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Localization Based Switching Adaptive Control for Time-Varying Discrete Time Systems

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Abstract

In this paper a new systematic switching control approach to adaptive stabilization of linear time-varying (LTV) discrete-time systems is presented. This approach is based on a localization method, and is conceptually different from existing switching adaptive control schemes. A feature of the localization based method is that the control switching converges rapidly. By utilizing this fast speed of localization and the rate of admissible parameter variation, we provide conditions under which the closed-loop system can be exponentially stabilized.

1 Introduction

Control design for linear dynamic systems with unknown parameters has been extensively studied over the last three decades. Despite significant advances in adaptive control and robust control in recent years, control of systems with large-size time-varying uncertainty remains a very difficult task. Not only in the time varying case are the control problems hard, so is the analysis of stability and performance.

It is well-known [5] that classical adaptive algorithms prior to 1980 were all based on the following set of standard assumptions or variations of them:

(i) An upper bound on the plant order is known;

(ii) The plant is minimum phase;

(iii) The sign of high frequency gain is known;

(iv) The uncertain parameters are constant, and the closed-loop system is free from measurement noise and input/output disturbance.

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The classical adaptive algorithms are known to suffer from various robustness problems [17]. A number of attempts have been made since 1980 to relax the assumptions above. A major breakthrough occurred in the mid 1980's [8, 10] for adaptive control of LTV plants with sufficiently small in the mean parameter variations. Later attempts were made for a broader class of systems (see, for example, [3, 16, 18]).

In a separate research line, a number of switching control algorithms have been proposed recently by several authors [2, 4, 9, 12, 13], thus significantly weakening the assumptions in (i)-(iv). Both continuous and discrete linear time-invariant systems were considered. Research in this direction was originated from the pioneering works by Nussbaum [15] and Martensson [9]. In particular, it was shown in [9] that the only *a priori* information which is needed for adaptive stabilization of a minimal linear time-invariant plant is the order of any stabilizing controller. Martensson's method is based on a "dense" search over the control parameter space, allows no measurement noise, and guarantees only asymptotic stability rather than exponential stability. These weaknesses were overcome by the work [4] where a finite switching control method was proposed for LTI systems with uncertain parameters satisfying some mild compactness assumptions.

A different switching control approach, called hysteresis switching, was reported in a number of papers [11, 19] in the context of adaptive control. The switching, in these cases, is used to avoid the "stabilizability" problem in adaptive controllers.

Conventional switching control techniques are based on some mechanism of an exhaustive search over the entire set of potential controllers (either a continuum set [9] or a finite set [4]). A major drawback is that the search may converge very slowly, resulting in excessive transients which render the system "unstable" in a practical sense. This phenomenon can take place even if the closed-loop system is exponentially stabilized. To alleviate this problem, several new switching control schemes have been proposed recently. The so-called supervisory control for adaptive set-point tracking is proposed by Morse [13] to improve the transient response. Very similar, in spirit, supervisory control schemes were analyzed in [6, 14].

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The main idea of the supervisory control schemes is to reduce the set of potentially stabilizing feedbacks based on certain on-line estimation. However, they do not guarantee a finite convergence of a switching controller.

In this paper, we present a new approach for switching adaptive control for LTV systems. This approach is based on a localization method, and is conceptually different from the supervisory control schemes and other switching schemes. The localization method was initially proposed by the authors for LTI systems [20]. This method has the unique feature of fast convergence for switching. That is, it can localize a suitable stabilizing controller very quickly, hence the name of localization. The main contribution of this paper is the generalization of the method to LTV systems. We show that this method is also easy to implement, has no bursting phenomenon, and works with or without a known bound on the exogenous disturbance.

