X_i \\ J_{2im} & b_2 X_i \end{bmatrix}$, the energy-to-peak filter design for networked singular MJSSs over a finite-time interval was investigated in [38]. It should be pointed out that the result in [36–38] is not only with some restrictions on free-weighting matrix but also limits the dimension of the filter to be of full-order. A natural research problem is how to establish a less conservative filter design condition for networked DMJSs without any constraint on the free-weighting matrix and the dimension of the filter.

In this paper, we investigate the asynchronous state estimation problem for discrete-time DMJSs with packet losses. By taking the packet losses and asynchronous modes into consideration, a novel filtering error system has been proposed. Firstly, based on a stochastic Lyapunov functional, a sufficient condition is obtained such that the filtering error system is stochastically admissible with the H_∞ noise attenuation performance index. Then by using the matrix inequality decoupling technique, the condition on the existence of the asynchronous filter is developed in terms of strict LMIs. The main novelty of the paper lies in the following aspects: 1) By using the compensation scheme of packet losses, the asynchronous state estimation problem for discrete-time DMJSs affected by packet losses is studied for the first time. 2) Compared with [18,21,36–38], the filter design method in the paper

Table 1

The notations and descriptions.

| Notation | Description |
|------------------------|--|
| $X \geq 0$ ($X > 0$) | Semi-positive definite (positive definite) matrix |
| I | Identity matrix with appropriate dimensions |
| 0 | Zero matrix with appropriate dimensions |
| $(\cdot)^T$ | Transpose of a matrix |
| $\text{diag}\{\dots\}$ | Block-diagonal matrix |
| $\ x\ $ | Euclidean norm of the vector x |
| $\mathbf{E}\{\cdot\}$ | Mathematical expectation |
| * | Ellipsis for the terms that are introduced by symmetry |
| $\text{sym}(X)$ | $X + X^T$ |
| * | Matrices that are not relevant in the discussion |
| $l_2[0, \infty)$ | The space of summable infinite sequence over $[0, \infty)$ |

shows less conservative results since the constraints on the free-weighting matrix and the dimension of the filter are overcome. Last, two numerical examples are provided to show the effectiveness of the proposed methods.

Notations: Throughout this paper, the following standard notations are used (Table 1).

2. Preliminaries

Consider the following discrete-time DMJSs:

$$\begin{cases} Ex(k+1) &= A(r_k)x(k) + B(r_k)\omega(k), \\ y(k) &= C(r_k)x(k) + D(r_k)\omega(k), \\ z(k) &= H(r_k)x(k), \end{cases} \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the system state, $\omega(k) \in \mathbb{R}^v$ is the disturbance signal that belongs to $l_2[0, \infty)$, $y(k) \in \mathbb{R}^l$ is the output vector, and $z(k) \in \mathbb{R}^p$ is the signal to be estimated, respectively. The matrix $E \in \mathbb{R}^{n \times n}$ is singular with $\text{rank}(E) = r \leq n$. For every $i \in \mathcal{J} = \{1, 2, \dots, S\}$, $r_k = i$, we denote $A(r_k) = A_i$, $B(r_k) = B_i$, $C(r_k) = C_i$, $D(r_k) = D_i$, $H(r_k) = H_i$. A_i , B_i , C_i , D_i and H_i are known compatible dimension constant matrices. Switching law $\{r_k, k \geq 0\}$ is a discrete-time Markov stochastic process taking values in a finite state space $\mathcal{J} = \{1, 2, \dots, S\}$, the evolution of $\{r_k, k \geq 0\}$ is governed by the following transition probabilities:

$$\Pr\{r_{k+1} = j | r_k = i\} = \mu_{ij},$$

where $\mu_{ij} \geq 0, \forall i, j \in \mathcal{J}$ with $\sum_{j=1}^S \mu_{ij} = 1$.

In this paper, we study the state estimation for system (1) over a lossy network, where the packet losses and asynchronous modes are simultaneously considered in the communication link between the plant and the estimator. In this case, we adopt the compensation scheme in [31] to cope with the effect of the packet losses:

$$\hat{y}(k) = (1 - \delta_k)y(k) + \delta_k \hat{y}(k-1). \quad (2)$$

The stochastic variable δ_k is a Bernoulli distributed white sequence with the following probability distribution laws:

$$\begin{aligned} \text{Prob}\{\delta_k = 1\} &= \mathbf{E}\{\delta_k\} = \delta, \\ \text{Prob}\{\delta_k = 0\} &= 1 - \mathbf{E}\{\delta_k\} = 1 - \delta, \end{aligned}$$

where $\delta \in [0, 1]$ is a known constant.

Remark 1. In (2), the stochastic variables δ_k represents the possibility of occurring networked induced packet losses. Specifically, if $\delta_k = 0$, the communication is perfect, while $\delta_k = 1$ corresponds to the packet is lost in the measurement channel. Different from [33,36,37], the previous measurement data will be used as the observation data if the current measurement data packet is lost during transmissions in our paper.

Then, we augment the system state and the system measurement output from (1) and (2) in the following compact form:

$$\begin{aligned} \hat{E}\hat{x}(k+1) &= \hat{A}_i(\delta_k)\hat{x}(k) + \hat{B}_i(\delta_k)\omega(k), \\ \hat{y}(k) &= \hat{C}_i(\delta_k)\hat{x}(k) + \hat{D}_i(\delta_k)\omega(k), \\ z(k) &= \hat{H}_i\hat{x}(k), \end{aligned} \quad (3)$$

where

$$\begin{aligned} \hat{x}(k) &= \begin{bmatrix} x(k) \\ \hat{y}(k-1) \end{bmatrix}, \quad \hat{E} = \begin{bmatrix} \hat{E} & 0 \\ 0 & I \end{bmatrix}, \quad \hat{A}_i(\delta_k) = \begin{bmatrix} A_i & 0 \\ (1-\delta_k)C_i & \delta_k I \end{bmatrix}, \\ \hat{B}_i(\delta_k) &= \begin{bmatrix} B_i \\ (1-\delta_k)D_i \end{bmatrix}, \quad \hat{C}_i(\delta_k) = \begin{bmatrix} (1-\delta_k)C_i & \delta_k I \end{bmatrix}, \\ \hat{D}_i(\delta_k) &= (1-\delta_k)D_i, \quad \hat{H}_i = \begin{bmatrix} H_i & 0 \end{bmatrix}. \end{aligned}$$

In this paper, we consider the following asynchronous filter for system (3):

$$\begin{cases} x_f(k+1) &= A_f(\sigma_k)x_f(k) + B_f(\sigma_k)\hat{y}(k), \\ z_f(k) &= C_f(\sigma_k)x_f(k), \end{cases} \quad (4)$$

where $x_f(k) \in \mathbb{R}^m$ is the filter state with $m \leq n$, $z_f(k) \in \mathbb{R}^p$ is the output vector of the filter, respectively. $A_f(\sigma_k)$, $B_f(\sigma_k)$ and $C_f(\sigma_k)$ are the filter gains to be designed with appropriate dimensions. Note that σ_k is assumed to be a nonstationary Markov chain, which is dependent on r_{k+1} and takes values in another finite state space $\Omega = \{1, 2, \dots, M\}$. Further, σ_k belongs to the following transition probabilities:

$$\Pr\{\sigma_{k+1} = n | \sigma_k = m\} = \pi_{mn}^{r_{k+1}},$$

where $\pi_{mn}^{r_{k+1}} \geq 0$ with $\sum_{n=1}^M \pi_{mn}^{r_{k+1}} = 1, \forall m, n \in \Omega$. It should be pointed out that the Markov chain r_k is assumed to be independent of $\mathcal{F}_{k-1} = \sigma\{\sigma_1, \sigma_2, \dots, \sigma_{k-1}\}$, where \mathcal{F}_{k-1} is a σ -algebra generated by $\{\sigma_1, \sigma_2, \dots, \sigma_{k-1}\}$. For the sake of simplicity, we denote $A_f(\sigma_k) = A_{fm}$, $B_f(\sigma_k) = B_{fm}$ and $C_f(\sigma_k) = C_{fm}$, for every $\sigma_k = m, m \in \Omega$.

