Proceedings of the 35th
Conference on Decision and Control
Kobe, Japan • December 1996

MINIMUM SWITCHING CONTROL FOR ADAPTIVE TRACKING

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ABSTRACT

The switching control approach has attracted a lot of attention recently for solving adaptive control problems. This approach relies on the condition that there exist a finite (or countable) number of non-switching controllers such that at least one of them will be able to control a given family of unknown (uncertain) plants. Once these non-switching controllers are found, a switching law is applied to adaptively select a correct controller. A serious problem of the switching control method is that the number of non-switching controllers can potentially be very large, especially for multi-input multi-output systems. In this paper, we consider a class of minimum-phase plants (MIMO) with some mild closedness assumptions. Given any polynomial reference input, we provide a switching control law which guarantees the exponentially stability of the closed-loop system with exponential tracking performance. The main contribution of the paper is that we give the minimum number of non-switching controllers required for switching. In particular, the number is equal to 2 for a single-input single-output plant (one for each sign of the high-frequency gain), and is equal to 2^m for an m-input m output plant. In particular, the number is independent of the degree and the relative degree of the plant.

1. INTRODUCTION

Most of the classical model reference adaptive control methods (works priori to 1980) are based on the following set of basic assumptions:

- The plant is of minimum phase;
- An upper bound of the plant degree is known;
- Its relative degree is known;
- Its sign of the high frequency gain is known;
- The reference model has the same relative degree as the plant.

This work was supported by the Australian Research Council.

See, for example, Goodwin and Sin [4] and Sastry and Bodson [15] for overviews. It is well recognized that this set of assumptions are often unrealistic in practical applications where the plant may be difficult to model. Adaptive controllers designed based on these assumptions may be very non-robust, as shown by well-known examples of Rohrs *et. al.* [14].

One of the important lines of adaptive control research over the last 15 years or so is to investigate the minimal set of assumptions needed for the plant so that it can be adaptively stabilized. This line of research can be traced back to a paper by Morse [8] which raised a number of open questions regarding the classical assumptions. The first breakthrough was given in a paper by Nussbaum [13] which provides a new adaptation method (called Nussbaum gain later) for treating the case where the sign of the high frequency gain of the plant is unknown. Nussbaum's result was generalized by Martensson [6] which shows surprisingly that asymptotic stabilization of a minimal plant can be achieved with a rather weak assumption, i.e., one only needs to know the degree of a stabilizing controller. In fact, even this condition can be relaxed. Because Martensson's approach involves an exhaustive on-line search over the space of candidate gain matrices before "latching on" to an appropriate stabilizer, two serious problems arise: 1) Lyapunov stability cannot be guaranteed and, consequently, an excessive overshoot may occur; 2) the output must be free of (even arbitrarily small) persistent measurement noises to avoid possible destabilization. These problems have been reported and carefully analyzed in a paper by Fu and Barmish [2].

An alternative approach called switching control to adaptive stabilization was proposed by Fu and Barmish [2] to assure Lyapunov stability (exponential stability in fact) and to permit small measurement noises. More explicitly, Fu and Barmish show that adaptive stabilization of a family of unknown multi-input multi-output (MIMO) plants Σ can be achieved if the following mild assumptions are satisfied:

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- The upper bound $n_{\max}$ of the degree of the plant family is known;
- Every member of Σ is stabilizable and detectable;
- For each $n \leq n_{\max}$, the set of state-space realization matrices of the subfamily Σ_n of plants with degree n is compact (i.e. bounded and closed), and the compact set is known;

Indeed, it is shown in [2] that there exists a finite number of fixed linear time-invariant controllers (which will be called non-switching controllers in the sequel) such that every member of Σ will be stabilized by at least one of them. Consequently, a switching mechanism is applied on-line to search for a correct controller for an arbitrary unknown member of Σ. The resulting controller is piecewise linear time-invariant with at most a finite number of switchings. The closed-loop system is guaranteed to be exponentially stable, and robust with respect to small measurement noises. Further, an extension of this result is given in [3] to treat the case where singular perturbations to the plant exist, i.e., the compactness assumption above is violated.

