Finite-Time H_∞ Filtering for Nonlinear Singular Systems With Nonhomogeneous Markov Jumps

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***Abstract*—This paper addresses the finite-time H_∞ filtering for a class of nonlinear singular nonhomogeneous Markov jump systems by T–S fuzzy approximation approach, where the transition probabilities (TPs) are time-varying and unknown. First, by considering a stochastic Lyapunov functional and rendering the time-varying TPs inside a polytope, a sufficient condition on singular stochastic H_∞ finite-time boundedness (SSH_∞FTB) for the filtering error systems is given. Then, by using the matrix inequality decoupling technique, a novel linear matrix inequality (LMI) condition on the existence of the finite-time H_∞ fuzzy filter is presented. The fuzzy filter is developed in terms of LMIs ensuring the filtering error system is SSH_∞FTB. Compared with the previous ones, the proposed design method in this paper has more freedom, leading to less conservative results. A tunnel diode circuit is provided to illustrate the effectiveness and advantage of the design approach proposed in this paper.**

***Index Terms*—Nonhomogeneous Markov chain, singular stochastic H_∞ finite-time boundedness (SSH_∞FTB), singular T–S fuzzy systems.**

I. Introduction

Markov jump systems (MJSs) are commonly used to characterize and model many types of practical systems, such as communication systems, networked control systems, economics systems, and so on. MJSs have drawn much attention from diverse fields due to its successful description for practical systems with abrupt changes in their structures [1]. So far, under the assumption that the transition probabilities (TPs) are homogeneous, the problem of stability analysis, the design of controller and filter for MJSs have been extensively investigated in [2]–[5]. For example, when the TPs are homogeneous, the necessary and sufficient criteria of stability and stabilization for MJSs were obtained in [2], and the adaptive sliding mode control problem of nonlinear MJSs was concerned in [3]. However, the assumption that the TPs are homogeneous is not realistic in many situations. For example, when the evolution between the operating modes of the dc motor device is determined by time-varying TPs, the authors studied finite-time stabilization for nonlinear discrete-time singular MJSs in [6]. In this case, the nonhomogeneous Markov chain or process is more suited for describing the practical systems. Another example of nonhomogeneous MJSs (NMJSs) arises in networked control systems [7], as packet losses and stochastic delays in networked control systems can be described by Markov process. But in practice, delays or packet losses are changing over time, which leads to the time-varying TPs, so it is meaningful to study NMJSs. For NMJSs, the problems of stability analysis and design are given in [8]–[13]. The filter design issue for NMJSs is studied in [13], where a slack matrix $X(i)$ is introduced to avoid the cross-coupling of matrix product terms. However, the structure of matrix $X(i)$ is constrained (i.e., setting $X(i) = \begin{bmatrix} R(i) & Y(i) \\ Z(i) & Y(i) \end{bmatrix}$), which brings a conservative result. It is noted that all the scenarios are established in the normal NMJSs. These results in those papers are not applicable when the considered system is singular.

Singular systems, also referred to as descriptor systems, differential-algebraic systems, generalized state-space systems or semi-state systems, have a strong ability to describe many practical systems, such as biological systems, network control systems, economic systems, power systems, and so on [14], [15]. The problem of mixed H_∞ and passive filtering for singular time-delay systems is studied in [16]. When abrupt changes occur to singular systems, we naturally model them as singular MJSs (SMJSs) [15]. Recently, many elegant results for SMJSs have been presented [17]–[25]. For example, the problem of stability and stabilization for SMJSs was concerned in [17]–[20]. By using the replacement of matrix variables, the sliding mode control problem for SMJSs was investigated in [21]. Wu *et al.* [22], Ma and Boukas [23], Li and Zhong [24], and Shen *et al.* [25] considered the problem of filter synthesis for a class of discrete-time SMJSs. To obtain the filter parameters, some matrix inequalities in [23] and special constraints on freedom matrix are used in [24] and [25]. Note that all these constraints will increase the conservatism of the designed results. How to overcome these constraints is one of the main concerns in this paper.

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On another research front, the T–S fuzzy model is recognized as an effective tool in approximating or describing a complex nonlinear system, which was first introduced in [26]. In the last several years, the fuzzy-model-based technique has been widely and successfully used in the modeling and control of nonlinear systems [27]–[31]. Recently, based on the T–S fuzzy model, the dissipativity analysis and filtering problem for nonlinear MJSs were discussed in [32]–[34]. For NMJSs, the filtering issue via fuzzy approach has been presented in [35]–[38]. Note that in order to design the filter parameters in terms of linear matrix inequalities (LMIs), Yin *et al.* [35], [36] and Sathishkumar *et al.* [38] imposed the restrictions on free matrices. Singular T–S fuzzy systems are more sophisticated since regularity and causality (or impulse-free for continuous-time systems) should be considered simultaneously, which is a distinguishing feature of normal T–S fuzzy systems [39]. Recently, based on the hypothesis that the TPs are homogeneous, the controller design problem of continuous-time singular T–S fuzzy MJSs has been addressed in [40]–[44]. To the author's knowledge, the issue of finite-time H_∞ filter design for discrete-time singular T–S fuzzy MJSs has not been addressed, especially when the TPs are nonhomogeneous. Thus from the point of the theory and practical applications, it deserves to be studied, which motives this paper.

In this paper, we study the finite-time H_∞ filtering problem for a type of nonlinear discrete-time singular NMJSs, where the nonlinearity is described by the T–S fuzzy approximation approach. First, based on a stochastic Lyapunov functional, a sufficient condition on singular stochastic finite-time boundedness (SSFTB) for T–S fuzzy singular NMJSs (FSNMJSs) is given. Then, a finite-time H_∞ fuzzy filter for discrete-time FSNMJSs is designed from a new perspective. With the fuzzy filter, the filter error system is singular stochastic H_∞ finite-time bounded. The main contributions are as follows.

1) A unified framework, which takes NMJSs, T–S fuzzy model and singular systems into account, is presented for designing the finite-time H_∞ filter for the first time.
2) Compared with [24], [25], [35], [36], and [38], a less conservative method is proposed by avoiding the restrictions on free matrices.
3) As a practical example, a tunnel diode circuit is applied to illustrate the validness and superiority of the proposed methods in this paper.

Notations: Throughout this paper, $\mathcal{R} = \{1, 2, \ldots, \nu\}$ with ν being the number of fuzzy IF-THEN rules. $X \geq 0$ ($X > 0$) means that the symmetric matrix X is semi-positive definite (positive definite). I and 0 represent, respectively, the identity matrix and zero matrix with appropriate dimensions. The superscript "T" denotes the transpose of a matrix. diag$\{\cdots\}$ represents a block-diagonal matrix. $\|x\|$ refers to Euclidean norm of the vector x. $\mathbf{E}[\,\cdot\,]$ stands for the mathematical expectation. In addition, in symmetric block matrices, $*$ represents as an ellipsis for the terms that are introduced by symmetry, and sym $\{X\}$ represents $X + X^{\mathrm{T}}$. $\bigstar$ represents matrices that are not relevant in the discussion.

II. Preliminaries

Consider a general nonlinear dynamic system as follows:

$$\begin{cases} Ex(k+1) = f(\tau_k, x(k), \omega(k)) \\ y(k) = g(\tau_k, x(k), \omega(k)) \\ z(k) = l(\tau_k, x(k), \omega(k)) \end{cases}$$

where $f(\cdot)$, $g(\cdot)$, and $l(\cdot)$ are the vector function of system nonlinearity. $x(k) \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^l$, and $z(k) \in \mathbb{R}^p$ are the system state, the measurement output, and the signal to be estimated, respectively. The matrix $E \in \mathbb{R}^{n\times n}$ is singular with rank$(E) = r_e \leq n$. In addition, the noise signal $\omega(k) \in \mathbb{R}^q$ satisfies

$$\mathbf{E}\left\{\sum_{k=0}^{N} \omega^{\mathrm{T}}(k)\omega(k)\right\} \leq d^2, d \geq 0. \tag{1}$$

In this paper, the switching law $\{\tau_k, k \geq 0\}$ is a discrete-time Markov stochastic process taking values in a finite state space $\mathcal{S} = \{1, 2, \ldots, S\}$, the evolution of $\{\tau_k, k \geq 0\}$ is governed by the following TPs:

$$\pi_{\mathrm{rm}}(k) = \Pr\{\tau_{k+1} = m | \tau_k = r\}$$

with $\pi_{\mathrm{rm}}(k) \geq 0$ and $\sum_{m=1}^{S} \pi_{\mathrm{rm}}(k) = 1$. $\pi_{\mathrm{rm}}(k)$ are the entries of the TP matrix $\Pi(k)$. $\Pi(k)$ is a time-varying matrix that resides in a polytope

$$\Pi(k) \in \mathrm{co}\left\{\Pi^s : s = 1, 2, \ldots, \mathcal{M}\right\} \tag{2}$$

where $\Pi^s : s = 1, 2, \ldots, \mathcal{M}$ are given constant TP matrices that are the vertices of the polytope and co stands for convex hull, namely

$$\Pi(k) = \sum_{s=1}^{\mathcal{M}} \alpha_s(k)\Pi^s \tag{3}$$

where $\alpha_s(k) \in [0, 1]$, $s = 1, 2, \ldots, \mathcal{M}$ and $\sum_{s=1}^{\mathcal{M}} \alpha_s(k) = 1$.