Define $\eta(k) = [\hat{x}^T(k) \quad x_f^T(k)]^T$ and $e(k) = z(k) - z_f(k)$, it follows from (3) and (4) that the corresponding filtering error systems can be written as

$$\begin{cases} \tilde{E}\eta(k+1) &= \mathcal{A}_{im}(\delta_k)\eta(k) + \mathcal{B}_{im}(\delta_k)\omega(k), \\ e(k) &= C_{im}\eta(k), \end{cases} \quad (5)$$

where

$$\begin{aligned} \tilde{E} &= \begin{bmatrix} \hat{E} & 0 \\ 0 & I \end{bmatrix}, \quad \mathcal{A}_{im}(\delta_k) = \begin{bmatrix} \hat{A}_i(\delta_k) & 0 \\ B_{fm}\hat{C}_i(\delta_k) & A_{fm} \end{bmatrix}, \\ \mathcal{B}_{im}(\delta_k) &= \begin{bmatrix} \hat{B}_i(\delta_k) \\ B_{fm}\hat{D}_i(\delta_k) \end{bmatrix}, \quad \mathcal{C}_{im} = \begin{bmatrix} \hat{H}_i & -C_{fm} \end{bmatrix}. \end{aligned} \quad (6)$$

Definition 1. [19]

- (i) System (5) with $\omega(k) = 0$ is said to be regular and causal, if the pairs $(\tilde{E}, \mathcal{A}_{im}(\delta))$ are regular and causal.
- (ii) System (5) with $\omega(k) = 0$ is said to be stochastically stable, if for any initial state $\eta(0)$ and initial mode r_0, σ_0 , the following inequality holds:

$$\mathbf{E}\left\{\sum_{k=0}^{\infty} \|\eta(k)\|^2 \mid \eta(0), r_0, \sigma_0\right\} < \infty. \quad (7)$$

- (iii) System (5) with $\omega(k) = 0$ is said to be stochastically admissible, if it is regular, causal and stochastically stable.

Definition 2. [19] For a prescribed level of noise attenuation $\gamma > 0$, system (5) is said to be stochastically admissible with the H_∞ performance level γ , if system (5) with $\omega(k) = 0$ is stochastically admissible, and under zero initial condition, the following inequality is satisfied

$$\mathbf{E}\left\{\sum_{k=0}^{\infty} e^T(k)e(k)\right\} < \gamma^2 \sum_{k=0}^{\infty} \omega^T(k)\omega(k). \quad (8)$$

In the following, the asynchronous filter in the form of (4) for system (1) with random packet losses is designed such that system (5) is stochastically admissible with the H_∞ performance level γ .

Before ending this section, the following lemma is introduced.

Lemma 1. [39] Given matrices A, P and a symmetric matrix T of appropriate dimensions. The inequality

$$T + \text{sym}\{PA\} < 0$$

is fulfilled if there exists a matrix L such that the following condition holds:

$$\begin{bmatrix} T & \beta P + A^T L^T \\ * & -\beta L - \beta L^T \end{bmatrix} < 0.$$

3. Main results

In this section, we investigate the asynchronous state estimation problem for discrete-time DMJSs with random packet losses. To this end, we first present a sufficient condition such that system (5) is stochastically admissible and has the H_∞ noise attenuation performance index.

Theorem 1. System (5) is stochastically admissible with the H_∞ performance level, if for a given scalar $\delta \in [0, 1]$, there exist symmetric matrix $\tilde{P}_{im} > 0$, matrices $M_{im}, N_{im}, Q_{im}, T_{im}, \forall i, j \in \mathcal{J}, m, n \in \Omega$, and a scalar γ , such that

$$\begin{bmatrix} \mathcal{G}_{im}^{11} & \mathcal{G}_{im}^{12} & M_{im}\mathcal{B}_{im}(\delta) & C_{im}^T \\ * & \mathcal{G}_{im}^{22} & N_{im}\mathcal{B}_{im}(\delta) & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad (9)$$

where

$$\begin{aligned} \mathcal{G}_{im}^{11} &= \text{sym}\{M_{im}(\mathcal{A}_{im}(\delta) - \tilde{E})\} + \tilde{E}^T X_{im} \tilde{E} - \tilde{E}^T \tilde{P}_{im} \tilde{E}, \\ \mathcal{G}_{im}^{12} &= -M_{im} + (\mathcal{A}_{im}(\delta) - \tilde{E})^T N_{im}^T + \tilde{E}^T X_{im} + Q_{im}^T S_i^T, \\ \mathcal{G}_{im}^{22} &= \text{sym}\{-N_{im} + S_i T_{im}\} + X_{im}, \end{aligned}$$

$$X_{im} = \sum_{j=1}^S \sum_{n=1}^M \mu_{ij} \pi_{mn}^j \tilde{P}_{jn},$$

$\mathcal{A}_{im}(\delta), \mathcal{B}_{im}(\delta)$ are defined in (6) with δ_k replaced by δ . $S_i \in \mathbb{R}^{(m+n+1) \times (n-r)}$ is the arbitrary matrix satisfying $\tilde{E}^T S_i = 0$ and $\text{rank}(S_i) = n - r$.

Proof. Firstly, we prove that system (5) with $\omega(k) = 0$ is regular and causal. From (9), it follows that

$$\begin{bmatrix} \mathcal{G}_{im}^{11} & \mathcal{G}_{im}^{12} \\ * & \mathcal{G}_{im}^{22} \end{bmatrix} < 0, \quad (10)$$

then, (10) is equivalent to:

$$\tilde{A}_{im}^T \hat{X}_{im} \hat{A}_{im} + \tilde{A}_{im}^T \hat{S}_i \hat{Q}_{im} + \hat{Q}_{im}^T \hat{S}_i^T \hat{A}_{im} - \hat{E}^T \hat{P}_{im} \hat{E} < 0, \quad (11)$$

where

$$\begin{aligned} \hat{E} &= \begin{bmatrix} \tilde{E} & 0 \\ 0 & 0 \end{bmatrix}, \quad \hat{A}_{im} = \begin{bmatrix} \tilde{E} & I \\ \mathcal{A}_{im}(\delta) - \tilde{E} & -I \end{bmatrix}, \quad \hat{P}_{im} = \begin{bmatrix} \tilde{P}_{im} & 0 \\ 0 & 0 \end{bmatrix}, \\ \hat{Q}_{im} &= \begin{bmatrix} Q_{im} & T_{im} \\ M_{im}^T & N_{im}^T \end{bmatrix}, \quad \hat{S}_i = \begin{bmatrix} S_i & 0 \\ 0 & I \end{bmatrix}. \end{aligned}$$

Since $X_{im} = \sum_{j=1}^S \sum_{n=1}^M \mu_{ij} \pi_{mn}^j \tilde{P}_{jn}$ and $\tilde{P}_{im} > 0$, we have $\hat{X}_{im} \geq 0$, it yields from (11) that

$$\tilde{A}_{im}^T \hat{S}_i \hat{Q}_{im} + \hat{Q}_{im}^T \hat{S}_i^T \hat{A}_{im} - \hat{E}^T \hat{P}_{im} \hat{E} < 0. \quad (12)$$

Since $\text{rank}(\widehat{E}) = m + r + l$, there exist two nonsingular matrices $\mathcal{M}, \mathcal{N} \in \mathbb{R}^{2(m+n+l) \times 2(m+n+l)}$ such that

$$\begin{aligned} \mathcal{M}\widehat{E}\mathcal{N} &= \begin{bmatrix} I_{m+r+l} & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{M}\widehat{A}_{im}\mathcal{N} = \begin{bmatrix} \widehat{\mathcal{A}}_{im}^1 & \widehat{\mathcal{A}}_{im}^2 \\ \widehat{\mathcal{A}}_{im}^3 & \widehat{\mathcal{A}}_{im}^4 \end{bmatrix}, \\ \mathcal{M}^{-T}\widehat{P}_{im}\mathcal{M}^{-1} &= \begin{bmatrix} \widehat{P}_{im}^1 & \widehat{P}_{im}^2 \\ * & \widehat{P}_{im}^3 \end{bmatrix}, \quad \mathcal{M}^{-T}\widehat{S}_i = \begin{bmatrix} \widehat{S}_i^1 \\ \widehat{S}_i^2 \end{bmatrix}, \\ \widehat{Q}_{im}\mathcal{N} &= \begin{bmatrix} \widehat{Q}_{im}^1 & \widehat{Q}_{im}^2 \end{bmatrix}. \end{aligned} \tag{13}$$