A somewhat different switching control approach, called hysteresis switching, is also reported in a series of papers by Middleton *et. al.* [7], Morse *et. al.* [11], and Weller and Goodwin [17] to solve the problem of model reference adaptive control. No compactness assumption is required for this approach. However, the family of plants Σ to be dealt with need to satisfy the following assumptions:

- Every member of Σ is of minimum phase;
- Every member of Σ is stabilizable and detectable;
- An upper bound $n_{\max}$ of the plant degree is known.

The basic idea in Morse *et. al.* [11] and Weller and Goodwin [17] involves two steps. The first step is to construct an estimator for each subfamily of plants with the same McMillian degree, the same relative degree, the same high-frequency gain sign, and the same "permutation" of outputs. Subsequently, a classical model reference adaptive controller is designed for each such subfamily. The second step is to use a so-called hysteresis switching algorithm based on Middleton *et. al.* to adaptively select a correct controller. The detailed switching mechanism is somewhat different from the switching control method in Fu and Barmish [2]. The main difficulty with this approach is that the number of non-switching controllers is excessively large. This number is in fact equal to $2^m \times m! \times mn_{\max}$ for m-input m-output plants. For example, when $m = 5$ and $n_{\max} = 10$, this number is equal to 192,000. The excessively large number of non-switching controllers mean that it may potentially take an extremely long dwell time before a correct controller can be found.

The interest in switching control has resurged very recently owing to new contributions made by Morse [9, 10], Narendra and Balakrishnan [12] and Hocherman *et. al.* [5]. The so-called supervisory control for adaptive set-point tracking is proposed by Morse [9, 10] to speed up the switching and to improve the transient response. Hocherman *et. al.* [5] studies the convergence of Morse' switching scheme. A different superviosry control scheme for model reference adaptive control is proposed by Narendra and Balakrishnan[12].

The purpose of this paper is to identify the minimum number of non-switching controllers required by the switching control method for a rather general class of minimum phase systems. More specifically, given a family of uncertain plants and a polynomial reference signal, we need to design an output feedback controller such that the closed-loop system corresponding to any plant in the family is exponentially stable and its output exponentially approaches the reference signal. The family of plants we consider in this paper are assumed to satisfy the same set of assumptions as required by the hysteresis switching control approach, plus an additional mild boundedness assumption. The reference signal is assumed to be a polynomial function. We show that the number of non-switching controllers can be reduced down to 2^m only for m-input m-output plants. For the same example above, this number is reduced from 192,000 to 32 only!

Our approach involves two key ideas: 1) We divide the family of plants into 2^m subfamilies, each can be robustly stabilized by a single linear time-invariant controller. This step is based on an important paper on robust stabilization by Wei and Barmish [16]. By slightly modifying their controller and relaxing the assumptions they use for the controller design, we show that each subfamily can be controlled by a single linear time-invariant controller such that the closed-loop system associated with any member of the subfamily is exponentially stable and its output exponentially tracks the reference signal. 2) Once these 2^m non-switching controllers have been determined, a simple switching algorithm similar to the one in Fu and Barmish [2] is employed on-line to search for a correct controller. After at most $2^m - 1$ switchings, a correct controller will be found for an arbitrary member of the plant family.

The rest of this paper is organized as follows: Section 2 formalizes the adaptive tracking problem and assumptions; Section 3 considers the design of non-switching controllers; Section 4 provides the switching algorithm and the main result on exponential stability and tracking; and Section 5 concludes with some remarks.

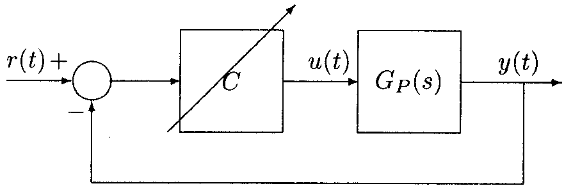

Figure 1: Adaptive Tracking Problem

2. PROBLEM FORMULATION AND ASSUMPTIONS

Let $\mathcal{R}^{m\times m}[s]$ denote the set of all $m \times m$ rational matrices. Given a set of rational matrices $\Sigma \subset \mathcal{R}^{m\times m}[s]$, representing a family of uncertain plants, and a polynomial time function $r(\cdot) : \mathcal{R} \to \mathcal{R}^m$, the adaptive tracking problem considered in this paper is as follows: *Find an adaptive controller C as depicted in Figure 1 such that for any $G_P(s) \in \Sigma$, the closed-loop system is exponentially stable and its output $y(t)$ will exponentially approach $r(t)$, i.e.*

$$||y(t) - r(t)|| \leq Me^{-\lambda t}, \quad \text{if } x(0) = 0, \tag{1}$$

for some $M > 0$ and $\lambda > 0$.