2 Problem Statement

We consider a general class of LTV discrete-time plants in the following form:

$$D(t,z^{-1})y(t) = N(t,z^{-1})u(t) + \xi(t) + \eta(t) \qquad (2.1)$$

where $u(t)$ is the input, $y(t)$ is the output, $\xi(t)$ is an exogenous disturbance, $\eta(t)$ represents some unmodelled dynamics (to be specified later), z^{-1} is the unit delay operator,

$$N(t,z^{-1}) = n_1(t)z^{-1} + \cdots + n_n(t)z^{-n} \qquad (2.2)$$

$$D(t,z^{-1}) = 1 + d_1(t)z^{-1} + \cdots + d_n(t)z^{-n} \qquad (2.3)$$

We will denote by $\theta(t)$ the vector of unknown time-varying parameters, i.e.,

$$\theta(t) = (n_n(t),\cdots,n_1(t),d_n(t),\cdots,d_1(t))^T \qquad (2.4)$$

Throughout this paper, we will use the following non-minimal description of the plant (2.1):

$$x(t+1) = A(\theta(t))x(t) + B(\theta(t))u(t) + E\ (\xi(t)+\eta(t)) \qquad (2.5)$$

where

$$x(t) = \left[\ u(t-n+1)\cdots u(t-1)|y(t-n+1)\cdots y(t))^T\ \right] \qquad (2.6)$$

and $A(\theta(t))$, $B(\theta(t))$ and E are matrices of appropriate dimensions. We also define

$$\phi(t) = \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} \qquad (2.7)$$

Then, (2.1) can be rewritten as

$$y(t) = \phi^T(t-1)\theta(t) + \xi(t) + \eta(t) \qquad (2.8)$$

The following assumptions are used throughout the paper:

(A1) The order n of plant (excluding the unmodelled dynamics) is known.

(A2) There exists a known compact set $\Omega \in \mathbf{R}^{2n_{\max}}$ such that $\theta(t) \in \Omega$ for all $t \in \mathbf{N}$.

(A3) The plant (2.1) with frozen parameters and zero unmodelled dynamics (i.e., $\eta(t) \equiv 0$) is uniformly stabilizable over Ω. That is, for any $\theta(t) \equiv \theta \in \Omega$, there exists a linear time-invariant controller $C(z^{-1})$ such that the closed-loop system is exponentially stable.

(A4) The exogenous disturbance ξ is uniformly bounded, i.e.,

$$\sup_{t\geq 0} |\xi(t)| \leq \bar{\xi} \qquad (2.9)$$

for some known constant $\bar{\xi}$.

(A5) The unmodelled dynamics is arbitrary subject to

$$||\eta(t)|| \leq \bar{\eta}(t) = \epsilon \sup_{0\leq k\leq t} \sigma^{t-k}||x(k)|| \qquad (2.10)$$

for some constants $\epsilon > 0$ and $0 \leq \sigma < 1$ which represent the "size" and "decay rate" of the unmodelled dynamics, respectively.

(A6) The uncertain parameters are allowed to have two types of time variations: (i) slow parameter drifting described by

$$||\theta(t) - \theta(t-1)|| \leq \alpha,\ \forall\, t > t_0 \qquad (2.11)$$

for some constant $\alpha > 0$, and (ii) infrequent large jumps constrained by

$$\sum_{i=t}^{t+\tau N} s_i \leq \tau \qquad (2.12)$$

for all $t \geq 0$, where $\tau > 0$ and $N > 0$ are constants with $1/N$ representing the "frequency" of large jumps, and

$$s_i = \begin{cases} 0 & \text{if } ||\theta(i) - \theta(i-1)|| \leq \alpha, \\ 1 & \text{otherwise} \end{cases} \qquad (2.13)$$

Remark 2.1 *Assumption (A1) can be relaxed to that only an upper bound $n_{\max}$ is known. Assumption (A4) will be used only in Sections 3-4 and will be relaxed to allow $\bar{\xi}$ to be unknown in Section 5 where an estimation scheme is given for $\bar{\xi}$.*

Remark 2.2 *By using simple algebraic manipulations, measurement noises and input disturbances are easily incorporated into the model (2.1). In this case,* $y(t)$, $u(t)$ *and* $\xi(t)$ *represent the measured output, computed input and (generalized) exogenous disturbance, respectively. For example, if a linear time-invariant discrete-time plant is described by*

$$y(z) = \frac{N(z^{-1})}{D(z^{-1})}(u(z) + d(z)) + q(z)$$

where $d(z)$ *and* $q(z)$ *are the input disturbance and plant noise, respectively, the plant can be rewritten as*

$$D(z^{-1})y(z) = N(z^{-1})u(z) + (N(z^{-1})d(z) + D(z^{-1})q(z^{-1}))$$

Therefore, the exogenous input $\xi(z)$ *is* $N(z^{-1})d(z) + D(z^{-1})q(z^{-1})$.