From $\widehat{E}^T S_i = 0$, it follows that $\widehat{E}^T \widehat{S}_i = 0$, which further implies that

$$\mathcal{M}^{-T}\widehat{S}_i = \begin{bmatrix} 0 \\ \widehat{S}_i^2 \end{bmatrix}, \tag{14}$$

where $\widehat{S}_i^2 \in \mathbb{R}^{(m+2n+l-r) \times (m+2n+l-r)}$ with $\text{rank}(\widehat{S}_i^2) = m + 2n + l - r$. Pre- and post-multiply (12) by \mathcal{N}^T and \mathcal{N} , from (13) and (14), it is obtained that

$$\begin{bmatrix} \star & \star \\ \star & \text{sym}\{(\widehat{\mathcal{A}}_{im}^4)^T \widehat{S}_i^2 \widehat{Q}_{im}^2\} \end{bmatrix} < 0. \tag{15}$$

From (15), it implies that $\text{sym}\{(\widehat{\mathcal{A}}_{im}^4)^T \widehat{S}_i^2 \widehat{Q}_{im}^2\} < 0$, then $\widehat{\mathcal{A}}_{im}^4$ is nonsingular. According to Definition 1 and [12], we have the pairs $(\widehat{E}, \widehat{A}_{im})$ are regular and causal. Further, based on the fact that $\det(z\widehat{E} - \widehat{A}_{im}) = \det(z\widehat{E} - A_{im}(\delta))$ for $\forall i \in \mathcal{J}, m \in \Omega$, it is obtained that system (5) with $\omega(k) = 0$ is regular and causal. □

Next, when $\omega(k) = 0$, we prove that system (5) is stochastically stable. We construct a stochastic Lyapunov functional as follows:

$$V(\eta(k), r_k, \sigma_k) = \eta^T(k) \widetilde{E}^T \widetilde{P}_{im} \widetilde{E} \eta(k), \tag{16}$$

where $\widetilde{P}_{im} > 0$. We have

$$\begin{aligned} \mathbf{E}[V(\eta(k+1), r_{k+1}, \sigma_{k+1}) | \eta(k), r_k, \sigma_k] &= \eta^T(k+1) \widetilde{E}^T \left(\sum_{j=1}^S \sum_{n=1}^M \widetilde{P}_{jn} \Pr\{r_{k+1} = j, \sigma_{k+1} = n | r_k = i, \sigma_k = m\} \right) \widetilde{E} \eta(k+1) \\ &= \eta^T(k+1) \widetilde{E}^T \left(\sum_{j=1}^S \sum_{n=1}^M \widetilde{P}_{jn} \Pr\{\sigma_{k+1} = n | r_{k+1} = j, r_k = i, \sigma_k = m\} \right. \\ &\quad \times \Pr\{r_{k+1} = j | r_k = i, \sigma_k = m\} \left. \right) \widetilde{E} \eta(k+1) \\ &= \eta^T(k+1) \widetilde{E}^T \left(\sum_{j=1}^S \sum_{n=1}^M \widetilde{P}_{jn} \pi_{mn}^j \Pr\{r_{k+1} = j | r_k = i, \sigma_k = m\} \right) \widetilde{E} \eta(k+1) \\ &= \eta^T(k+1) \widetilde{E}^T X_{im} \widetilde{E} \eta(k+1). \end{aligned}$$

Let $\chi(k) = \widetilde{E} \eta(k+1) - \widetilde{E} \eta(k)$, one has

$$\begin{aligned} \mathbf{E}[V(\eta(k+1), r_{k+1}, \sigma_{k+1}) | \eta(k), r_k, \sigma_k] &= (\widetilde{E} \eta(k) + \chi(k))^T X_{im} (\widetilde{E} \eta(k) + \chi(k)). \end{aligned} \tag{17}$$

Using the equations $\chi(k) = \widetilde{E} \eta(k+1) - \widetilde{E} \eta(k)$ and $\widetilde{E} \eta(k+1) = A_{im}(\delta_k) \eta(k)$, it is obtained that

$$\mathbf{E}[(A_{im}(\delta_k) - \widetilde{E}) \eta(k) - \chi(k) | \eta(k)] = (A_{im}(\delta) - \widetilde{E}) \eta(k) - \chi(k) = 0, \tag{18}$$

then it follows that

$$0 = 2[\eta^T(k) \quad \chi^T(k)] \mathcal{T}_{im}^T [(A_{im}(\delta) - \widetilde{E}) \eta(k) - \chi(k)], \tag{19}$$

where $\mathcal{T}_{im} = \begin{bmatrix} M_{im}^T & N_{im}^T \\ 0 & 0 \end{bmatrix}$.

For any matrix $Q_{im}, T_{im} \in \mathbb{R}^{(n-r) \times (m+n+l)}$, $S_i \in \mathbb{R}^{(m+n+l) \times (n-r)}$ is any full column rank matrix satisfying $\widetilde{E}^T S_i = 0$, along with $\chi(k) = \widetilde{E} \eta(k+1) - \widetilde{E} \eta(k)$, one has

$$0 = 2\chi^T(k) S_i [Q_{im} \eta(k) + T_{im} \chi(k)]. \tag{20}$$

Adding (19) and (20) into (17), it is obtained that

$$\mathbf{E}[V(\eta(k+1), r_{k+1}, \sigma_{k+1}) | \eta(k), r_k, \sigma_k] = \vartheta(k) \Theta_{im} \vartheta^T(k), \tag{21}$$

where

$$\begin{aligned} \vartheta(k) &= \begin{bmatrix} \eta^T(k) & \chi^T(k) \end{bmatrix}, \\ \Theta_{im} &= \begin{bmatrix} \text{sym}\{M_{im}(A_{im}(\delta) - \widetilde{E})\} + \widetilde{E}^T X_{im} \widetilde{E} & \mathcal{G}_{im}^{12} \\ * & \mathcal{G}_{im}^{22} \end{bmatrix}. \end{aligned}$$

From (9) and (21), it follows that

$$\begin{aligned} \Delta V(\eta(k), r_k, \sigma_k) &= \mathbf{E}[V(\eta(k+1), r_{k+1}, \sigma_{k+1}) | \eta(k), r_k, \sigma_k] \\ &\quad - V(\eta(k), r_k, \sigma_k) \\ &= \vartheta(k) \mathcal{G}_{im} \vartheta^T(k) \leq -\alpha_0 \eta^T(k) \eta(k), \end{aligned} \tag{22}$$

where

$$\begin{aligned} \mathcal{G}_{im} &= \begin{bmatrix} \mathcal{G}_{im}^{11} & \mathcal{G}_{im}^{12} \\ * & \mathcal{G}_{im}^{22} \end{bmatrix}, \\ \alpha_0 &= \lambda_{\min}\{-\mathcal{G}_{im}, i, j \in \mathcal{J}, m, n \in \Omega\}. \end{aligned}$$

Then based on (22), it is obtained that

$$\begin{aligned} \mathbf{E}[V(\eta(k), r_k, \sigma_k) | \eta(k-1), r_{k-1}, \sigma_{k-1})] \\ - V(\eta(k-1), r_{k-1}, \sigma_{k-1}) \leq -\alpha_0 \eta^T(k-1) \eta(k-1), \end{aligned}$$

one can continue the iterative procedure to obtain

$$\begin{aligned} \mathbf{E}[V(\eta(k), r_k, \sigma_k) | \eta(0), r_0, \sigma_0] - V(\eta(0), r_0, \sigma_0) \\ \leq -\alpha_0 \mathbf{E} \left[\sum_{l=0}^{k-1} \eta^T(l) \eta(l) | \eta(0), r_0, \sigma_0 \right], \end{aligned}$$

which implies that

$$\mathbf{E} \left[\sum_{l=0}^{\infty} \eta^T(l) \eta(l) | \eta(0), r_0, \sigma_0 \right] \leq \frac{V(\eta(0), r_0, \sigma_0)}{\alpha_0} < \infty.$$

Thus when $\omega(k) = 0$, we have system (5) is stochastically stable.