Remark 1: It should be pointed out that (3) expresses a time-dependent stochastic process, and this class of Markov chain is called a nonhomogeneous Markov process. When $\Pi(k) = \Pi$ for some constant matrix Π, it will be reduced to a so-called homogeneous one.

We use the following T–S fuzzy model to approximate the nonlinearities.

Plant Rule i:

IF $\varepsilon_1(k)$ is M_{i1},..., $\varepsilon_p(k)$ is M_{ip}, THEN

$$\begin{cases} \mathrm{Ex}(k+1) = A_i(\tau_k)x(k) + F_i(\tau_k)\omega(k) \\ y(k) = H_i(\tau_k)x(k) + D_i(\tau_k)\omega(k) \\ z(k) = C_i(\tau_k)x(k) \end{cases} \tag{4}$$

where $\varepsilon_\iota(k)$ denotes the premise variable and $M_{i\iota}$ ($\iota = 1, 2, \ldots, p$, $i \in \mathcal{R}$) is the fuzzy set. The fuzzy basis functions are given by

$$h_i(\varepsilon(k)) = \frac{\prod_{\iota=1}^{p} M_{i\iota}(\varepsilon_\iota(k))}{\sum_{i=1}^{\nu}\prod_{\iota=1}^{p} M_{i\iota}(\varepsilon_\iota(k))}$$

with $M_{i\iota}(\varepsilon_\iota(k))$ being the grade of membership of $\varepsilon_\iota(k)$ in $M_{i\iota}$. Therefore, for all k, we have $h_i(\varepsilon(k)) \geq 0$ and $\sum_{i=1}^{\nu} h_i(\varepsilon(k)) = 1$.

By using the "fuzzy blending" method [26], a more compact presentation of discrete-time FSNMJSs is inferred as follows:

$$\begin{cases} Ex(k+1) = \sum_{i=1}^{\nu} h_i(\varepsilon(k))[A_i(\tau_k)x(k) + F_i(\tau_k)\omega(k)] \\ y(k) = \sum_{i=1}^{\nu} h_i(\varepsilon(k))[H_i(\tau_k)x(k) + D_i(\tau_k)\omega(k)] \\ z(k) = \sum_{i=1}^{\nu} h_i(\varepsilon(k))C_i(\tau_k)x(k). \end{cases} \tag{5}$$

For simplicity, we denote $\tau_k = r$, $A_i(\tau_k) = A_i(r)$, $F_i(\tau_k) = F_i(r)$, $H_i(\tau_k) = H_i(r)$, $C_i(\tau_k) = C_i(r)$, $D_i(\tau_k) = D_i(r)$, $\forall\ \tau_k \in \mathcal{S}$. $A_i(r)$, $F_i(r)$, $H_i(r)$, $C_i(r)$, $D_i(r)$ are known compatible dimension constant matrices.

Remark 2: If $E = I$, system (5) reduces to T–S fuzzy NMJSs studied in [35]–[38]. Thus, the situation considered in this paper is general.

In this paper, we consider the following fuzzy filter for system (5), its ith rule is given by:

IF $\varepsilon_1(k)$ is M_{i1},..., $\varepsilon_p(k)$ is M_{ip}, THEN

$$\begin{cases} \hat{x}(k+1) = A_{fi}(r)\hat{x}(k) + B_{fi}(r)y(k) \\ \hat{z}(k) = C_{fi}(r)\hat{x}(k). \end{cases} \tag{6}$$

Through the parallel distributed compensation, the overall dynamical fuzzy filter model can be constructed as

$$\begin{cases} \hat{x}(k+1) = \sum_{i=1}^{\nu} h_i(\varepsilon(k))\big[A_{fi}(r)\hat{x}(k) + B_{fi}(r)y(k)\big] \\ \hat{z}(k) = \sum_{i=1}^{\nu} h_i(\varepsilon(k))C_{fi}(r)\hat{x}(k) \end{cases} \tag{7}$$

where $\hat{x}(k) \in \mathbb{R}^m$ and $\hat{z}(k) \in \mathbb{R}^u$ are, respectively, the state and the output signal of the filter. $A_{fi}(r)$, $B_{fi}(r)$, and $C_{fi}(r)$ are the filter parameters to be designed later.

Define

$$\eta(k) = \big[\, x^{\mathrm{T}}(k) \quad \hat{x}^{\mathrm{T}}(k) \,\big]^{\mathrm{T}}, \quad e(k) = z(k) - \hat{z}(k) \tag{8}$$

then, the filtering error system formed by system (5) and filter (7) can be written as

$$\begin{cases} \widehat{E}\eta(k+1) = \widehat{A}(h)\eta(k) + \widehat{F}(h)\omega(k) \\ e(k) = \widehat{C}(h)\eta(k) \end{cases} \tag{9}$$

where

$$\widehat{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}$$

$$\widehat{A}(h) = \sum_{i=1}^{\nu} h_i(\varepsilon(k)) \sum_{j=1}^{\nu} h_j(\varepsilon(k)) \begin{bmatrix} A_i(r) & 0 \\ B_{fj}(r)H_i(r) & A_{fj}(r) \end{bmatrix}$$

$$\widehat{F}(h) = \sum_{i=1}^{\nu} h_i(\varepsilon(k)) \sum_{j=1}^{\nu} h_j(\varepsilon(k)) \begin{bmatrix} F_i(r) \\ B_{fj}(r)D_i(r) \end{bmatrix}$$

$$\widehat{C}(h) = \sum_{i=1}^{\nu} h_i(\varepsilon(k)) \sum_{j=1}^{\nu} h_j(\varepsilon(k)) \big[\, C_i(r) \quad -C_{fj}(r) \,\big]. \tag{10}$$

Definition 1 [15]: System (9) with $\omega(k) = 0$ is said to be:

1) regular if $\det(s\widehat{E} - \widehat{A}(h)) \not\equiv 0$ for $\forall\ r \in \mathcal{S}$;

2) causal if degree $\{\det(s\widehat{E} - \widehat{A}(h))\}= \operatorname{rank}(\widehat{E})$ for $\forall\ r \in \mathcal{S}$.

Definition 2 (SSFTB) [19]: System (9) is said to be SSFTB with respect to $(c_1, c_2, G(r), N, d)$, where $0 < c_1 < c_2$, $G(r) > 0$ and $N \in \mathbb{Z}$, if system (9) is regular and causal, and satisfies

$$\mathbf{E}\big\{\eta^{\mathrm{T}}(0)\widehat{E}^{\mathrm{T}}G(r)\widehat{E}\eta(0)\big\} \le c_1^2$$
$$\Rightarrow \mathbf{E}\big\{\eta^{\mathrm{T}}(k)\widehat{E}^{\mathrm{T}}G(r)\widehat{E}\eta(k)\big\} < c_2^2,\ \forall k \in 1, 2, \ldots, N. \tag{11}$$

Definition 3 (Singular Stochastic H_∞ Finite-Time Boundedness (SSH_∞FTB)) [19]: System (9) is said to be SSH_∞FTB with respect to $(c_1, c_2, G(r), N, d, \overline{\gamma})$, where $\overline{\gamma}$ is a prescribed positive scalar. If system (9) is SSFTB with respect to $(c_1, c_2, G(r), N, d)$ and under zero initial condition, the filtering error $e(k)$ satisfies

$$\mathbf{E}\left\{\sum_{k=0}^{N} e^{\mathrm{T}}(k)e(k)\right\} < \overline{\gamma}^2 \sum_{k=0}^{N} \omega^{\mathrm{T}}(k)\omega(k). \tag{12}$$

The purpose of this paper is to design the finite-time H_∞ fuzzy filter in the form of (7) for system (5) such that system (9) is SSH_∞FTB with respect to $(c_1, c_2, G(r), N, d, \overline{\gamma})$.

In the following, we introduce a useful lemma for deriving our main results.

Lemma 1 [27]: Given matrices A, M and a symmetric matrix T of appropriate dimensions. The inequality

$$T + A^{\mathrm{T}}M^{\mathrm{T}} + MA < 0$$

is fulfilled if there exist matrix N and a scalar β such that the following holds:

$$\begin{bmatrix} T & \beta M + A^{\mathrm{T}}N^{\mathrm{T}} \\ * & -\beta N - \beta N^{\mathrm{T}} \end{bmatrix} < 0.$$

III. Main Results

In this section, the finite-time H_∞ filtering problem for FSNMJSs is investigated. To this end, we first provide a novel synthesis approach for SSFTB of FSNMJSs, which is the foundation of the subsequent theorems.