Before we introduce the assumptions on the plant family, we need to introduce some notation.

Definition 1 *[16] Given $G(s) \in R^{m\times m}[s]$ and two $m \times m$ polynomial matrices $N(s)$ and $D(s)$, the pair $(N(s), D(s))$ is called a* row Hermite factorization *if the following conditions hold:*

1. *$D(s)$ is invertible and $G(s) = N(s)D^{-1}(s)$;*
2. *$N(s)$ and $D(s)$ are coprime in the closed right-half plane;*
3. *$D_{ii}(s)$ is a monic polynomial for $i = 1, ..., m$;*
4. *$D_{ij}(s) = 0$ for all $i < j$;*
5. *$\deg D_{ij}(s) < \deg D_{ii}(s)$ for all $i > j$,*

where $D_{ij}(s)$ is the ij-th element of $D(s)$.

Remark 1 *It is known that there always exists a row Hermite factorization for any rational matrix; see [16]. This factorization is even unique if the coprimeness condition above is strengthened to include the open left-half plane. The reason we use a weaker coprimeness condition is to allow a simpler factorization for parameterized rational matrices. For example, a row Hermite factorization of*

$$G(s,q) = \frac{s+1}{s+q}, \quad q \in [1/2, 2]$$

is given by

$$N(s,q) = s+1; \quad D(s,q) = s+q$$

when the weaker version of coprimeness condition is used. For the stronger version of coprimeness condition, the row Hermite factorization of $G(s,q)$ at $q = 1$ must be given by $N(s) = 1$ and $D(s) = 1$, which causes discontinuity.

Based on the remark above, we can express Σ in an equivalent form:

$$(\mathcal{N}_P, \mathcal{D}_P) = \{(N_P(s), D_P(s)) : \text{a row Hermite factorization for } G_P(s) \in \Sigma\} \tag{2}$$

However, for notational simplicity, we will also denote $(\mathcal{N}_P, \mathcal{D}_P)$ by Σ unless confusion arises.

Remark 2 *Using duality, we can define the column Hermite factorization $(\tilde{D}(s), \tilde{N}(s))$ for every $G(s)$, i.e., $G(s) = \tilde{D}^{-1}(s)\tilde{N}(s)$. All the properties above about the row Hermite factorization also apply using duality to the column Hermite factorization. In this paper, the row Hermite factorization will be used for the plant and the column Hermite factorization, for the controller.*

Definition 2 *A given family of polynomials $\mathcal{P}$ is called of* degree d *if every polynomial $p(s) \in \mathcal{P}$ is of degree d. $\mathcal{P}$ (possibly with different degrees) is called* bounded *if the set of zeros of $\mathcal{P}$ is a bounded set. The* closure *of $\mathcal{P}$ is defined to be the set of all limiting polynomials convergent from a sequence of polynomials of the same degree in $\mathcal{P}$. A family of polynomial matrices is called bounded if every matrix element family is bounded.*

Remark 3 *A few comments on the boundedness condition are in order. If a family of polynomials $\mathcal{P}$ contains a zero polynomial (which is identically equal to zero), then our definition of boundedness implies that $\mathcal{P}$ is not bounded because the zero polynomial has zeros everywhere. In fact, $\mathcal{P}$ with maximum degree $\bar{d}$ is bounded if and only if the following conditions hold:*

- *It does not contain a zero polynomial;*
- *For every $1 \leq d < \bar{d}$, the subfamily of polynomials in $\mathcal{P}$ with degree d has the following property:*
  - *The set of polynomial coefficients is bounded in $\mathcal{R}^{d+1}$;*
  - *There exists some $\delta > 0$ such that the absolute value of the leading coefficient of every polynomial in the subfamily is δ or more.*