Finally, we prove that (8) holds under zero initial condition. To this end, we set

$$J_{z\omega}(k) = \mathbf{E} \left\{ \sum_{k=0}^T [e^T(k) e(k) - \gamma^2 \omega^T(k) \omega(k)] \right\}, \quad T \in [0, \infty).$$

From (22), it follows that

$$\begin{aligned} J_{z\omega}(k) &= \mathbf{E} \left\{ \sum_{k=0}^T [e^T(k) e(k) - \gamma^2 \omega^T(k) \omega(k) + \Delta V(\eta(k), r_k, \sigma_k)] \right\} \\ &\quad - \mathbf{E} \left\{ \sum_{k=0}^T \Delta V(\eta(k), r_k, \sigma_k) \right\} \\ &\leq \mathbf{E} \left\{ \sum_{k=0}^T [e^T(k) e(k) - \gamma^2 \omega^T(k) \omega(k) + \Delta V(\eta(k), r_k, \sigma_k)] \right\} \\ &\leq \mathbf{E} \left\{ \sum_{k=0}^T \zeta(k) \widetilde{\Theta}_{im} \zeta^T(k) \right\}, \end{aligned} \tag{23}$$

where

$$\begin{aligned} \zeta(k) &= \begin{bmatrix} \eta^T(k) & \chi^T(k) & \omega^T(k) \end{bmatrix}, \\ \widetilde{\Theta}_{im} &= \begin{bmatrix} \mathcal{G}_{im}^{11} & \mathcal{G}_{im}^{12} & M_{im} \mathcal{B}_{im}(\delta) \\ * & \mathcal{G}_{im}^{22} & N_{im} \mathcal{B}_{im}(\delta) \\ * & * & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\gamma^2 I \end{bmatrix} \\ &\quad + \begin{bmatrix} \mathcal{C}_{im}^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \mathcal{C}_{im}^T \\ 0 \\ 0 \end{bmatrix}^T. \end{aligned}$$

According to the Schur complement, it is obtained that (9) is equivalent to $\widetilde{\Theta}_{im} < 0$. Then from (3), it follows that (8) holds.

Therefore, system (5) is stochastically admissible with the H_∞ performance level γ . The proof is completed.

Remark 2. In Theorem 1, the asynchronous H_∞ state estimation problem for discrete-time DMJSs with packet losses has been studied. With the help of free weighting matrices method, slack variables M_{im} and N_{im} are introduced. In this case, the coupling between Lyapunov variables and system matrices is eliminated. In contrast to [38], where the traditional inequality $-\mathcal{J}_{im}\overline{\Theta}_{im}^{-1}\mathcal{J}_{im}^T \leq \mathcal{Y}_{im}\overline{\Theta}_{im}\mathcal{Y}_{im}^T - \mathcal{Y}_{im}\mathcal{J}_{im}^T - \mathcal{J}_{im}\mathcal{Y}_{im}^T$ is introduced, this alternative controller synthesis method will give less conservative results by means of the incremental flexibility from Lyapunov variables and additional slack variables.

In the following theorem, a sufficient condition in terms of LMIs is presented for system (5) to be stochastically admissible with the H_∞ performance level.

Theorem 2. System (5) is stochastically admissible with the H_∞ performance level, if for given scalars $\delta \in [0, 1]$, β_i and matrices U_{im} , there exist symmetric matrix $\begin{bmatrix} \tilde{P}_{im1} & \tilde{P}_{im2} \\ * & \tilde{P}_{im3} \end{bmatrix} > 0$, matrices $M_{im1}, M_{im2}, M_{im3}, M_{im4}, N_{im1}, N_{im2}, N_{im3}, N_{im4}, Q_{im1}, Q_{im2}, T_{im1}, T_{im2}, L_m, \hat{A}_{fm}, \hat{B}_{fm}, \hat{C}_{fm}, \forall i, j \in \mathcal{J}, m, n \in \Omega$, and a scalar γ , such that

$$\begin{bmatrix} \Theta_{im}^1 & \Theta_{im}^2 & \Theta_{im}^3 & \Theta_{im}^4 & \Pi_{im}^7 \\ * & \Pi_{im}^5 & \Theta_{im}^6 & 0 & \Pi_{im}^8 \\ * & * & -\gamma^2 I & 0 & \hat{D}_i^T(\delta)\hat{B}_{fm}^T \\ * & * & * & -I & 0 \\ * & * & * & * & -\beta_i L_m - \beta_i L_m^T \end{bmatrix} < 0, \quad (24)$$

where

$$\begin{aligned} \Theta_{im}^1 &= \begin{bmatrix} \Theta_{im}^{11} & \Theta_{im}^{12} \\ * & \text{sym}\{\hat{A}_{fm} - M_{im4}\} + X_{im3} - \tilde{P}_{im3} \end{bmatrix}, \\ \Theta_{im}^2 &= \begin{bmatrix} \Theta_{im}^{21} & \Theta_{im}^{22} \\ \Theta_{im}^{23} & -M_{im4} + \hat{A}_{fm}^T - N_{im4}^T + X_{im3} \end{bmatrix}, \\ \Theta_{im}^3 &= \begin{bmatrix} M_{im1}\hat{B}_i(\delta) + U_{im}\hat{B}_{fm}\hat{D}_i(\delta) \\ M_{im3}\hat{B}_i(\delta) + \hat{B}_{fm}\hat{D}_i(\delta) \end{bmatrix}, \quad \Theta_{im}^4 = \begin{bmatrix} \hat{H}_i^T \\ -\hat{C}_{fm}^T \end{bmatrix}, \\ \Pi_{im}^5 &= \begin{bmatrix} \Pi_{im}^{51} & \Pi_{im}^{52} \\ * & \text{sym}\{-N_{im4}\} + X_{im3} \end{bmatrix}, \\ \Pi_{im}^7 &= \begin{bmatrix} \beta_i(M_{im2} - U_{im}L_m) + \hat{C}_i^T(\delta)\hat{B}_{fm}^T \\ \beta_i(M_{im4} - L_m) + \hat{A}_{fm}^T \end{bmatrix}, \\ \Theta_{im}^6 &= \begin{bmatrix} N_{im1}\hat{B}_i(\delta) + U_{im}\hat{B}_{fm}\hat{D}_i(\delta) \\ N_{im3}\hat{B}_i(\delta) + \hat{B}_{fm}\hat{D}_i(\delta) \end{bmatrix}, \\ \Pi_{im}^8 &= \begin{bmatrix} \beta_i(N_{im2} - U_{im}L_m) \\ \beta_i(N_{im4} - L_m) \end{bmatrix}, \\ \Theta_{im}^{11} &= \text{sym}\{M_{im1}\hat{A}_i(\delta) - M_{im1}\hat{E} + U_{im}\hat{B}_{fm}\hat{C}_i(\delta)\} + \hat{E}^T X_{im1} \hat{E} - \hat{E}^T \tilde{P}_{im1} \hat{E}, \\ \Theta_{im}^{12} &= U_{im}\hat{A}_{fm} + \hat{A}_i^T(\delta)M_{im3}^T + \hat{C}_i^T(\delta)\hat{B}_{fm}^T - M_{im2} - \hat{E}^T M_{im3}^T + \hat{E}^T X_{im2} - \hat{E}^T \tilde{P}_{im2}, \\ \Theta_{im}^{21} &= -M_{im1} + \hat{A}_i^T(\delta)N_{im1}^T + \hat{C}_i^T(\delta)\hat{B}_{fm}^T U_{im}^T - \hat{E}^T N_{im1}^T + \hat{E}^T X_{im1} + Q_{im1}^T \begin{bmatrix} R_i^T & 0 \end{bmatrix}, \\ \Theta_{im}^{22} &= -M_{im2} + \hat{A}_i^T(\delta)N_{im3}^T + \hat{C}_i^T(\delta)\hat{B}_{fm}^T - \hat{E}^T N_{im3}^T + \hat{E}^T X_{im2}, \\ \Theta_{im}^{23} &= -M_{im3} + \hat{A}_{fm}^T U_{im}^T - N_{im2}^T + X_{im2}^T + Q_{im2}^T \begin{bmatrix} R_i^T & 0 \end{bmatrix}, \\ \Pi_{im}^{51} &= \text{sym}\{-N_{im1} + \begin{bmatrix} R_i \\ 0 \end{bmatrix} T_{im1}\} + X_{im1}, \\ \Pi_{im}^{52} &= -N_{im2} - N_{im3}^T + X_{im2} + \begin{bmatrix} R_i \\ 0 \end{bmatrix} T_{im2}. \end{aligned}$$