Remark 2.3 *Note that assumptions similar to (A4) have been used in adaptive control for systems with unmodelled dynamics; see, e.g., [10, 7, 3].*

The switching controller to be designed will be of the following form:

$$u(t) = K_{i(t)}x(t) \tag{2.14}$$

where $K_{i(t)}$ is the control gain applied at time t, and $i(t)$ is the switching index at time t, taking value in a finite index set I. The objective of the control design is to determine the set of control gains

$$K_I = \{K_i, i \in I\} \tag{2.15}$$

and an on-line switching algorithm for $i(t)$ so that the closed-loop system will be "stable" in some sense.

The specific notion of stability to used in this paper is described below:

Definition 2.1 *The system (2.1) satisfying (A4)-(A5) is said to be globally* $\bar{\xi}$*-exponentially stabilized by the controller (2.14) if there exist constants* $M_1 > 0$, $0 < \rho < 1$, *and a function* $M_2(\cdot) : \mathbf{R}_+ \to \mathbf{R}_+$ *with* $M_2(0) = 0$ *such that*

$$||x(t)|| \leq M_1 \rho^{(t-t_0)} ||x(t_0)|| + M_2(\bar{\xi}) \tag{2.16}$$

holds for all $t_0 \geq 0$, $x(t_0)$, $\bar{\xi} \geq 0$, *and* $\xi(\cdot)$ *and* $\eta(\cdot)$ *satisfying (A4)-(A5), respectively.*

3 Localization Technique for LTI Plant

The switching algorithms to be used in this paper are based on a *localization* technique. This technique is used in [20] for LTI plants, which potentially allows us to discard incorrect controllers very quickly while guaranteeing exponential stability of the closed-loop system. In this section, we provide a localization technique for LTI plants which is slightly different from [20] but will be readily extended to LTV plants.

First, we decompose the parameter set Ω to obtain a *finite cover* $\{\Omega_i\}_{i=1}^{L}$ which satisfies the following conditions:

C1. $\Omega_i \subset \Omega$, $\Omega_i \neq \{\ \}, i = 1, \cdots, L$;

C2. $\bigcup_{i=1}^{L} \Omega_i = \Omega$;

C3. For each $i = 1, \cdots, L$, let θ_i and $r_i > 0$ denote the "center" and "radius" of Ω_i, i.e., $\theta_i \in \Omega_i$ and $||\theta - \theta_i|| \leq r_i$ for all $\theta \in \Omega_i$. Then, there exist K_i, $i = 1, \cdots, L$, such that

$$\begin{array}{l} |\lambda_{\max}(A(\theta) + B(\theta)K_i)| < 1, \\ \forall\ ||\theta - \theta_i|| \leq r_i, i = 1, \cdots, L \end{array} \tag{3.1}$$

It is well-known that such a finite-cover can be found under assumptions (A1)-(A3); see, e.g., [4]. More specifically, there exist (sufficiently large) L, (sufficiently small) r_i, and suitable K_i, $i = 1, \cdots, L$, such that (C1)-(C3) hold.

The key observation used in the localization technique is the following fact: Given any parameter vector $\theta \in \Omega_j$ and a control gain $K_{i(t)}$ for some $i(t), j = 1, \cdots, L$, if $i(t) = j$, then

$$y(t) = \theta^T \phi(t-1) + \xi(t) + \eta(t) \tag{3.2}$$

It follows that

$$|\theta_j^T \phi(t-1) - y(t)| \leq r_j ||\phi(t-1)|| + \bar{\xi} + \bar{\eta}(t) \tag{3.3}$$

This observation leads to a simple localization scheme by elimination: If the above inequality is violated at any time instant, we know that the switching index $i(t)$ is wrong (i.e., $i(t) \neq j$), so it can be eliminated. The unique feature of the localization technique comes from the fact that violation of (3.3) allows us not only to eliminate $i(t)$, but many others. As a result, a correct controller can be found very quickly.