Theorem 1: For given scalars $\beta \ge 1$, $c_1 > 0$, $N > 0$, $d > 0$ and matrices $G(r) > 0$, system (9) is SSFTB with respect to $(c_1, c_2, G(r), N, d)$, if there exist constants $c_2 > 0$, $\lambda_2 > 0$, a set of positive definite symmetric matrices $\widehat{P}_s(r)$ and $\mathcal{P}(r)$, matrices $U(r)$, $V(r)$, $Q(r)$, $T(r)$, $\mathcal{R}$, $\forall r \in \mathcal{S}$, such that

$$\begin{bmatrix} \mathscr{W}^{11}(r) & \mathscr{W}^{12}(r) & U(r)\widehat{F}(h) \\ * & \mathscr{W}^{22}(r) & V(r)\widehat{F}(h) \\ * & * & -\mathcal{R} \end{bmatrix} < 0 \tag{13}$$

$$G(r) < \widehat{P}_s(r) < \lambda_2 G(r) \tag{14}$$

$$\lambda_2 c_1^2 + \delta d^2 < \beta^{-N} c_2^2 \tag{15}$$

where

$$\mathscr{W}^{11}(r) = \operatorname{sym}\big\{U(r)\big(\widehat{A}(h) - \widehat{E}\big)\big\} + \widehat{E}^{\mathrm{T}}\mathcal{P}(r)\widehat{E} - \beta\widehat{E}^{\mathrm{T}}\widehat{P}_s(r)\widehat{E}$$

$$\mathscr{W}^{12}(r) = -U(r) + \big(\widehat{A}(h) - \widehat{E}\big)^{\mathrm{T}}V^{\mathrm{T}}(r) + \widehat{E}^{\mathrm{T}}\mathcal{P}(r) + Q^{\mathrm{T}}(r)R^{\mathrm{T}}(r)$$

$$\mathscr{W}^{22}(r) = \operatorname{sym}\{-V(r) + R(r)T(r)\} + \mathcal{P}(r)$$

$$\widehat{P}_s(r) = \sum_{s=1}^{\mathcal{M}} \alpha_s(k)\widehat{P}^s(r)$$

$$\mathcal{P}(r) = \sum_{m=1}^{S}\sum_{s=1}^{\mathcal{M}}\sum_{l=1}^{\mathcal{M}} \alpha_s(k)\xi_l(k)\pi_{\mathrm{rm}}^{s}\widehat{P}^l(m).$$

$R(r) \in \mathbb{R}^{(m+n)\times(n-r_e)}$ is the arbitrary matrix satisfying $\widehat{E}^{\mathrm{T}}R(r) = 0$ and rank $(R(r)) = n - r_e$.

Proof: First, we prove that system (9) is regular and causal. From (13), it follows that:

$$\begin{bmatrix} \mathscr{W}^{11}(r) & \mathscr{W}^{12}(r) \\ * & \mathscr{W}^{22}(r) \end{bmatrix} < 0 \tag{16}$$

then (16) can be equivalently rewritten as

$$\mathcal{A}^{\mathrm{T}}(r)X(r)\mathcal{A}(r) + \mathrm{sym}\{\mathcal{A}^{\mathrm{T}}(r)\widetilde{S}(r)\widetilde{Q}(r)\} - \beta\widetilde{E}^{\mathrm{T}}\widetilde{P}(r)\widetilde{E} < 0 \tag{17}$$

where

$$\widetilde{E} = \begin{bmatrix} \widehat{E} & 0 \\ 0 & 0 \end{bmatrix},\ \mathcal{A}(r) = \begin{bmatrix} \widehat{E} & I \\ \widehat{A}(h) - \widehat{E} & -I \end{bmatrix}$$
$$\widetilde{P}(r) = \begin{bmatrix} \widehat{P}_s(r) & 0 \\ 0 & 0 \end{bmatrix},\ \widetilde{Q}(r) = \begin{bmatrix} Q(r) & T(r) \\ U^{\mathrm{T}}(r) & V^{\mathrm{T}}(r) \end{bmatrix}$$
$$\widetilde{S}(r) = \begin{bmatrix} R(r) & 0 \\ 0 & I \end{bmatrix},\ X(r) = \begin{bmatrix} \mathcal{P}(r) & 0 \\ 0 & 0 \end{bmatrix}.$$

From (17), it follows that:

$$\mathrm{sym}\{\mathcal{A}^{\mathrm{T}}(r)\widetilde{S}(r)\widetilde{Q}(r)\} - \beta\widetilde{E}^{\mathrm{T}}\widetilde{P}(r)\widetilde{E} < 0. \tag{18}$$

Since $\mathrm{rank}(\widetilde{E}) = m + r$, there exist two nonsingular matrices $\mathscr{P}, \mathscr{G} \in \mathbb{R}^{(2(m+n))\times(2(m+n))}$ such that

$$\mathscr{P}\widetilde{E}\mathscr{G} = \begin{bmatrix} I_{m+r} & 0 \\ 0 & 0 \end{bmatrix}$$
$$\mathscr{P}\mathcal{A}(r)\mathscr{G} = \begin{bmatrix} \mathcal{A}^1(r) & \mathcal{A}^2(r) \\ \mathcal{A}^3(r) & \mathcal{A}^4(r) \end{bmatrix}$$
$$\mathscr{P}^{-T}\widetilde{P}(r)\mathscr{P}^{-1} = \begin{bmatrix} \widetilde{P}^1(r) & \widetilde{P}^2(r) \\ * & \widetilde{P}^3(r) \end{bmatrix}$$
$$\mathscr{P}^{-T}\widetilde{S}(r) = \begin{bmatrix} \widetilde{S}^1(r) \\ \widetilde{S}^2(r) \end{bmatrix}$$
$$\widetilde{Q}(r)\mathscr{G} = \begin{bmatrix} \widetilde{Q}^1(r) & \widetilde{Q}^2(r) \end{bmatrix}. \tag{19}$$

From $\widetilde{E}^{\mathrm{T}}\widetilde{S}(r) = 0$, it follows that $\widetilde{S}^1(r) = 0$ and $\widetilde{S}^2(r) \in \mathbb{R}^{(m+2n-r)\times(m+2n-r)}$ has full rank, namely:

$$\mathscr{P}^{-T}\widetilde{S}(r) = \begin{bmatrix} 0 \\ \widetilde{S}^2(r) \end{bmatrix},\ \widetilde{S}^2(r) \in \mathbb{R}^{(m+2n-r)\times(m+2n-r)}$$
$$\mathrm{rank}\left(\widetilde{S}^2(r)\right) = m + 2n - r. \tag{20}$$

Pre- and post-multiplying (18) by $\mathscr{W}^{\mathrm{T}}$ and $\mathscr{W}$, along with (19) and (20), it is obtained that

$$\begin{bmatrix} \bigstar & \bigstar \\ \bigstar & \mathrm{sym}\left\{\left(\mathcal{A}^4(r)\right)^{\mathrm{T}}\widetilde{S}^2(r)\widetilde{Q}^2(r)\right\} \end{bmatrix} < 0$$

which further implies that $\mathrm{sym}\{(\mathcal{A}^4(r))^{\mathrm{T}}\widetilde{S}^2(r)\widetilde{Q}^2(r)\} < 0$, then $\mathcal{A}^4(r)$ is nonsingular. From Definition 1, it follows that the pair $(\widetilde{E}, \mathcal{A}(r))$ is regular and causal. Further, based on the fact that $\det(z\widehat{E} - \widehat{A}(h)) = \det(z\widetilde{E} - \mathcal{A}(r))$ for every $r \in \mathcal{S}$, it is obtained that system (9) is regular and causal.