$R_i \in \mathbb{R}^{n \times (n-r)}$ is the arbitrary matrix satisfying $E^T R_i = 0$ and $\text{rank}(R_i) = n - r$. Then the desired asynchronous filters are given by

$$A_{fm} = L_m^{-1} \hat{A}_{fm}, \quad B_{fm} = L_m^{-1} \hat{B}_{fm}, \quad C_{fm} = \hat{C}_{fm}. \quad (25)$$

Proof. Firstly, we assign

$$\begin{aligned} M_{im} &= \begin{bmatrix} M_{im1} & M_{im2} \\ M_{im3} & M_{im4} \end{bmatrix}, \quad N_{im} = \begin{bmatrix} N_{im1} & N_{im2} \\ N_{im3} & N_{im4} \end{bmatrix}, \\ \tilde{P}_{im} &= \begin{bmatrix} \tilde{P}_{im1} & \tilde{P}_{im2} \\ * & \tilde{P}_{im3} \end{bmatrix}, \quad Q_{im} = \begin{bmatrix} Q_{im1} & Q_{im2} \end{bmatrix}, \\ T_{im} &= \begin{bmatrix} T_{im1} & T_{im2} \end{bmatrix}. \end{aligned} \quad (26)$$

□

From $\tilde{E}^T S_i = 0$ and $\text{rank}(S_i) = n - r$, it follows that

$$S_i = \begin{bmatrix} R_i \\ 0 \\ 0 \end{bmatrix}, \quad (27)$$

where $E^T R_i = 0$ and $\text{rank}(R_i) = n - r$.

Then, substituting (26), (27), (6) with δ_k replaced by δ into (9), it follows that

$$\begin{bmatrix} \Pi_{im}^1 & \Pi_{im}^2 & \Pi_{im}^3 & \Pi_{im}^4 \\ * & \Pi_{im}^5 & \Pi_{im}^6 & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad (28)$$

where

$$\begin{aligned} \Pi_{im}^1 &= \begin{bmatrix} \Pi_{im}^{11} & \Pi_{im}^{12} \\ * & \text{sym}\{M_{im4}A_{fm} - M_{im4}\} + X_{im3} - \tilde{P}_{im3} \end{bmatrix}, \\ \Pi_{im}^2 &= \begin{bmatrix} \Pi_{im}^{21} & \Pi_{im}^{22} \\ \Pi_{im}^{23} & -M_{im4} + A_{fm}^T N_{im4}^T - N_{im4}^T + X_{im3} \end{bmatrix}, \\ \Pi_{im}^3 &= \begin{bmatrix} M_{im1}\hat{B}_i(\delta) + M_{im2}B_{fm}\hat{D}_i(\delta) \\ M_{im3}\hat{B}_i(\delta) + M_{im4}B_{fm}\hat{D}_i(\delta) \end{bmatrix}, \\ \Pi_{im}^4 &= \begin{bmatrix} \hat{H}_i^T \\ -\hat{C}_{fm}^T \end{bmatrix}, \\ \Pi_{im}^6 &= \begin{bmatrix} N_{im1}\hat{B}_i(\delta) + N_{im2}B_{fm}\hat{D}_i(\delta) \\ N_{im3}\hat{B}_i(\delta) + N_{im4}B_{fm}\hat{D}_i(\delta) \end{bmatrix}, \\ \Pi_{im}^{11} &= \text{sym}\{M_{im1}\hat{A}_i(\delta) - M_{im1}\hat{E} + M_{im2}B_{fm}\hat{C}_i(\delta)\} + \hat{E}^T X_{im1} \hat{E} - \hat{E}^T \tilde{P}_{im1} \hat{E}, \\ \Pi_{im}^{12} &= M_{im2}A_{fm} + \hat{A}_i^T(\delta)M_{im3}^T + \hat{C}_i^T(\delta)B_{fm}^T M_{im4}^T - M_{im2} - \hat{E}^T M_{im3}^T + \hat{E}^T X_{im2} - \hat{E}^T \tilde{P}_{im2}, \\ \Pi_{im}^{21} &= -M_{im1} + \hat{A}_i^T(\delta)N_{im1}^T + \hat{C}_i^T(\delta)B_{fm}^T N_{im2}^T - \hat{E}^T N_{im1}^T + \hat{E}^T X_{im1} + Q_{im1}^T \begin{bmatrix} R_i^T & 0 \end{bmatrix}, \\ \Pi_{im}^{22} &= -M_{im2} + \hat{A}_i^T(\delta)N_{im3}^T + \hat{C}_i^T(\delta)B_{fm}^T N_{im4}^T - \hat{E}^T N_{im3}^T + \hat{E}^T X_{im2}, \\ \Pi_{im}^{23} &= -M_{im3} + A_{fm}^T N_{im2}^T - N_{im2}^T + X_{im2}^T + Q_{im2}^T \begin{bmatrix} R_i^T & 0 \end{bmatrix}. \end{aligned}$$

Then, decoupling some product terms in (28), it is equivalent to

$$\begin{bmatrix} \Psi_{im}^1 & \Psi_{im}^2 & \Psi_{im}^3 & \Pi_{im}^4 \\ * & \Pi_{im}^5 & \Psi_{im}^6 & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} + \text{sym} \left\{ \begin{bmatrix} M_{im2} \\ M_{im4} \\ N_{im2} \\ N_{im4} \\ 0 \\ 0 \end{bmatrix} \mathcal{H}_{im} \right\} < 0, \quad (29)$$

where

$$\begin{aligned} \Psi_{im}^1 &= \begin{bmatrix} \Psi_{im}^{11} & \Psi_{im}^{12} \\ * & \text{sym}\{-M_{im4}\} + X_{im3} - \tilde{P}_{im3} \end{bmatrix}, \\ \Psi_{im}^2 &= \begin{bmatrix} \Psi_{im}^{21} & \Psi_{im}^{22} \\ \Psi_{im}^{23} & -M_{im4} - N_{im4}^T + X_{im3} \end{bmatrix}, \\ \Psi_{im}^3 &= \begin{bmatrix} M_{im1}\hat{B}_i(\delta) \\ M_{im3}\hat{B}_i(\delta) \end{bmatrix}, \quad \Psi_{im}^6 = \begin{bmatrix} N_{im1}\hat{B}_i(\delta) \\ N_{im3}\hat{B}_i(\delta) \end{bmatrix}, \\ \Psi_{im}^{11} &= \text{sym}\{M_{im1}\hat{A}_i(\delta) - M_{im1}\hat{E}\} + \hat{E}^T X_{im1} \hat{E} - \hat{E}^T \tilde{P}_{im1} \hat{E}, \\ \Psi_{im}^{12} &= \hat{A}_i^T(\delta)M_{im3}^T - M_{im2} - \hat{E}^T M_{im3}^T + \hat{E}^T X_{im2} - \hat{E}^T \tilde{P}_{im2}. \end{aligned}$$

$$\begin{aligned}\Psi_{im}^{21} &= -M_{im1} + \hat{A}_i^T(\delta)N_{im1}^T - \hat{E}^T N_{im1}^T + \hat{E}^T X_{im1} + Q_{im1}^T \begin{bmatrix} R_i^T & 0 \end{bmatrix}, \\ \Psi_{im}^{22} &= -M_{im2} + \hat{A}_i^T(\delta)N_{im3}^T - \hat{E}^T N_{im3}^T + \hat{E}^T X_{im2}, \\ \Psi_{im}^{23} &= -M_{im3} - N_{im2}^T + X_{im2}^T + Q_{im2}^T \begin{bmatrix} R_i^T & 0 \end{bmatrix}, \\ \mathcal{H}_{im} &= \begin{bmatrix} B_{fm}\hat{C}_i(\delta) & A_{fm} & 0 & 0 \\ 0 & 0 & B_{fm}\hat{D}_i(\delta) & 0 \end{bmatrix}.\end{aligned}$$

Further, together with (25), (29) is equivalent to

$$\Theta_{im} + \text{sym}\{\mathcal{P}_{im}\mathcal{H}_{im}\} < 0, \quad (30)$$

where

$$\Theta_{im} = \begin{bmatrix} \Theta_{im}^1 & \Theta_{im}^2 & \Theta_{im}^3 & \Theta_{im}^4 \\ * & \Pi_{im}^5 & \Theta_{im}^6 & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix}, \quad \mathcal{P}_{im} = \begin{bmatrix} M_{im2} - U_{im}L_m \\ M_{im4} - L_m \\ N_{im2} - U_{im}L_m \\ N_{im4} - L_m \\ 0 \\ 0 \end{bmatrix}.$$

Applying Lemma 1 to (24), then it is obtained that (30) holds, which further implies (9) holds. Therefore, based on Theorem 1, if (24) holds, system (5) is stochastically admissible with the H_∞ performance level.