Denote by $z_i(s)$ the i-th lower principal minor of $N_P(s)$, i.e. $z_i(s)$ is the determinant of the part of $N_P(s)$ with the first $(i-1)$ rows and columns deleted. Further denote the families of polynomials

$$\mathcal{Z}_i = \{z_i(s) : N_P(s) \in \mathcal{N}_P\}; \quad i = 1, \cdots, m \tag{3}$$

We will adopt the following set of assumptions in the rest of the paper:

A1. (Minimum Phase Invariance) $\det N_P(s)$ is Hurwitz for every member $N_P(s)$ in the closure of $\mathcal{N}_P$.

A2. (Upper bound of Degree) The upper bound of the degree $d_{\max}$ of $D_{ij}(s)$ over $\mathcal{D}_P$ is known.

A3. (Boundedness of Numerator) $\mathcal{Z}_i$ is bounded for every $i = 1, \cdots, m$.

A4. (Boundedness of Denominator) $\mathcal{D}_P$ is bounded.

Remark 4 *The Assumptions A1-A2 are similar to the ones used in [11, 17]. The boundedness assumptions will enable us to significantly reduce the number of non-switching controllers and to guarantee other nice properties such as exponential stability and linear piecewise-invariant control. Note that the boundedness assumptions are rather weak.*

Finally, we define the *maximum degree* of the reference signal $r(t)$ to be

$$n_r = \max\{n_1, \cdots, n_m\} \tag{4}$$

where n_i the polynomial degree of the i-th component of $r(t), i = 1, \cdots, m$.

3. DESIGN OF NON-SWITCHING CONTROLLERS

Our method for designing non-switching controllers is motivated by a robust stabilization approach of Wei and Barmish [16]. These authors consider a family of uncertain plants satisfying assumptions similar to A1-A4 (slightly stronger though) and the following additional one:

A5. The leading coefficient of every principal minor $z_i(s)$ of $N_P(s)$ is either positive invariant or negative invariant over $\mathcal{N}_P$.

With this additional assumption, it is shown in [16] that there exists a single linear time-invariant controller to robustly stabilize the whole family of plants.

Our first result, Lemma 1 shows that a given family of plants satisfying Assumptions A1-A4 can be decomposed into 2^m subfamilies such that each subfamily will satisfy not only A1-A4 but also A5. Using the design approach of Wei and Barmish [16], we can find a linear time-invariant stabilizer for each of the 2^m subfamilies of plants. Consequently, 2^m non-switching controllers can be designed to cover the whole family Σ.

To also achieve the tracking requirement, we apply a standard "trick" which converts the tracking problem into a stabilization problem. More precisely, the plant is cascaded with an integrator matrix of sufficiently high order before the stabilization design. This cascaded part is a part of the controller but treated as a part of the plant in the design.

Lemma 1 *Given a family of transfer matrices Σ in (2) satisfying Assumptions A1-A4, let $Z_i, i = 1, \cdots, m$ be given by (3) and define*

$$\begin{aligned} \mathcal{Z}_i^+ &= \{z_i(s) : z_i(s) \in \mathcal{Z}_i \text{ with the leading} \\ &\qquad \text{coefficient being positive}\} \qquad (5) \\ \mathcal{Z}_i^- &= \{z_i(s) : z_i(s) \in \mathcal{Z}_i \text{ with the leading} \\ &\qquad \text{coefficient being negative}\} \qquad (6) \end{aligned}$$

Given any sign vector

$$\alpha = (\alpha_1, \cdots, \alpha_m), \alpha_i \in \{-, +\}, i = 1, \cdots, m, \tag{7}$$

define

$$\Sigma_\alpha = \{(N_P(s), D_P(s)) \in \Sigma, z_i(s) \in \mathcal{Z}_i^{\alpha_i}\} \tag{8}$$

Then, we have the following properties:

i).