Next, we prove that system (9) is SSFTB, i.e., (11) holds. To this end, we assume that the left hand side of (11) holds for the given $c_1 > 0$. We construct the Lyapunov functional as

$$\begin{aligned} V(\eta(k), \tau_k, \alpha_k) &= \eta^{\mathrm{T}}(k)\widehat{E}^{\mathrm{T}}\widehat{P}_s(r)\widehat{E}\eta(k) \\ &= \eta^{\mathrm{T}}(k)\widehat{E}^{\mathrm{T}}\left(\sum_{s=1}^{\mathcal{M}}\alpha_s(k)\widehat{P}^s(r)\right)\widehat{E}\eta(k) \end{aligned} \tag{21}$$

where $\widehat{P}_s(r) > 0$. Then it is obtained that

$$\begin{aligned} &\mathbf{E}\big[V(\eta(k+1), \tau_{k+1}, \alpha_{k+1})|\tau_k, \alpha_k\big] \\ &= \eta^{\mathrm{T}}(k+1)\widehat{E}^{\mathrm{T}}\sum_{m=1}^{S}\pi_{\mathrm{rm}}(\alpha_k)\widehat{P}_m(\alpha_{k+1})\widehat{E}\eta(k+1) \\ &= \eta^{\mathrm{T}}(k+1)\widehat{E}^{\mathrm{T}}\mathcal{P}(r)\widehat{E}\eta(k+1) \end{aligned}$$

where

$$\pi_{\mathrm{rm}}(\alpha_k) = \sum_{s=1}^{\mathcal{M}}\alpha_s(k)\pi_{\mathrm{rm}}^{s}$$
$$\widehat{P}_m(\alpha_{k+1}) = \sum_{s=1}^{\mathcal{M}}\alpha_s(k+1)\widehat{P}^s(m) = \sum_{l=1}^{\mathcal{M}}\xi_l(k)\widehat{P}^l(m)$$
$$\mathcal{P}(r) = \sum_{m=1}^{S}\sum_{s=1}^{\mathcal{M}}\sum_{l=1}^{\mathcal{M}}\alpha_s(k)\xi_l(k)\pi_{\mathrm{rm}}^{s}\widehat{P}^l(m).$$

Let $y(k) = \widehat{E}\eta(k+1) - \widehat{E}\eta(k)$, one has

$$\begin{aligned} &\mathbf{E}\big[V(\eta(k+1), \tau_{k+1}, \alpha_{k+1})|\tau_k, \alpha_k\big] \\ &\quad= \left(\widehat{E}\eta(k) + y(k)\right)^{\mathrm{T}}\mathcal{P}(r)\left(\widehat{E}\eta(k) + y(k)\right). \end{aligned} \tag{22}$$

Using the equations $y(k) = \widehat{E}\eta(k+1) - \widehat{E}\eta(k)$ and $\widehat{E}\eta(k+1) = \widehat{A}(h)\eta(k) + \widehat{F}(h)\omega(k)$, it is obtained that

$$\left(\widehat{A}(h) - \widehat{E}\right)\eta(k) - y(k) + \widehat{F}(h)\omega(k) = 0 \tag{23}$$

then it follows that:

$$0 = 2\vartheta(k)\mathcal{M}^{\mathrm{T}}(r)\left\{\left(\widehat{A}(h) - \widehat{E}\right)\eta(k) - y(k) + \widehat{F}(h)\omega(k)\right\} \tag{24}$$

where

$$\vartheta(k) = \begin{bmatrix} \eta^{\mathrm{T}}(k) & y^{\mathrm{T}}(k) & \omega^{\mathrm{T}}(k) \end{bmatrix}$$
$$\mathcal{M}(r) = \begin{bmatrix} U^{\mathrm{T}}(r) & V^{\mathrm{T}}(r) & 0 \end{bmatrix}.$$

For any matrix $Q(r), T(r) \in \mathbb{R}^{(n-r_e)\times(m+n)}$, $R(r) \in \mathbb{R}^{(m+n)\times(n-r_e)}$ is any full column rank matrix satisfying $\widehat{E}^{\mathrm{T}}R(r) = 0$, along with $y(k) = \widehat{E}\eta(k+1) - \widehat{E}\eta(k)$, one has

$$0 = 2y^{\mathrm{T}}(k)R(r)\big[Q(r)\eta(k) + T(r)y(k)\big]. \tag{25}$$

Adding (24) and (25) into (22), it is obtained that

$$\mathbf{E}\big[V(\eta(k+1), \tau_{k+1}, \alpha_{k+1})|\tau_k, \alpha_k\big] = \vartheta(k)\Theta(r)\vartheta^{\mathrm{T}}(k) \tag{26}$$

where

$$\Theta(r) = \begin{bmatrix} \Theta_1(r) & \mathscr{W}^{12}(r) & U(r)\widehat{F}(h) \\ * & \mathscr{W}^{22}(r) & V(r)\widehat{F}(h) \\ * & * & 0 \end{bmatrix}$$
$$\Theta_1(r) = \mathrm{sym}\left\{U(r)\left(\widehat{A}(h) - \widehat{E}\right)\right\} + \widehat{E}^{\mathrm{T}}\mathcal{P}(r)\widehat{E}.$$

From (13), it follows that:

$$\mathbf{E}\big[V(\eta(k+1),\tau_{k+1},\alpha_{k+1})|\tau_k,\alpha_k\big] \leq \beta V(\eta(k),\tau_k,\alpha_k)+\omega^{\mathrm{T}}(k)\mathcal{R}\omega(k)$$

which is equivalent to

$$\mathbf{E}\big[V(\eta(k+1),\tau_{k+1},\alpha_{k+1})\big] \leq \beta\mathbf{E}[V(\eta(k),\tau_k,\alpha_k)]+\mathbf{E}\big[\omega^{\mathrm{T}}(k)\mathcal{R}\omega(k)\big]. \tag{27}$$

Based on (2) and (27), it is obtained that

$$\begin{aligned}\mathbf{E}[V(\eta(k),\tau_k,\alpha_k)] &\leq \beta\mathbf{E}\big[V(\eta(k-1),\tau_{k-1},\alpha_{k-1})\big] + \mathbf{E}\big[\omega^{\mathrm{T}}(k-1)\mathcal{R}\omega(k-1)\big]\\ &\quad\cdots\cdots\\ &\leq \beta^k\mathbf{E}[V(\eta(0),\tau_0,\alpha_0)]+\delta\mathbf{E}\left[\sum_{n=0}^{k-1}\beta^{k-1-n}\omega^{\mathrm{T}}(n)\omega(n)\right]\\ &\leq \beta^k\mathbf{E}[V(\eta(0),\tau_0,\alpha_0)]+\delta\beta^k d^2\end{aligned} \tag{28}$$

where $\delta=\max_{r\in\mathcal{S}}\sigma_{\max}\mathcal{R}$.

Define $\overline{P}_s(r) = G^{-(1/2)}(r)\widehat{P}_s(r)G^{-(1/2)}(r)$, then based on $\mathbf{E}\{\eta^{\mathrm{T}}(0)\widehat{E}^{\mathrm{T}}G(r)\widehat{E}\eta(0)\}\leq c_1^2$ and (14), it follows that:

$$\begin{aligned}\mathbf{E}[V(\eta(0),\tau_0,\alpha_0)] &= \mathbf{E}\big[\eta^{\mathrm{T}}(0)\widehat{E}^{\mathrm{T}}\widehat{P}_s(r)\widehat{E}\eta(0)\big]\\ &= \mathbf{E}\Big[\eta^{\mathrm{T}}(0)\widehat{E}^{\mathrm{T}}G^{\frac12}(r)\overline{P}_s(r)G^{\frac12}(r)\widehat{E}\eta(0)\Big]\\ &\leq \max_{r\in\mathcal{S}}\lambda_{\max}\big(\overline{P}_s(r)\big)\mathbf{E}\big\{\eta^{\mathrm{T}}(0)\widehat{E}^{\mathrm{T}}G(r)\widehat{E}\eta(0)\big\}\\ &\leq \lambda_2 c_1^2.\end{aligned} \tag{29}$$

On the other hand, from (14), we have

$$\begin{aligned}\mathbf{E}[V(\eta(k),\tau_k,\alpha_k)] &= \mathbf{E}\big[\eta^{\mathrm{T}}(k)\widehat{E}^{\mathrm{T}}\widehat{P}_s(r)\widehat{E}\eta(k)\big]\\ &= \mathbf{E}\Big[\eta^{\mathrm{T}}(k)\widehat{E}^{\mathrm{T}}G^{\frac12}(r)\overline{P}_s(r)G^{\frac12}(r)\widehat{E}\eta(k)\Big]\\ &\geq \mathbf{E}\big\{\eta^{\mathrm{T}}(k)\widehat{E}^{\mathrm{T}}G(r)\widehat{E}\eta(k)\big\}.\end{aligned} \tag{30}$$

From (28) to (30), it follows that:

$$\mathbf{E}\big\{\eta^{\mathrm{T}}(k)\widehat{E}^{\mathrm{T}}G(r)\widehat{E}\eta(k)\big\} < \beta^k\Big(\lambda_2c_1^2+\delta d^2\Big) < \beta^N\Big(\lambda_2c_1^2+\delta d^2\Big).$$

Then, from (15), it follows that $\mathbf{E}\{\eta^{\mathrm{T}}(k)\widehat{E}^{\mathrm{T}}G(r)\widehat{E}\eta(k)\} < c_2^2,\ \ \forall k\in\{1,2,\ldots,N\}$. Therefore, system (9) is SSFTB with respect to $(c_1,c_2,G(r),N,d)$. The proof is completed. ■

Remark 3: A sufficient condition on SSFTB for a class of FSNMJSs is presented in Theorem 1 for the first time. It is noted that two zero equations (24) and (25) have been added to prove the SSFTB of FSNMJSs, which can increase the freedom of the finite-time fuzzy filter design in the sequel.