We now describe the localization algorithm. Let $I(t)$ denote the set of "admissible" control gain indices at time t and initialize it to be

$$I(t_0) = \{1, 2, \cdots, L\} \tag{3.4}$$

Choose any initial switching index $i(t_0) \in I(t_0)$. For $t > t_0$, define

$$\hat{I}(t) = \{j : (3.3) \text{ holds }, j = 1, \cdots, L\} \tag{3.5}$$

Then, the localization algorithm is simply given by

$$I(t) = I(t-1) \bigcap \hat{I}(t), \quad \forall t > t_0 \tag{3.6}$$

The switching index is updated by taking

$$i(t) = \begin{cases} i(t-1) \text{ if } t > t_0 \text{ and } i(t-1) \in I(t) \\ \text{any member of } I(t) \text{ otherwise} \end{cases} \tag{3.7}$$

The following result holds:

Lemma 3.1 *Given the uncertain system (2.1) satisfying Assumptions (A1)-(A5), suppose a finite cover* $\{\Omega_i\}_{i=1}^L$ *be a finite cover of* Ω *satisfying Conditions (C1)-(C3). Then, the localization algorithm given in (3.4)-(3.7) applied to a LTI plant (2.1) possesses the following properties:*

(i) $I(t) \neq \{\ \}, \quad \forall t \geq t_0$;

(ii) *There exists a switching index* $j \in I(t)$ *for all* $t \geq t_0$ *such that the closed-loop system with* $u(t) = K_j x(t)$ *is globally exponentially stable.*

To guarantee exponential stability of the closed-loop system, we need a further property of the finite cover of Ω.

Definition 3.1 *A given set of matrices* $\{A(\theta) : \theta \in \Omega\}$ *is called quadratically stable [1] if there exist symmetric and positive-definite matrices* H, Q *such that*

$$A^T(\theta) H A(\theta) - H \leq -Q, \quad \forall\, \theta \in \Omega \tag{3.8}$$

It is obvious that the finite cover $\{\Omega\}_{i=1}^L$ of Ω can always be made such that each Ω_i is "small" enough so that the corresponding family of "closed-loop" matrices $\{A(\theta)+B(\theta)K_i : \theta \in \Omega_i\}$ is quadratically stable for some K_i.

In view of the observation above, we replace the Condition (C3) with the following:

C3'. For each $i = 1, \cdots, L$, let θ_i and $r_i > 0$ denote the "center" and "radius" of Ω_i, i.e., $\theta_i \in \Omega_i$ and $||\theta - \theta_i|| \leq r_i$ for all $\theta \in \Omega_i$. Then, there exist a positive scalar q, control gain matrices K_i, and symmetric and positive-definite matrices H_i and Q_i, $i = 1, \cdots, L$, such that

$$(A(\theta) + B(\theta)K_i)^T H(A(\theta) + B(\theta)K_i) - H_i \leq -Q_i, \quad \forall\, ||\theta - \theta_i|| \leq r_i + q,\ i = 1, \cdots, L \tag{3.9}$$

Remark 3.1 *We also note that a finite cover which satisfies (C1)-(C2) and (C3') is guaranteed to exist.*

The following theorem contains the main result for the LTI case:

Theorem 3.1 *Given a LTI plant (2.1) satisfying Assumptions (A1)-(A5), let the* $\{\Omega_i\}_{i=1}^L$ *be a finite cover of* Ω *satisfying Conditions (C1)-(C2) and (C3'), Then, any localization algorithm given in (3.4)-(3.7) will guarantee the following properties when* ϵ *(i.e., the "size" of unmodelled dynamics) is sufficiently small:*

(i) *The closed-loop system is globally exponentially stable, i.e., there exists constants* $M_1 > 0$, $0 < \rho < 1$, *and a function* $M_2(\cdot) : \mathbf{R}_+ \to \mathbf{R}_+$ *with* $M_2(0) = 0$ *independent of the initial conditions such that*

$$||x(t)|| \leq M_1 \rho^{(t-t_0)} ||x(t_0)|| + M_2(\bar{\xi}) \tag{3.10}$$

holds for all $t \geq t_0$ *and* $x(t_0)$;

(ii) *The switching sequence* $\{i(t_0), i(t_0+1), \cdots\}$ *is finitely convergent, i.e.,* $i(t) =$ *const,* $\forall\, t \geq t^*$ *for some* t^*.