$$\Sigma = \cup_\alpha \Sigma_\alpha \tag{9}$$

ii). *Each subfamily Σ_α satisfies Assumptions A1-A5.*

Theorem 1 *Consider a family of uncertain plants $\Sigma_\alpha \subset \mathcal{R}^{m \times m}$ satisfying Assumptions A1-A5. Then, for any reference signal polynomial reference signal $r(\cdot) : \mathcal{R} \to \mathcal{R}^m$ of degree $n = (n_1, \ldots, n_m)$ such that the closed-loop system associated with any uncertain plant $G_P(s) \in \Sigma_\alpha$ is Hurwitz (exponentially) stable and its output $y(t)$ exponentially tracks $r(t)$.*

Remark 5 *The number of non-switching controllers is 2^m, which is the number of possible sign vectors in (7). We point out that this number is minimal. To see this, consider the following family of plants:*

$$\begin{aligned} \Sigma &= \Big\{ \operatorname{diag}\{\alpha_1 \frac{1}{s-1}, \cdots, \alpha_m \frac{1}{s-1}\}, \\ &\qquad \alpha_i \in \{-1, 1\}, i = 1, \cdots, m\} \qquad (10) \end{aligned}$$

Obviously, it requires 2^m linear time-invariant stabilizers to cover the whole family, one for each combinations of the signs of the high-frequency gains.

The proof of Theorem 1 is given in a rather informal way. We simply need to show how to construct a single controller for each subfamily of plants $\Sigma_\alpha \subset \mathcal{R}^{m \times m}[s]$ satisfying Assumptions A1-A5. Our design procedure involves two simple steps: The first step is to cascade the plant by an integrator matrix so that the tracking problem becomes a robust stabilization problem. The second step is to apply the design procedure of Wei and Barmish [16] to achieve robust stabilization. We only explain how the first step is carried out because the details of the second step can be found in [16].

Recall that the maximum degree of the reference signal is given by n_r. Define the integrator matrix:

$$I(s) = \text{diag}\left\{s^{-n_r}, \ldots, s^{-n_r}\right\} \in \mathcal{R}^{m\times m}[s], \qquad (11)$$

the family of cascaded plants

$$\begin{aligned}\hat{\Sigma} &= \{\hat{G}_P(s) = I(s)G_P(s) : G_P(s) \in \Sigma\} \\ &= \{(N(s), D(s)I(s)) : (N(s), D(s)) \in \Sigma\}, \qquad (12)\end{aligned}$$

and their subfamilies $\hat{\Sigma}_\alpha$.

Then, it is easy to verify that $\hat{\Sigma}_\alpha$ still satisfies Assumptions A1-A5. Further, the following fact is well-known: If there exists a controller $\hat{C}_\alpha(s) \in \mathcal{R}^{m\times m}[s]$ which robustly exponentially stabilizes $\hat{\Sigma}_\alpha$, then the following controller

$$C_\alpha(s) = \hat{C}_\alpha(s)I(s) \qquad (13)$$

will robustly exponentially stabilize Σ_α and guarantee the exponential tracking requirement for any reference signal $r(t)$ with maximum degree n_r.

4. SWITCHING ALGORITHM

Once the 2^m non-switching controllers $\{C_\alpha(s)\}$ are designed, the next step is to specify a switching algorithm which, when applied on-line, is able to adaptively find a correct controller for any given plant $G_P(s) \in \Sigma$. The switching algorithm is the same as in Fu and Barmish [2].

We index all the subfamilies of plants Σ_α and the controllers $C_\alpha(s)$ by Σ_i and $C_i(s)$, respectively, $i = 1, \cdots, 2^m$.

First, we use the output of the plant to generate the following signal:

$$\dot{\phi}(t) = ||e(t)||^2 \qquad (14)$$

where

$$e(s) = I(s)(y(s) - r(s)) \qquad (15)$$

Define a test function

$$V(t,\tau) = \phi(t) - \phi(t-\tau) = \int_{t-\tau}^{t} ||e(t)||^2 dt \qquad (16)$$

for $t \leq 0$ and $\tau \in [0, t]$.

Given any plant $G_P(s) \in \Sigma$, suppose the controller $C_i(s)$ is applied at some time t_{i-1}. If $G_P(s) \in \Sigma_i$, then a nice property of $e(t)$ is that it converges to zero exponentially. It follows that there exists a *dwell time* $\tau_i > 0$ such that $V(t, \tau_i)$ has the following monotonic decreasing property:

$$V(t,\tau_i) \leq \rho V(t-\tau_i, \tau_i), \; \forall t \geq t_{i-1} + 2\tau_i \qquad (17)$$

for any prescribed $\rho \in (0, 1)$.