Based on the obtained results in Theorem 1, we now in the position to derive the sufficient condition on SSH_∞FTB of system (9), which is given in the following theorem.

Theorem 2: For given scalars $\beta\geq 1$, $c_1>0$, $N>0$, $d>0$ and matrices $G(r)>0$, system (9) is SSH_∞FTB with respect to $(c_1,c_2,G(r),N,d,\overline{\gamma})$, where $\overline{\gamma}=\sqrt{\gamma\beta^N}$, if there exist constants $c_2>0$, $\lambda_2>0$, $\gamma>0$, a set of positive definite symmetric matrices $\widehat{P}_s(r)$ and $\mathcal{P}(r)$, matrices $U(r)$, $V(r)$, $Q(r)$, $T(r)$, $\forall r\in\mathcal{S}$, such that (14) and

$$\begin{bmatrix}\mathscr{W}^{11}(r) & \mathscr{W}^{12}(r) & U(r)\widehat{F}(h) & \widehat{C}^{\mathrm{T}}(h)\\ * & \mathscr{W}^{22}(r) & V(r)\widehat{F}(h) & 0\\ * & * & -\gamma I & 0\\ * & * & * & -I\end{bmatrix}<0 \tag{31}$$

$$\lambda_2c_1^2+\gamma d^2<\beta^{-N}c_2^2 \tag{32}$$

where $\mathscr{W}^{11}(r)$, $\mathscr{W}^{12}(r)$, $\mathscr{W}^{22}(r)$, $\mathcal{P}(r)$, $R(r)$ are the same as Theorem 1.

Proof: In view of (31), it is obtained that

$$\begin{bmatrix}\mathscr{W}^{11}(r) & \mathscr{W}^{12}(r) & U(r)\widehat{F}(h)\\ * & \mathscr{W}^{22}(r) & V(r)\widehat{F}(h)\\ * & * & -\gamma I\end{bmatrix}<0$$

by setting $R(r)=\gamma I$, from (31) and (13), according to Theorem 1, we have system (9) is SSFTB with respect to $(c_1,c_2,G(r),N,d)$.

In addition, from (31), we have

$$\mathbf{E}\big[V(\eta(k+1),\tau_{k+1},\alpha_{k+1})\big] \leq \beta V(\eta(k),\tau_k,\alpha_k)+\gamma\omega^{\mathrm{T}}(k)\omega(k)-e^{\mathrm{T}}(k)e(k). \tag{33}$$

Based on this, we have the following iteration process:

$$\begin{aligned}\mathbf{E}[V(\eta(k),\tau_k,\alpha_k)] &\leq \beta\mathbf{E}\big[V(\eta(k-1),\tau_{k-1},\alpha_{k-1})\big] + \gamma\mathbf{E}\big[\omega^{\mathrm{T}}(k-1)\omega(k-1)\big]-\mathbf{E}\big[e^{\mathrm{T}}(k-1)e(k-1)\big]\\ &\quad\cdots\\ &\leq \beta^k\mathbf{E}[V(\eta(0),\tau_0,\alpha_0)]+\gamma\mathbf{E}\left[\sum_{n=0}^{k-1}\beta^{k-1-n}\omega^{\mathrm{T}}(n)\omega(n)\right]\\ &\quad -\mathbf{E}\left[\sum_{n=0}^{k-1}\beta^{k-1-n}e^{\mathrm{T}}(n)e(n)\right].\end{aligned} \tag{34}$$

Then under zero initial condition, since $V(\eta(k),\tau_k,\alpha_k)\geq 0$, it is obtained that

$$\gamma\mathbf{E}\left[\sum_{n=0}^{k-1}\beta^{k-1-n}\omega^{\mathrm{T}}(n)\omega(n)\right]\geq\mathbf{E}\left[\sum_{n=0}^{k-1}\beta^{k-1-n}e^{\mathrm{T}}(n)e(n)\right]$$

when $\beta\geq 1$, it implies that

$$\begin{aligned}\mathbf{E}\left[\sum_{n=0}^{k-1}e^{\mathrm{T}}(n)e(n)\right] &\leq \mathbf{E}\left[\sum_{n=0}^{k-1}\beta^{k-1-n}e^{\mathrm{T}}(n)e(n)\right]\\ &\leq \gamma\mathbf{E}\left[\sum_{n=0}^{k-1}\beta^{k-1-n}\omega^{\mathrm{T}}(n)\omega(n)\right]\\ &\leq \gamma\beta^{k-1}\mathbf{E}\left[\sum_{n=0}^{k-1}\omega^{\mathrm{T}}(n)\omega(n)\right]\end{aligned} \tag{35}$$

which further implies that

$$\mathbf{E}\left[\sum_{n=0}^{N}e^{\mathrm{T}}(n)e(n)\right]\leq\overline{\gamma}^2\mathbf{E}\left[\sum_{n=0}^{N}\omega^{\mathrm{T}}(n)\omega(n)\right]$$

with $\overline{\gamma}=\sqrt{\gamma\beta^N}$. Thus, according to Definition 3, system (9) is SSH_∞FTB with respect to $(c_1,c_2,G(r),N,d,\overline{\gamma})$. The proof is completed. ■

Remark 4: It is worth mentioning that the conditions in Theorem 2 are nonlinear matrix inequalities with respect to the freedom matrices $U(r)$, $V(r)$ and the desired finite-time H_∞ fuzzy filter parameter matrices since some product terms of these parameters to be determined appear in a nonlinear fashion. If we set $U(r) = \begin{bmatrix} U_1(r) & 0 \\ U_2(r) & Y(r) \end{bmatrix}$ and $V(r) = \begin{bmatrix} V_1(r) & 0 \\ V_2(r) & aY(r) \end{bmatrix}$ by using the method of [24], where a is a given scalar, the finite-time H_∞ fuzzy filter parameters can be designed using LMIs directly. Alternatively, the parameters can be designed when setting $U(r) = \begin{bmatrix} U_1(r) & Y(r) \\ U_2(r) & Y(r) \end{bmatrix}$ and $V(r) = \begin{bmatrix} V_1(r) & Y(r) \\ V_2(r) & Y(r) \end{bmatrix}$ by using the method of [25]. However, it should be pointed out that the result obtained is conservative due to the fact that the block structures of $U(r)$ and $V(r)$ are constrained. Then a new design method is given to design a finite-time H_∞ fuzzy filter for FSNMJSs, which is less conservative and more general than the existing ones.

In order to take the advantage of the existing LMI toolbox in MATLAB [28], an alternative theorem is presented by strict LMI conditions in the following statement.

Theorem 3: For given scalars $\beta \geq 1$, $c_1 > 0$, $N > 0$, $d > 0$, $\kappa(r)$ and matrices $G(r) > 0$, system (9) is SSH_∞FTB with respect to $(c_1, c_2, G(r), N, d, \overline{\gamma})$, where $\overline{\gamma} = \sqrt{\gamma\beta^N}$, if there exist constants $c_2 > 0$, $\lambda_2 > 0$, $\gamma > 0$, matrices $\widehat{P}_1^s(r)$, $\widehat{P}_2^s(r)$, $\widehat{P}_3^s(r)$, $U_1(r)$, $U_2(r)$, $U_3(r)$, $U_4(r)$, $V_1(r)$, $V_2(r)$, $V_3(r)$, $V_4(r)$, $Q_1(r)$, $Q_2(r)$, $T_1(r)$, $T_2(r)$, $L(r)$, $\overline{A}_{fi}(r)$, $\overline{B}_{fi}(r)$, $\overline{C}_{fi}(r)$, $\forall r \in \mathcal{S}$, such that (32) and

$$\Lambda_{ii}(r) < 0, \quad i = 1, 2, \ldots, \nu \tag{36}$$

$$\Lambda_{ij}(r) + \Lambda_{ji}(r) < 0, \quad j < i = 1, 2, \ldots, \nu \tag{37}$$

$$G(r) < \begin{bmatrix} \widehat{P}_1^s(r) & \widehat{P}_2^s(r) \\ * & \widehat{P}_3^s(r) \end{bmatrix} < \lambda_2 G(r) \tag{38}$$

where

$$\Lambda_{ij}(r) = \begin{bmatrix} \Lambda_{ij}^{11}(r) & \Lambda_{ij}^{12}(r) & \Lambda_{ij}^{13}(r) & \Lambda_{ij}^{14}(r) & \Lambda_{ij}^{15}(r) \\ * & \Lambda^{22}(r) & \Lambda_{ij}^{23}(r) & 0 & \Lambda_{ij}^{25}(r) \\ * & * & -\gamma I & 0 & \Lambda_{ij}^{35}(r) \\ * & * & * & -I & 0 \\ * & * & * & * & \Lambda_{ij}^{55}(r) \end{bmatrix}$$