Remark 3. Note that U_{im} is chosen first. By the choice of the U_{im} , the dimension of the filter designed in the paper can be of full-order or reduced-order.

Remark 4. In Theorem 2, a sufficient condition on the issue of the asynchronous H_∞ filter design for discrete-time DMJSs with measurement losses was proposed. Then in order to compare with previous results, such as [21,36,37], the H_∞ filtering issue for DMJSs with packet losses is given. We consider the following filter for system (3) without the asynchronous modes:

$$\begin{cases} x_f(k+1) = A_{fi}x_f(k) + B_{fi}\hat{y}(k), \\ z_f(k) = C_{fi}x_f(k). \end{cases} \quad (31)$$

Based on Theorem 2, the following corollary is obtained.

Corollary 1. System (31) is stochastically admissible with the H_∞ performance level, if for given scalars $\delta \in [0, 1]$, β_i and matrices U_i , there exist symmetric matrix $\begin{bmatrix} \tilde{P}_{i1} & \tilde{P}_{i2} \\ * & \tilde{P}_{i3} \end{bmatrix} > 0$, matrices M_{i1} , M_{i2} , M_{i3} ,

M_{i4} , N_{i1} , N_{i2} , N_{i3} , N_{i4} , Q_{i1} , Q_{i2} , T_{i1} , T_{i2} , L_i , \hat{A}_{fi} , \hat{B}_{fi} , \hat{C}_{fi} , $\forall i, j \in \mathcal{J}$, and a scalar γ , such that

$$\begin{bmatrix} \Theta_i^1 & \Theta_i^2 & \Theta_i^3 & \Theta_i^4 & \Pi_i^7 \\ * & \Pi_i^5 & \Theta_i^6 & 0 & \Pi_i^8 \\ * & * & -\gamma^2 I & 0 & \hat{D}_i^T(\delta)\hat{B}_{fi}^T \\ * & * & * & -I & 0 \\ * & * & * & * & -\beta_i L_i - \beta_i L_i^T \end{bmatrix} < 0, \quad (32)$$

where

$$\begin{aligned}\Theta_i^1 &= \begin{bmatrix} \Theta_i^{11} & \Theta_i^{12} \\ * & \text{sym}\{\hat{A}_{fi} - M_{i4}\} + X_{i3} - \tilde{P}_{i3} \end{bmatrix}, \\ \Theta_i^2 &= \begin{bmatrix} \Theta_i^{21} & \Theta_i^{22} \\ \Theta_i^{23} & -M_{i4} + \hat{A}_{fi}^T - N_{i4}^T + X_{i3} \end{bmatrix}, \\ \Theta_i^3 &= \begin{bmatrix} M_{i1}\hat{B}_i(\delta) + U_i\hat{B}_{fi}\hat{D}_i(\delta) \\ M_{i3}\hat{B}_i(\delta) + \hat{B}_{fi}\hat{D}_i(\delta) \end{bmatrix}, \quad \Theta_i^4 = \begin{bmatrix} \hat{H}_{fi}^T \\ -\hat{C}_{fi}^T \end{bmatrix}, \\ \Pi_i^5 &= \begin{bmatrix} \Pi_i^{51} & \Pi_i^{52} \\ * & \text{sym}\{-N_{i4}\} + X_{i3} \end{bmatrix}, \\ \Pi_i^7 &= \begin{bmatrix} \beta_i(M_{i2} - U_iL_i) + \hat{C}_i^T(\delta)\hat{B}_{fi}^T \\ \beta_i(M_{i4} - L_i) + \hat{A}_{fi}^T \end{bmatrix},\end{aligned}$$

$$\Theta_i^6 = \begin{bmatrix} N_{i1}\hat{B}_i(\delta) + U_i\hat{B}_{fi}\hat{D}_i(\delta) \\ N_{i3}\hat{B}_i(\delta) + \hat{B}_{fi}\hat{D}_i(\delta) \end{bmatrix},$$

$$\Pi_i^8 = \begin{bmatrix} \beta_i(N_{i2} - U_iL_i) \\ \beta_i(N_{i4} - L_i) \end{bmatrix},$$

$$\Theta_i^{11} = \text{sym}\{M_{i1}\hat{A}_i(\delta) - M_{i1}\hat{E} + U_i\hat{B}_{fi}\hat{C}_i(\delta)\} + \hat{E}^T X_{i1}\hat{E} - \hat{E}^T \tilde{P}_{i1}\hat{E},$$

$$\Theta_i^{12} = U_i\hat{A}_{fi} + \hat{A}_i^T(\delta)M_{i3}^T + \hat{C}_i^T(\delta)\hat{B}_{fi}^T - M_{i2} - \hat{E}^T M_{i3}^T + \hat{E}^T X_{i2} - \hat{E}^T \tilde{P}_{i2},$$

$$\Theta_i^{21} = -M_{i1} + \hat{A}_i^T(\delta)N_{i1}^T + \hat{C}_i^T(\delta)\hat{B}_{fi}^T U_i^T - \hat{E}^T N_{i1}^T + \hat{E}^T X_{i1} + Q_{i1}^T \begin{bmatrix} R_i^T & 0 \end{bmatrix},$$

$$\Theta_i^{22} = -M_{i2} + \hat{A}_i^T(\delta)N_{i3}^T + \hat{C}_i^T(\delta)\hat{B}_{fi}^T - \hat{E}^T N_{i3}^T + \hat{E}^T X_{i2},$$

$$\Theta_i^{23} = -M_{i3} + \hat{A}_{fi}^T U_i^T - N_{i2}^T + X_{i2}^T + Q_{i2}^T \begin{bmatrix} R_i^T & 0 \end{bmatrix},$$

$$\Pi_i^{51} = \text{sym}\{-N_{i1} + \begin{bmatrix} R_i \\ 0 \end{bmatrix} T_{i1}\} + X_{i1},$$

$$\Pi_i^{52} = -N_{i2} - N_{i3}^T + X_{i2} + \begin{bmatrix} R_i \\ 0 \end{bmatrix} T_{i2}.$$

$R_i \in \mathbb{R}^{n \times (n-r)}$ is the arbitrary matrix satisfying $E^T R_i = 0$ and $\text{rank}(R_i) = n - r$. Then the desired mode-dependent H_∞ filters are given by

$$A_{fi} = L_i^{-1}\hat{A}_{fi}, \quad B_{fi} = L_i^{-1}\hat{B}_{fi}, \quad C_{fi} = \hat{C}_{fi}.$$

Remark 5. In Corollary 1, a sufficient condition on the H_∞ filter design issue for discrete-time DMJSs with packet losses was given. It should be noted that similar problems have been studied in [36,37], where the structures on freedom matrices must be $C_{4i}^{-1}G_{3i} = I$ in [36] and $X_i = \begin{bmatrix} X_{2i} & Y \\ X_{2i} & Y \end{bmatrix}$ in [37], respectively. However, these hard constraints are avoided in this paper, which bring less conservative results.

Remark 6. When $\mathcal{J} = \{1\}$, the problem studied in Corollary 1 reduces to the H_∞ filtering problem for discrete-time descriptor systems with packet losses. Similar research has been reported in [33], which shows that the problem studied in our paper is more general.