4 Localization Technique for LTV Plants

We note that the localization algorithm (3.4)-(3.7) does not generally guarantee condition (i) in Lemma 3.1 if the plant parameters vary in time. Consequently, this localization algorithm cannot be directly applied to LTV plants. In this section, we describe a modified localization technique which is applicable to both LTI and LTV plants. This modified technique guarantees global exponential stability of the closed-loop system, provided that the parameters do not drift faster than a certain maximum rate and that the number of parameter jumps is sufficiently infrequent.

Before we present a switching control algorithm for LTV systems, we note that discrete-time systems generally do not allow for arbitrarily fast parameter variations. This property is sharply different from the continuous-time case, where in certain cases, it may be possible to stabilize systems with arbitrarily rapid variations.

4.1 Switching Controller Design

The general structure of the switching control is similar to the time-invariant case except that the localization algorithm needs some modification. More specifically, the switching index set $I(t)$ is initialized with (3.4). At each $t > t_0$, a set $\hat{I}(t)$ is computed using (3.3) and (3.5) where r_i is replaced by $(r_i + q)$, $\forall i$ and $I(t)$ is updated by

$$I(t) = \begin{cases} I(t-1) \bigcap \hat{I}(t) & \text{if } I(t-1) \bigcap \hat{I}(t) \neq \{\ \} \\ \hat{I}(t) & \text{otherwise} \end{cases} \tag{4.1}$$

The control law is taken to be

$$u(t) = K_{i(t)} x(t) \tag{4.2}$$

where the switching index $i(t)$ is given by (3.7).

The following result shows that the modified localization scheme above also guarantees global exponential stability of the closed-loop system when the parameters drift slowly enough and/or the occurence of parameter jumps is not too rapid.

Theorem 4.1 *Consider the uncertain LTV system (2.1) satisfying Assumptions (A1)-(A6). Let $\{\Omega_i\}_{i=1}^{L}$ be a finite cover of Ω satisfying Conditions (C1)-(C2) and (C3'). Then, the localization scheme described above guarantees the following properties:*
(i) The closed-loop system is globally exponentially stable if

$$M_1^{[\frac{N\alpha}{q}]\ell}\rho^N < 1 \tag{4.3}$$

where M_1 and ρ are constants in (3.10), α, N, q are constants used in Assumption (A6) to describe the "rate" of parameter variations and the "frequency" of large parameters jumps, q is given in Condition (C3'), and ℓ denotes the maximum number of switchings for the case when the parameters are time-invariant.
(ii) The switching sequence $\{i(t_0), i(t_0+1), \cdots\}$ is finitely convergent if the parameters are constant.

4.2 Optimal Localization Algorithm

The localization scheme described above allows an arbitrary new switching index to be used when a switching occurs. The problem of optimal localization addresses the issue of optimal selection of the new switching index at each switching instance so that the set of admissible switching indices $I(t)$ can be pruned down as quickly as possible. A complete solution to this problem is provided in the full version of this paper.

5 Localization in the Presence of Unknown Disturbance Bound

In this section we further relax Assumption (A4) to allow the disturbance bound $\bar{\xi}$ to be unknown. That is, we replace (A4) with

(A4') The exogenous disturbance ξ is uniformly bounded,

$$\sup_{t\geq t_0} |\xi(t)| \leq \bar{\xi} \tag{5.1}$$

for some *unknown* constant $\bar{\xi}$.

As a tradeoff, we need to restrict the parameter variations to slow drifting only, i.e., we replace (A6) with

(A6') The uncertain parameters are allowed to have slow drift described by (2.11).

Following the results presented in previous sections, we introduce a generalized localization algorithm to tackle the new difficulty. The key feature of the algorithm is the use of an on-line estimate of $\bar{\xi}$. This estimate starts with a small (or zero) initial value, and is gradually increased when it is invalidated by the observations of the output. With the tradeoff between a larger number of switchings and a higher complexity, the new localization algorithm guarantees similar properties for the closed-loop system as for the case of known disturbance bound.