$$\Lambda_{ij}^{11}(r) = \text{sym}\left\{\begin{bmatrix} \Lambda_{ij}^{11}(1r) & \overline{A}_{fj}(r) - U_2(r) \\ \Lambda_{ij}^{11}(3r) & \overline{A}_{fj}(r) - U_4(r) \end{bmatrix}\right\} + \begin{bmatrix} E^{\mathrm{T}}\widehat{\mathcal{P}}_1(r)E - \beta E^{\mathrm{T}}\widehat{P}_1^s(r)E & E^{\mathrm{T}}\widehat{\mathcal{P}}_2(r) - \beta E^{\mathrm{T}}\widehat{P}_2^s(r) \\ * & \widehat{\mathcal{P}}_3(r) - \beta\widehat{P}_3^s(r) \end{bmatrix}$$

$$\Lambda_{ij}^{12}(r) = \begin{bmatrix} \Lambda_{ij}^{12}(1r) & \Lambda_{ij}^{12}(2r) \\ \Lambda_{ij}^{12}(3r) & \overline{A}_{fj}^{\mathrm{T}}(r) - V_4^{\mathrm{T}}(r) - U_4(r) + \widehat{\mathcal{P}}_3(r) \end{bmatrix}$$

$$\Lambda_{ij}^{13}(r) = \begin{bmatrix} U_1(r)F_i(r) + \overline{B}_{fj}(r)D_i(r) \\ U_3(r)F_i(r) + \overline{B}_{fj}(r)D_i(r) \end{bmatrix}$$

$$\Lambda_{ij}^{14}(r) = \begin{bmatrix} C_i^{\mathrm{T}}(r) \\ -\overline{C}_{fj}^{\mathrm{T}}(r) \end{bmatrix}$$

$$\Lambda_{ij}^{15}(r) = \begin{bmatrix} \kappa(r)(U_2(r) - L(r)) + H_i^{\mathrm{T}}(r)\overline{B}_{fj}^{\mathrm{T}}(r) \\ \kappa(r)(U_4(r) - L(r)) + \overline{A}_{fj}^{\mathrm{T}}(r) \end{bmatrix}$$

$$\Lambda^{22}(r) = \begin{bmatrix} \Lambda^{22}(1r) & \Lambda^{22}(2r) \\ * & -V_4(r) - V_4^{\mathrm{T}}(r) + \widehat{\mathcal{P}}_3(r) \end{bmatrix}$$

$$\Lambda_{ij}^{23}(r) = \begin{bmatrix} V_1(r)F_i(r) + \overline{B}_{fj}(r)D_i(r) \\ V_3(r)F_i(r) + \overline{B}_{fj}(r)D_i(r) \end{bmatrix}$$

$$\Lambda_{ij}^{25}(r) = \kappa(r)\begin{bmatrix} V_2(r) - L(r) \\ V_4(r) - L(r) \end{bmatrix}$$

$$\Lambda_{ij}^{35}(r) = D_i^{\mathrm{T}}(r)\overline{B}_{fj}^{\mathrm{T}}(r)$$

$$\Lambda_{ij}^{55}(r) = -\kappa(r)L(r) - \kappa(r)L^{\mathrm{T}}(r)$$

$$\Lambda_{ij}^{11}(1r) = U_1(r)(A_i(r) - E) + \overline{B}_{fj}(r)H_i(r)$$

$$\Lambda_{ij}^{11}(3r) = U_3(r)(A_i(r) - E) + \overline{B}_{fj}(r)H_i(r)$$

$$\Lambda_{ij}^{12}(1r) = (A_i(r) - E)^{\mathrm{T}}V_1^{\mathrm{T}}(r) + H_i^{\mathrm{T}}(r)\overline{B}_{fj}^{\mathrm{T}}(r) - U_1(r) + E^{\mathrm{T}}\widehat{\mathcal{P}}_1(r) + Q_1^{\mathrm{T}}(r)S^{\mathrm{T}}(r)$$

$$\Lambda_{ij}^{12}(2r) = (A_i(r) - E)^{\mathrm{T}}V_3^{\mathrm{T}}(r) + H_i^{\mathrm{T}}(r)\overline{B}_{fj}^{\mathrm{T}}(r) - U_2(r) + E^{\mathrm{T}}\widehat{\mathcal{P}}_2(r)$$

$$\Lambda_{ij}^{12}(3r) = \overline{A}_{fj}^{\mathrm{T}}(r) - V_2^{\mathrm{T}}(r) - U_3(r) + \widehat{\mathcal{P}}_2^{\mathrm{T}}(r) + Q_2^{\mathrm{T}}(r)S^{\mathrm{T}}(r)$$

$$\Lambda^{22}(1r) = \text{sym}\{-V_1(r) + S(r)T_1(r)\} + \widehat{\mathcal{P}}_1(r)$$

$$\Lambda^{22}(2r) = -V_2(r) - V_3^{\mathrm{T}}(r) + \widehat{\mathcal{P}}_2(r) + S(r)T_2(r)$$

$$\widehat{\mathcal{P}}_\varsigma(r) = \sum_{m=1}^{S}\pi_{\mathrm{rm}}^s\widehat{P}_\varsigma^l(m), \ \varsigma = 1, 2, 3.$$

$S(r) \in \mathbb{R}^{n\times(n-r_e)}$ is the arbitrary matrix satisfying $E^{\mathrm{T}}S(r) = 0$ and rank $(S(r)) = n - r_e$. Then the parameters of the desired mode-dependent finite-time H_∞ fuzzy filter can be obtained by

$$A_{fi}(r) = L^{-1}(r)\overline{A}_{fi}(r), \quad B_{fi}(r) = L^{-1}(r)\overline{B}_{fi}(r)$$
$$C_{fi}(r) = \overline{C}_{fi}(r). \tag{39}$$

Proof: First, let

$$U(r) = \begin{bmatrix} U_1(r) & U_2(r) \\ U_3(r) & U_4(r) \end{bmatrix}, \ V(r) = \begin{bmatrix} V_1(r) & V_2(r) \\ V_3(r) & V_4(r) \end{bmatrix}$$
$$\widehat{P}_s(r) = \begin{bmatrix} \widehat{P}_{1s}(r) & \widehat{P}_{2s}(r) \\ * & \widehat{P}_{3s}(r) \end{bmatrix}, \ \mathcal{P}(r) = \begin{bmatrix} \mathcal{P}_1(r) & \mathcal{P}_2(r) \\ * & \mathcal{P}_3(r) \end{bmatrix}$$
$$Q(r) = \begin{bmatrix} Q_1(r) & Q_2(r) \end{bmatrix}, \ T(r) = \begin{bmatrix} T_1(r) & T_2(r) \end{bmatrix}$$
$$R(r) = \begin{bmatrix} S(r) \\ 0 \end{bmatrix}. \tag{40}$$

Then we substitute (10) and (40) into (31), it follows that:

$$\sum_{i=1}^{\nu} h_i(\varepsilon(k)) \sum_{j=1}^{\nu} h_j(\varepsilon(k))\Sigma_{ij}(r) < 0 \tag{41}$$

where

$$\Sigma_{ij}(r) = \begin{bmatrix} \Sigma_{ij}^{11}(r) & \Sigma_{ij}^{12}(r) & \Sigma_{ij}^{13}(r) & \Sigma_{ij}^{14}(r) \\ * & \Sigma^{22}(r) & \Sigma_{ij}^{23}(r) & 0 \\ * & * & -\gamma I & 0 \\ * & * & * & -I \end{bmatrix}$$

$$\Sigma_{ij}^{11}(r) = \text{sym}\left\{\begin{bmatrix} \Sigma_{ij}^{11}(1r) & U_2(r)(A_{fj}(r) - I) \\ \Sigma_{ij}^{11}(3r) & U_4(r)(A_{fj}(r) - I) \end{bmatrix}\right\} + \otimes(r)$$

$$\Sigma_{ij}^{12}(r) = \begin{bmatrix} \Sigma_{ij}^{12}(1r) & \Sigma_{ij}^{12}(2r) \\ \Sigma_{ij}^{12}(3r) & \Sigma_{ij}^{12}(4r) \end{bmatrix}$$