Remark 7. In contrast to [36–38], where the dimension of the designed filter must be of full-order, the dimension of the filter designed in the paper can be of full-order or reduced-order.

Remark 8. It should be pointed out that the mode-dependent filter (31) is a powerful tool to cope with the H_∞ filtering problem for discrete-time DMJSs with completely available mode information. Specifically, in the communication network medium, sometimes the mode information cannot be transmitted to the filter successfully due to communication issues. In such situations, the mode-dependent filter design approach is not suitable, and thus, it is necessary and significant to develop a mode-independent filter design. When we replace L_i , \hat{A}_{fi} , \hat{B}_{fi} , \hat{C}_{fi} with L , \hat{A}_f , \hat{B}_f , \hat{C}_f in (32), the design of the mode-independent H_∞ filter for discrete-time DMJSs with packet losses can be obtained.

Remark 9. If the parameters β_i and δ in Theorem 2 or Corollary 1 are not chosen first and are also unknown variables, then Eqs. (24) and (32) are not LMIs. Hence, in Theorem 2 or Corollary 1, β_i and δ are chosen first, and the optimal values of β_i and δ can be found by the approach stated in [43] (Remark 5). The numerical solution to this problem can be obtained by using a numerical optimization algorithm, such as the program `fminsearch` in the Optimization Toolbox of Matlab.

Remark 10. From Theorem 2, a minimum H_∞ performance level $\bar{\gamma}$ can be obtained by solving the following optimisation problem:

$$\begin{aligned} & \text{Minimize } \bar{\gamma} \\ & \text{Subject to LMI (24), } \forall i, j \in \mathcal{J}, m, n \in \Omega \end{aligned}$$

and $\bar{\gamma} = \gamma^2$. Therefore, the corresponding optimal H_∞ noise attenuation performance is $\gamma_{\min} = \sqrt{\bar{\gamma}_{\min}}$.

4. Examples

In this section, two numerical examples are given to show the validness and advantage of the proposed methodology.

Example 1. We consider the discrete-time DMJSs (1) with two subsystems, the associated parameters are given by

- Subsystem 1

$$A_1 = \begin{bmatrix} 1.1 & 0.8 & 0.2 & 0 \\ 0 & 1.2 & -0.3 & 0.5 \\ 0.3 & 0 & 0.7 & 0.4 \\ 0.9 & 0.1 & 0.4 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0.2 \\ 0.6 \\ 0.7 \end{bmatrix}, H_1 = \begin{bmatrix} 0.6 \\ 0 \\ 0.1 \\ 0.5 \end{bmatrix},$$

$$C_1 = [1.1 \ 0.6 \ 0.2 \ 0], D_1 = 0.3.$$

- Subsystem 2

$$A_2 = \begin{bmatrix} 0.9 & 1.2 & 0.4 & 0 \\ 0 & 0.4 & -1 & 0.5 \\ 0.2 & 0.4 & 0 & 1.2 \\ 0.5 & 0.1 & 0.5 & -0.7 \end{bmatrix}, B_2 = \begin{bmatrix} 0.6 \\ 0.2 \\ 0 \\ 0.7 \end{bmatrix}, H_2 = \begin{bmatrix} 0.4 \\ 0.8 \\ -0.1 \\ 0.9 \end{bmatrix}, +$$

$$C_2 = [1.2 \ 0.7 \ 0.7 \ -0.1], D_2 = -0.1.$$

The singular matrix and transition probability matrix of system (1) are given by

$$E = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 3 \end{bmatrix}, [\mu_{ij}]_{2 \times 2} = \begin{bmatrix} 0.43 & 0.57 \\ 0.66 & 0.34 \end{bmatrix}.$$

Choose $R_1 = R_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\beta_1 = \beta_2 = 1.4$, $\delta = 0.6$, $U_1 = U_2 =$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Solving LMI (32), it is obtained that the

optimal H_∞ performance is $\gamma = 2.4464$ and the mode-dependent filter gain matrices are given by

$$A_{f1} = \begin{bmatrix} 0.2679 & 0.0044 & -0.0105 & 0.0017 \\ 0.0033 & 0.2755 & -0.0096 & 0.0117 \\ -0.0089 & -0.0076 & 0.2634 & 0.0191 \\ 0.0008 & 0.0108 & 0.0197 & 0.2558 \end{bmatrix},$$

$$B_{f1} = \begin{bmatrix} -0.1215 \\ 0.0242 \\ -0.0559 \\ -0.0395 \end{bmatrix}, C_{f1} = \begin{bmatrix} -0.0715 \\ -0.0214 \\ 0.0345 \\ -0.0258 \end{bmatrix}^T,$$

$$A_{f2} = \begin{bmatrix} 0.2855 & 0.0064 & -0.0094 & 0.0078 \\ 0.0074 & 0.2812 & -0.0217 & 0.0197 \\ -0.0057 & -0.0169 & 0.2626 & 0.0241 \\ 0.0048 & 0.0157 & 0.0239 & 0.2685 \end{bmatrix},$$

$$B_{f2} = \begin{bmatrix} -0.2082 \\ -0.0765 \\ -0.0807 \\ 0.0399 \end{bmatrix}, C_{f2} = \begin{bmatrix} -0.0098 \\ -0.0026 \\ 0.0423 \\ -0.0744 \end{bmatrix}^T.$$

Table 2
Minimum H_∞ performance γ when $\delta = 0$.

| | Corollary 1 in our paper | [18] | [21] |
|----------|--------------------------|--------|--------|
| γ | 2.8796 | 3.0064 | 6.8284 |

Table 3
Minimum H_∞ performance γ for different δ .

| δ | 0.3 | 0.7 | 0.98 |
|--------------------------|--------|--------|--------|
| [36,37] | 6.5819 | 6.7331 | 8.5746 |
| Corollary 1 in our paper | 3.0152 | 3.2190 | 7.2344 |

In order to compare with the existing design methods, the minimum H_∞ performance γ for $\delta = 0$ and different δ is given in Tables 2 and 3, respectively.

On the other hand, if we choose $U_1 = U_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Solv-

ing LMI (32), it is obtained that the optimal H_∞ performance is $\gamma = 2.5929$ and the mode-dependent filter gain matrices are given by

$$A_{f1} = \begin{bmatrix} 0.2709 & 0.0062 & -0.0109 \\ 0.0051 & 0.2791 & -0.0101 \\ -0.0091 & -0.0077 & 0.2649 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.1180 \\ 0.0231 \\ -0.0509 \end{bmatrix},$$

$$C_{f1} = [-0.0713 \ -0.0180 \ 0.0378],$$

$$A_{f2} = \begin{bmatrix} 0.2867 & 0.0065 & -0.0104 \\ 0.0075 & 0.2828 & -0.0234 \\ -0.0065 & -0.0181 & 0.2637 \end{bmatrix}, B_{f2} = \begin{bmatrix} -0.2060 \\ -0.0799 \\ -0.0799 \end{bmatrix},$$

$$C_{f2} = [-0.0080 \ 0.0011 \ 0.0471].$$

If we choose $U_1 = U_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$. Solving LMI (32), it is obtained

that the optimal H_∞ performance is $\gamma = 3.4386$ and the mode-dependent filter gain matrices are given by

$$A_{f1} = \begin{bmatrix} 0.2976 & 0.0079 \\ 0.0074 & 0.2825 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.0965 \\ 0.1113 \end{bmatrix},$$

$$C_{f1} = [-0.0453 \ 0.0607],$$

$$A_{f2} = \begin{bmatrix} 0.3041 & 0.0068 \\ 0.0068 & 0.2887 \end{bmatrix}, B_{f2} = \begin{bmatrix} -0.1708 \\ 0.1209 \end{bmatrix},$$

$$C_{f2} = [0.0245 \ 0.0151].$$

Remark 11. It can be seen from this example that the dimension of the filter can be of full-order or reduced-order. Unfortunately, the proposed filter design approaches in [36–38] are not suitable for these two kinds of filter structures due to limit that the filter must be of full-order. Importantly, due to the lower dimensions of the reduced filter, the cost for constructing the filter is lower in the practical engineering applications. Thus, the condition on the dimension of the filter in this paper is milder. In addition, from Tables 2 and 3, we can clearly see that the proposed design method in our paper is less conservative than the ones given in [18,21,36,37].