On the other hand, if $G_P(s) \notin \Sigma_i$, one of the three cases will happen:

1) The property (17) fails at $t = t_{i-1} + 2\tau_i$ immediately;

2) (17) holds for a little while after $t = t_{i-1} + 2\tau_i$ and then fails at, say, t_i;

3) (17) holds forever.

In the first case, we will know immediately (at $t_{i-1} + 2\tau_i$) that $G_P(s) \notin \Sigma_i$, so another controller should be selected. In the second case, we will not know that the controller is wrong until t_i. Again, switching is needed at t_i. However, the controller $C_i(s)$ has managed to decrease the test function for the period of time from $t_{i-1} + 2\tau_i$ to t_i. In the third case, we will never find out that $G_P(s) \notin \Sigma_i$, so $C_i(s)$ will be applied to $G_P(s)$ forever. It follows from (17) that the test function will decay to zero exponentially, and so will the error signal $e(t)$ (see (16)). That is, the tracking requirement is satisfied. Also implied is the exponential stability of the closed-loop system due to the coprimeness of $N_P(s)$ and $D_P(s)$.

Based on the analysis above, we are ready to build a switching function. Initially, we apply $C_1(s)$ and set the switching time $t_0 = 0$. Now, for $i = 1, 2, ..., 2^m - 1$, define the new switching instant

$$t_i = \sup\{t : t \geq t_{i-1} + 2\tau_i; V(t,\tau_i) \leq \rho V(t-\tau_i, \tau_i)\} \qquad (18)$$

and the switching index function

$$h(t) = i, \quad \text{for } t \in [t_{i-1}, t_i) \qquad (19)$$

Then, choose the switching control law is given by

$$C = C_{h(t)} \qquad (20)$$

In case $t_i = \infty$ for some $i < 2^m - 1$, the generation of t_i is terminated and the controller remains to be $C_i(t)$ indefinitely.

We make a few observations about the switching algorithm above. First, there are only a finite number of switchings and the switching index $h(t)$ converges to a constant. In fact, suppose $G_P(s) \in \Sigma_j$, $1 \leq j \leq 2^m$, then switching stops when or before the switching index reaches j. Secondly, for each switching index $h(t) = i$, the testing function diverges for at most $2\tau_i$ time long. So the overall behaviour of the testing function is that it decays exponentially everywhere (except for a bounded finite period of time which is negligible). Consequently, the error function $e(t)$ exponentially converges to zero. This, in turn, guarantees the exponential stability of the closed-loop system. The detailed analysis can be found in Fu and Barmish [2].

In summary, we have the following result:

Theorem 2 *Given a family of uncertain plants $\Sigma \subset \mathcal{R}^{m\times m}[s]$ satisfying Assumptions A1-A4 and a reference signal $r(\cdot) : \mathcal{R} \to \mathcal{R}^m$ with maximum degree n_r. Let the non-switching controllers $C_i(s), i = 1, \cdots, 2^m$ be designed according to the procedure in Section 3 and the switching control law be given by (14)-(20). Then, for any (unknown) member plant $G_P(s) \in \Sigma$, the closed-loop system is exponentially stable and the tracking error converges to zero exponentially.*

5. CONCLUSIONS

In this paper, we have considered an adaptive tracking problem for a family of uncertain m-input m-output plants which satisfy Assumptions A1-A4. We have shown that at most 2^m non-switching controllers are required such that any plant in the given plant family can be exponentially stabilized with exponential tracking performance by one of these controllers. This 2^m number is also shown to be the minimum number, without further restrictions on the plant family. This number is also significantly less than obtained in [11, 17] where a MRAC problem is considered. Once these 2^m controllers are found, a simple switching algorithm is established using the idea in [2]. This switching algorithm guarantees that a correct controller will be found adaptively. More importantly, both exponential stability and exponential tracking are guaranteed for the closed-loop system. The resulting switching controller is a piecewise linear time-invariant one with at most $2^m - 1$ switchings.

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