$$\Sigma_{ij}^{13}(r) = \begin{bmatrix} U_1(r)F_i(r) + U_2(r)B_{fj}(r)D_i(r) \\ U_3(r)F_i(r) + U_4(r)B_{fj}(r)D_i(r) \end{bmatrix}$$
$$\Sigma_{ij}^{14}(r) = \begin{bmatrix} C_i^{\mathrm{T}}(r) \\ -C_{fj}^{\mathrm{T}}(r) \end{bmatrix}$$
$$\Sigma^{22}(r) = \begin{bmatrix} \Sigma^{22}(1r) & \Sigma^{22}(2r) \\ * & -V_4(r) - V_4^{\mathrm{T}}(r) + \mathcal{P}_3(r) \end{bmatrix}$$
$$\Sigma_{ij}^{23}(r) = \begin{bmatrix} V_1(r)F_i(r) + V_2(r)B_{fj}(r)D_i(r) \\ V_3(r)F_i(r) + V_4(r)B_{fj}(r)D_i(r) \end{bmatrix}$$
$$\Sigma_{ij}^{11}(1r) = U_1(r)(A_i(r) - E) + U_2(r)B_{fj}(r)H_i(r)$$
$$\Sigma_{ij}^{11}(3r) = U_3(r)(A_i(r) - E) + U_4(r)B_{fj}(r)H_i(r)$$
$$\otimes(r) = \begin{bmatrix} \otimes_1(r) & E^{\mathrm{T}}\mathcal{P}_2(r) - \beta E^{\mathrm{T}}\widehat{P}_{2s}(r) \\ * & \mathcal{P}_3(r) - \beta\widehat{P}_{3s}(r) \end{bmatrix}$$
$$\otimes_1(r) = E^{\mathrm{T}}\mathcal{P}_1(r)E - \beta E^{\mathrm{T}}\widehat{P}_{1s}(r)E$$
$$\Sigma_{ij}^{12}(1r) = (A_i(r) - E)^{\mathrm{T}}V_1^{\mathrm{T}}(r) + H_i^{\mathrm{T}}(r)B_{fj}^{\mathrm{T}}(r)V_2^{\mathrm{T}}(r) - U_1(r) + E^{\mathrm{T}}\mathcal{P}_1(r) + Q_1^{\mathrm{T}}(r)S^{\mathrm{T}}(r)$$
$$\Sigma_{ij}^{12}(2r) = (A_i(r) - E)^{\mathrm{T}}V_3^{\mathrm{T}}(r) + H_i^{\mathrm{T}}(r)B_{fj}^{\mathrm{T}}(r)V_4^{\mathrm{T}}(r) - U_2(r) + E^{\mathrm{T}}\mathcal{P}_2(r)$$
$$\Sigma_{ij}^{12}(3r) = \big(A_{fj}(r) - I\big)^{\mathrm{T}}V_2^{\mathrm{T}}(r) - U_3(r) + \mathcal{P}_2^{\mathrm{T}}(r) + Q_2^{\mathrm{T}}(r)S^{\mathrm{T}}(r)$$
$$\Sigma_{ij}^{12}(4r) = \big(A_{fj}(r) - I\big)^{\mathrm{T}}V_4^{\mathrm{T}}(r) - U_4(r) + \mathcal{P}_3(r)$$
$$\Sigma^{22}(1r) = \mathrm{sym}\{-V_1(r) + S(r)T_1(r)\} + \mathcal{P}_1(r)$$
$$\Sigma^{22}(2r) = -V_2(r) - V_3^{\mathrm{T}}(r) + \mathcal{P}_2(r) + S(r)T_2(r)$$

then, decoupling some product terms in (41), (41) is equivalent to

$$\sum_{i=1}^{\nu} h_i(\varepsilon(k)) \sum_{j=1}^{\nu} h_j(\varepsilon(k))\Psi_{ij}(r) < 0 \tag{42}$$

where

$$\Psi_{ij}(r) = \begin{bmatrix} \Psi_{ij}^{11}(r) & \Psi_{ij}^{12}(r) & \Psi_{ij}^{13}(r) & \Sigma_{ij}^{14}(r) \\ * & \Sigma^{22}(r) & \Psi_{ij}^{23}(r) & 0 \\ * & * & -\gamma I & 0 \\ * & * & * & -I \end{bmatrix} + \mathrm{sym}\{\mathbb{H}^{\mathrm{T}}(r)\mathcal{U}^{\mathrm{T}}(r)\}$$
$$\Psi_{ij}^{11}(r) = \mathrm{sym}\left\{\begin{bmatrix} U_1(r)(A_i(r) - E) & -U_2(r) \\ U_3(r)(A_i(r) - E) & -U_4(r) \end{bmatrix}\right\} + \otimes(r)$$
$$\Psi_{ij}^{12}(r) = \begin{bmatrix} \Psi_{ij}^{12}(1r) & \Psi_{ij}^{12}(2r) \\ \Psi_{ij}^{12}(3r) & -V_4^{\mathrm{T}}(r) - U_4(r) + \mathcal{P}_3(r) \end{bmatrix}$$
$$\Psi_{ij}^{13}(r) = \begin{bmatrix} U_1(r)F_i(r) \\ U_3(r)F_i(r) \end{bmatrix}$$
$$\Psi_{ij}^{23}(r) = \begin{bmatrix} V_1(r)F_i(r) \\ V_3(r)F_i(r) \end{bmatrix}$$
$$\mathbb{H}^{\mathrm{T}}(r) = \begin{bmatrix} \begin{bmatrix} H_i^{\mathrm{T}}(r)B_{fj}^{\mathrm{T}}(r) \\ A_{fj}^{\mathrm{T}}(r) \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ D_i^{\mathrm{T}}(r)B_{fj}^{\mathrm{T}}(r) \\ 0 \end{bmatrix}$$
$$\mathcal{U}^{\mathrm{T}}(r) = \begin{bmatrix} \mathcal{U}_1(r) & \begin{bmatrix} V_2^{\mathrm{T}}(r) & V_4^{\mathrm{T}}(r) \end{bmatrix} & 0 & 0 \end{bmatrix}$$
$$\Psi_{ij}^{12}(1r) = (A_i(r) - E)^{\mathrm{T}}V_1^{\mathrm{T}}(r) - U_1(r) + E^{\mathrm{T}}\mathcal{P}_1(r) + Q_1^{\mathrm{T}}(r)S^{\mathrm{T}}(r)$$
$$\Psi_{ij}^{12}(2r) = (A_i(r) - E)^{\mathrm{T}}V_3^{\mathrm{T}}(r) - U_2(r) + E^{\mathrm{T}}\mathcal{P}_2(r)$$
$$\Psi_{ij}^{12}(3r) = -V_2^{\mathrm{T}}(r) - U_3(r) + \mathcal{P}_2^{\mathrm{T}}(r) + Q_2^{\mathrm{T}}(r)S^{\mathrm{T}}(r)$$
$$\mathcal{U}_1(r) = \begin{bmatrix} U_2^{\mathrm{T}}(r) & U_4^{\mathrm{T}}(r) \end{bmatrix}.$$

Further, (42) is equivalent to

$$\sum_{i=1}^{\nu} h_i(\varepsilon(k)) \sum_{j=1}^{\nu} h_j(\varepsilon(k))\Pi_{ij}(r) < 0 \tag{43}$$

where

$$\Pi_{ij}(r) = \begin{bmatrix} \Pi_{ij}^{11}(r) & \Pi_{ij}^{12}(r) & \Pi_{ij}^{13}(r) & \Sigma_{ij}^{14}(r) \\ * & \Sigma^{22}(r) & \Pi_{ij}^{23}(r) & 0 \\ * & * & -\gamma I & 0 \\ * & * & * & -I \end{bmatrix} + \mathrm{sym}\{\mathbb{H}^{\mathrm{T}}(r)\mathcal{L}^{\mathrm{T}}(r)\}$$
$$\Pi_{ij}^{11}(r) = \mathrm{sym}\left\{\begin{bmatrix} L(r)B_{fj}(r)H_i(r) & L(r)A_{fj}(r) \\ L(r)B_{fj}(r)H_i(r) & L(r)A_{fj}(r) \end{bmatrix}\right\} + \Psi_{ij}^{11}(r)$$
$$\Pi_{ij}^{12}(r) = \begin{bmatrix} H_i^{\mathrm{T}}(r)B_{fj}^{\mathrm{T}}(r)L^{\mathrm{T}}(r) & H_i^{\mathrm{T}}(r)B_{fj}^{\mathrm{T}}(r)L^{\mathrm{T}}(r) \\ A_{fj}^{\mathrm{T}}(r)L^{\mathrm{T}}(r) & A_{fj}^{\mathrm{T}}(r)L^{\mathrm{T}}(r) \end{bmatrix} + \Psi_{ij}^{12}(r)$$
$$\Pi_{ij}^{13}(r) = \begin{bmatrix} U_1(r)F_i(r) + L(r)B_{fj}(r)D_i(r) \\ U_3(r)F_i(r) + L(r)B_{fj}(r)D_i(r) \end{bmatrix}$$
$$\Pi_{ij}^{23}(r) = \begin{bmatrix} V_1(r)F_i(r) + L(r)B_{fj}(r)D_i(r) \\ V_3(r)F_i(r) + L(r)B_{fj}(r)D_i(r) \end{bmatrix}$$
$$\mathcal{L}^{\mathrm{T}}(r) = \begin{bmatrix} \mathcal{L}_1(r) & \mathcal{L}_2(r) & 0 & 0 \end{bmatrix}$$
$$\mathcal{L}_1(r) = \begin{bmatrix} U_2^{\mathrm{T}}(r) - L^{\mathrm{T}}(r) & U_4^{\mathrm{T}}(r) - L^{\mathrm{T}}(r) \end{bmatrix}$$
$$\mathcal{L}_2(r) = \begin{bmatrix} V_2^{\mathrm{T}}(r) - L^{\mathrm{T}}(r) & V_4^{\mathrm{T}}(r) - L^{\mathrm{T}}(r) \end{bmatrix}.$$