Example 2. Consider the following oil catalytic cracking process as shown in [11,15]:

$$\begin{aligned} \dot{x}_1(t) &= R_{11}x_1(t) + R_{12}x_2(t) + B_1u(t) + C_1\omega(t), \\ 0 &= R_{21}x_1(t) + R_{22}x_2(t) + B_2u(t) + C_2\omega(t), \end{aligned}$$

where $x_1(t)$ is a vector to be regulated, such as valve position, regenerate temperature, or blower capacity; $x_2(t)$ is the vector reflecting business benefits, policy, administration and so on. $u(t)$ describes the regulation value, $\omega(t)$ shows extra disturbances. Similar to [15], we study the case of $u(t) = 0$. Due to abrupt failures and sudden environmental changes, parameters of the oil catalytic cracking process system will be affected inevitably, where the phenomenon of parameters variation can be described by a Markov chain. Thus, by setting certain sampling time $T_s = \frac{T}{10}$, the oil catalytic cracking process system can be depicted by the DMJS in Eq. (1) with the following parameters:

• Subsystem 1

$$A_1 = \begin{bmatrix} 1.32 & 1.32 & 0.15 & 0.15 \\ 0 & 0.96 & -0.24 & 0.4 \\ 1.53 & 0.96 & 0.73 & 0.28 \\ 1.35 & 0.15 & 0.6 & -1.5 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1.26 \\ 0.16 \\ 1.62 \\ 1.05 \end{bmatrix}, H_1 = \begin{bmatrix} 0.72 \\ 0 \\ 0.79 \\ 0.75 \end{bmatrix},$$

$$C_1 = [1.1 \quad 0.6 \quad 0.2 \quad 0], D_1 = -1.3.$$

• Subsystem 2

$$A_2 = \begin{bmatrix} 1.08 & 1.56 & 0.18 & 0.15 \\ 0 & 0.32 & -0.8 & 0.4 \\ 1.22 & 1.72 & 0.48 & 0.84 \\ 0.75 & 0.15 & 0.75 & -1.05 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.78 \\ 0.16 \\ 0.72 \\ 1.05 \end{bmatrix}, H_2 = \begin{bmatrix} 0.4 \\ 0.8 \\ -0.1 \\ 0.9 \end{bmatrix},$$

$$C_2 = [1.2 \quad 0.7 \quad 0.7 \quad -0.1], D_2 = 1.$$

The singular matrix and transition probability matrix of system (1) are given by

$$E = \begin{bmatrix} 0 & 0.6 & 0 & 0 \\ 0 & 1.6 & 0 & 0 \\ 0 & 0 & 1.4 & 0 \\ 0 & 0 & 3 & 4.5 \end{bmatrix}, [\mu_{ij}]_{2 \times 2} = \begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{bmatrix}.$$

Choose $R_1 = R_2 = \begin{bmatrix} 0.8333 \\ -0.3125 \\ 0 \\ 0 \end{bmatrix}$, $\beta_1 = \beta_2 = 1.4$, $\delta = 0.6$,

$$U_{11} = U_{12} = U_{21} = U_{22} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}. \text{ The transition matrices}$$

of Markov chain σ_k are taken as $[\pi_{mn}]_{2 \times 2}^1 = \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix}$,

$[\pi_{mn}]_{2 \times 2}^2 = \begin{bmatrix} 0.5 & 0.5 \\ 0.33 & 0.67 \end{bmatrix}$. Solving LMI (24), it is obtained that the optimal H_∞ performance is $\gamma = 18.2290$ and the asynchronous

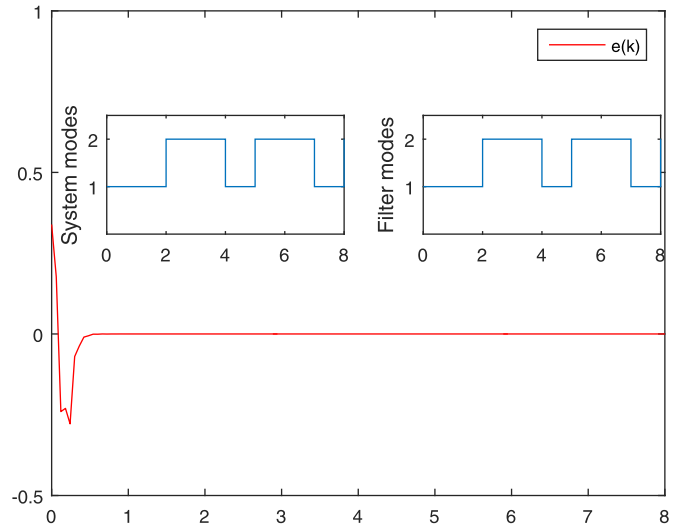


Fig. 1. The filtering error $e(k)$ of system (5).

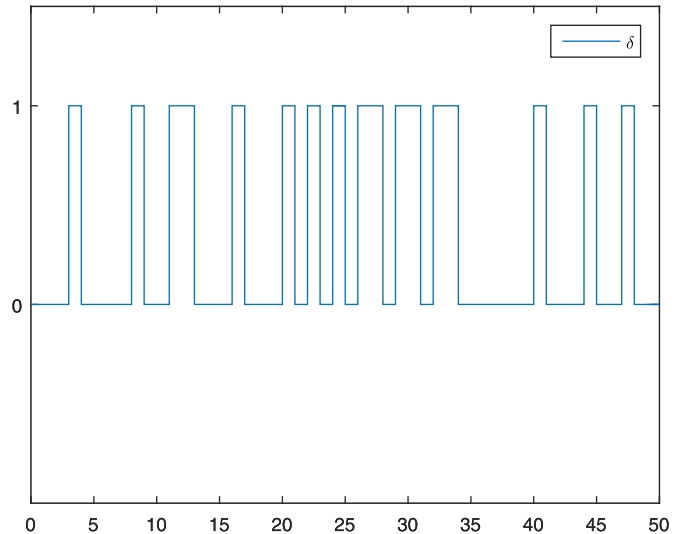


Fig. 2. Data packet losses.

filter gain matrices are given by

$$A_{f1} = \begin{bmatrix} 0.3041 & -0.0540 & -0.0227 \\ -0.0478 & 0.2512 & -0.0364 \\ -0.0156 & -0.0285 & 0.3224 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.1958 \\ -0.0240 \\ -0.0403 \end{bmatrix},$$

$$C_{f1} = [0.2823 \quad 0.7800 \quad 0.5386],$$

$$A_{f2} = \begin{bmatrix} 0.2820 & -0.0155 & -0.0129 \\ -0.0165 & 0.2739 & -0.0197 \\ -0.0084 & -0.0135 & 0.2950 \end{bmatrix}, B_{f2} = \begin{bmatrix} -0.2129 \\ -0.0547 \\ -0.0727 \end{bmatrix},$$

$$C_{f2} = [0.1102 \quad 0.7552 \quad 0.7146].$$

For simulation, taking exogenous disturbance $\omega(k) = e^{-k}$, the simulation of the filtering error $e(k)$ is presented in Fig. 1. The data packet losses are generated randomly according to $\delta = 0.6$, which is shown in Fig. 2. From Figs. 1 and 2, it is obtained that the proposed method in the paper is an effective one.

5. Conclusions

In this paper, the asynchronous H_∞ filter design problem for discrete-time DMJSs with packet losses has been investigated. The

transmission between the plant and the filter is assumed to be imperfect, which leads to the simultaneous occurrence of stochastic data packet losses and asynchronous modes. A new method has been developed to design the asynchronous H_∞ filter for this type of DMJSs. Sufficient conditions are presented guaranteeing that the filtering error system is stochastically admissible while preserving a prescribed H_∞ performance level. Then the desired asynchronous filter parameters are computed in terms of LMIs from a new perspective, which can be of full-order or reduced-order. In our future works, we will extend the proposed methods to asynchronous state estimation for DMJSs with complex transition probability, such as descriptor semi-MJSs [40], descriptor nonhomogeneous MJSs [41], and DMJSs with partly known transition descriptions [42].

Acknowledgments

This work is supported by National Natural Science Foundation of China (61473173), Major International (Regional) Joint Research Project of the National Natural Science Foundation of China (NSFC)(61320106011).

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