Let $\bar{\xi}(t)$ be the estimate for $\bar{\xi}$ at time t. Define

$$\begin{aligned}\hat{I}(t,\bar{\xi}(t)) = \{j : |\theta_j^T\phi(t-1) - y(t)| \leq \\ r_i\|\phi(t-1)\| + \bar{\xi}(t) + \bar{\eta}(t),\ j = 1,\cdots,L\}\end{aligned} \tag{5.2}$$

That is, $\hat{I}(t,\bar{\xi}(t))$ is the index set of parameter subsets which can not be invalidated by the estimate $\bar{\xi}(t)$ of the disturbance bound, at time t.

Denote the most recent switching instance by $s(t)$. We define $s(t)$ and $\bar{\xi}(t)$ as follows:

$$s(t_0) = t_0,\ \bar{\xi}(t_0) = 0 \tag{5.3}$$

$$s(t) = \begin{cases} t \text{ if } \begin{cases} \cap_{k=s(t-1)}^{t}\hat{I}(k,\bar{\xi}(k)) = \{\} \\ \text{and } t - s(t) \geq t_d \end{cases} \\ s(t-1) \text{ otherwise} \end{cases} \tag{5.4}$$

$$\bar{\xi}(t) = \begin{cases} \bar{\xi}(t-1) + \delta(t)\mu \text{ if} \\ \quad \begin{cases} \cap_{k=s(t)}^{t}\hat{I}(k,\bar{\xi}(k-1)) = \{\} \\ \text{and } t - s(t) < t_d \end{cases} \\ \bar{\xi}(t-1) \text{ otherwise} \end{cases} \tag{5.5}$$

where t_d is some positive integer representing a dwell time μ is any small positive constant representing a steady state residual (to be clarified later), and $\delta(t)$ is an integer function defined as follows:

$$\delta(t) = \min\left\{\delta : \cap_{k=s(t)}^{t}\hat{I}(k,\bar{\xi}(k-1)+\delta\mu) \neq \{\}, \delta \in \mathbf{N}\right\} \tag{5.6}$$

where $\mathbf{N}$ represents the set of non-negative integers. The switching index function $i(t)$ is defined the same way as before. The algorithm of localization is modified as follows:

$$I(t) = \cap_{k=s(t-1)}^{t}\hat{I}(k,\bar{\xi}(k)) \tag{5.7}$$

But the switching index $i(t)$ is still defined as in (3.7).

The key properties of the algorithm above are given as follows:

Theorem 5.1 *For any constant $\mu > 0$, there exist a parameter drift bound, $\alpha > 0$, a size of unmodelled dynamics, $\epsilon > 0$ (both sufficiently small), and dwell time t_d (sufficiently large), such that the localization algorithm described above, when applied to the plant (2.1) with Assumptions (A1)-(A3), (A4') and (A5), possesses the following properties:*
(1) $I(t) \neq \{\}$ for all $t \geq t_0$;
(2) $\sup_{t\geq t_0}\bar{\xi}(t) \leq \bar{\xi} + \mu$.
Consequently, the following properties hold:
(3) The closed-loop system is globally $(\bar{\xi}+\mu)$-exponentially stable, i.e., there exists constants $M_1 > 0$, $0 < \rho < 1$, and a function $M_2(\cdot) : \mathbf{R}_+ \to \mathbf{R}_+$ with $M_2(0) = 0$ such that

$$||x(t)|| \leq M_1\rho^{(t-t_0)}||x(t_0)|| + M_2(\bar{\xi}+\mu) \tag{5.8}$$

holds for all $t \geq t_0$ and $x(t_0)$;
(4) The switching sequence $\{i(t_0), i(t_0+1), \cdots\}$ is finitely convergent, i.e., $i(t)$ =const, $\forall\, t \geq t^*$ for some t^* if the uncertain parameters are constant.

We note that even though the value μ can be arbitrarily chosen, the estimate of the disturbance bound, $\bar{\xi}(t)$, can theoretically be larger than $\bar{\xi}$ by the margin μ.

6 Conclusions

In this paper we have presented a new systematic switching control based approach for adaptive stabilization of linear time-varying discrete-time systems. Our approach is based on a localization method which is conceptually different from supervisory adaptive control schemes and other existing switching adaptive schemes. Under some mild conditions exponential stability of the system is proved. Detailed technical information and simulation results are given in the full version of the paper [21] which is available on request.

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