On the other hand, it follows from (39) that:

$$\overline{A}_{fi}(r) = L(r)A_{fi}(r), \quad \overline{B}_{fi}(r) = L(r)B_{fi}(r)$$
$$\overline{C}_{fi}(r) = C_{fi}(r). \tag{44}$$

Substituting (44) into (36), (37), and considering the nature of the convex combination, it is obtained that

$$\sum_{i=1}^{\nu} h_i(\varepsilon(k)) \sum_{j=1}^{\nu} h_j(\varepsilon(k))\Upsilon_{ij}(r) < 0 \tag{45}$$

where

$$\Upsilon_{ij}(r) = \begin{bmatrix} \Pi_{ij}^{11}(r) & \Pi_{ij}^{12}(r) & \Pi_{ij}^{13}(r) & \Sigma_{ij}^{14}(r) & \Pi_{ij}^{15}(r) \\ * & \Sigma^{22}(r) & \Pi_{ij}^{23}(r) & 0 & \Lambda_{ij}^{25}(r) \\ * & * & -\gamma I & 0 & \Pi_{ij}^{35}(r) \\ * & * & * & -I & 0 \\ * & * & * & * & \Lambda_{ij}^{55}(r) \end{bmatrix}$$
$$\Pi_{ij}^{15}(r) = \begin{bmatrix} \kappa(r)(U_2(r) - L(r)) + H_i^{\mathrm{T}}(r)B_{fj}^{\mathrm{T}}(r)L^{\mathrm{T}}(r) \\ \kappa(r)(U_4(r) - L(r)) + A_{fj}^{\mathrm{T}}(r)L^{\mathrm{T}}(r) \end{bmatrix}$$
$$\Pi_{ij}^{35}(r) = D_i^{\mathrm{T}}(r)B_{fj}^{\mathrm{T}}(r)L^{\mathrm{T}}(r).$$

Applying Lemma 1 to (45), then it is obtained that (43) holds, which further implies (31) holds. Therefore, based on Theorem 2, if (32) and (36)–(38) hold, system (9) is SSH_∞FTB with respect to $(c_1, c_2, G(r), N, d, \overline{\gamma})$. ■

Remark 5: In Theorem 3, a novel sufficient condition on SSH_∞FTB of system (9) is obtained in terms of LMIs. When $E = I$, Theorem 3 degenerates to finite-time H_∞ fuzzy filter design results for T–S fuzzy NMJSs. A similar problem has been studied in [38], where the freedom matrix $U(r) = \begin{bmatrix} S(r) & Y(r) \\ Z(r) & Y(r) \end{bmatrix}$ is used. However, the structural constraints on the freedom matrix variables have been relaxed in this paper. Further, if we set $\beta = 1$ without conditions (32) and (38), the problem studied in this paper reduces to the H_∞ fuzzy filter design for T–S fuzzy NMJSs, which has been studied in [35] and [36]. It should be noted that the authors set $X(r) = \begin{bmatrix} R(r) & Y(r) \\ Z(r) & Y(r) \end{bmatrix}$ in [35] and [36]. All these hard constraints on slack variables have been dealt with in this paper. Thus, the method proposed in this paper is less conservative and more general.

Remark 6: By using the T–S fuzzy approximation approach, the SSH_∞FTB for a type of nonlinear SNMJSs has been investigated. In fact, in practical systems, the ideal assumption of complete availability of the stochastic modes can be limited by several factors such as cost, physical constraints or difficulty of measuring. Thus, it is appropriate to design mode-independent filters for such practical systems. When we set $L(r) = L$, $A_{fi}(r) = A_{fi}$, $B_{fi}(r) = B_{fi}$, $C_{fi}(r) = C_{fi}$ in (36) and (37), the desired fuzzy filter will become the mode-independent ones. Thus, from the practical point of view, the results in this paper are more powerful and desirable.

Remark 7: Note that the proposed main results are based on the LMI approach. One of the main problems while using the LMI approach is the computational issue especially if the size of LMI becomes large. Fortunately, all the computations of main results are off-line and so with the help of the MATLAB LMI toolbox, solving the LMIs (32), (36)–(38) will not be a big deal. When the LMIs (32), (36)–(38) have a solution, the filter gains are obtained directly and the filter to the estimation system is implemented.

Remark 8: From Theorem 3, a minimum H_∞ performance $\overline{\gamma}$ can be obtained by solving the following optimization problem:

$$\text{Minimize } \overline{\gamma}$$
$$\text{Subject to LMIs (32) and (36)–(38) } \forall\, r \in \mathcal{S}.$$

IV. Examples

In this section, a numerical example is given to show the advantage and applicability of the proposed theoretical methods.

We consider a tunnel diode circuit modified from [28], which is shown in Fig. 1. The tunnel diode is subject to abrupt failures, and the equipment is altered to take these failures into account according to the stochastic switching. In the circuit, the switch of "$\mathcal{S}$" occupies two positions in a random way. The switching mode is known to be cumbersome in some circumstances but is assumed in [29] to be precisely known. In this paper, we assume that the switching mode is not precisely known, which follows a nonhomogeneous Markov chain. C is a capacitor, $R1$ and $R2$ are resistors, $D1$ and $D2$ are tunnel diodes. The tunnel diodes and resistors are characterized by

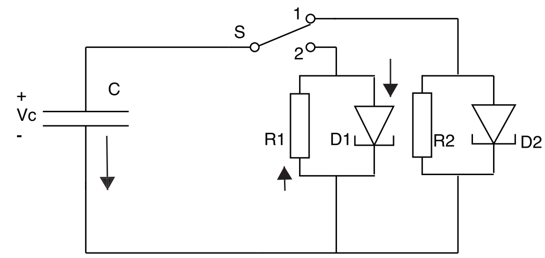

Fig. 1. Tunnel diode circuit.

$$i_{D\tau_t}(t) = \begin{cases} M(1)v_D(t) + N(1)v_D^3(t) & \text{if } \tau_t = 1 \\ M(2)v_D(t) + N(2)v_D^3(t) & \text{if } \tau_t = 2 \end{cases}$$
$$R(\tau_t) = \begin{cases} R1 & \text{if } \tau_t = 1 \\ R2 & \text{if } \tau_t = 2. \end{cases}$$

Based on Kirchhoff's laws, then the nonlinear differential-algebraic equations for the tunnel diode circuit are given as

$$\begin{aligned} C\dot{v}_C(t) &= i_{D\tau_t}(t) + i_R(t) \\ 0 &= -v_C(t) - R(\tau_t)i_R(t) + \omega(t) \\ y(t) &= Li_{D\tau_t}(t) + \omega(t) \\ z(t) &= Hv_D(t) \end{aligned} \tag{46}$$

where L and H are system parameters. Denote $x^{\mathrm{T}}(t) = \begin{bmatrix} v_C^{\mathrm{T}}(t) & i_R^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$, we can transform system (46) into

$$\begin{aligned} \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix}\dot{x}(t) &= \begin{bmatrix} M(\tau_t) & 1 \\ 0 & -R(\tau_t) \end{bmatrix}x(t) \\ &\quad + \begin{bmatrix} N(\tau_t) & 0 \\ 0 & 0 \end{bmatrix}x^3(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\omega(t) \\ y(t) &= L\begin{bmatrix} M(\tau_t) & 0 \end{bmatrix}x(t) \\ &\quad + L\begin{bmatrix} N(\tau_t) & 0 \end{bmatrix}x^3(t) + \omega(t) \\ z(t) &= \begin{bmatrix} H & 0 \end{bmatrix}x(t). \end{aligned}$$

By setting a certain sampling time, such as $T_s = T/10$, we can discretize the obtained continuous-time singular nonlinear NMJSs. The system parameters are as follows.

1) Mode 1

$$\begin{aligned} &A_1(1) = \begin{bmatrix} 1 & 0.9 \\ 0.8 & -0.3 \end{bmatrix},\ A_2(1) = \begin{bmatrix} 1 & 1.3 \\ 0.7 & 0.5 \end{bmatrix} \\ &F_1(1) = \begin{bmatrix} -0.3 \\ 0.25 \end{bmatrix},\ F_2(1) = \begin{bmatrix} -0.1 \\ 0.32 \end{bmatrix} \\ &H_1(1) = \begin{bmatrix} 0.5 & -0.4 \end{bmatrix},\ H_2(1) = \begin{bmatrix} 0.25 & -0.2 \end{bmatrix} \\ &C_1(1) = \begin{bmatrix} 0.3 & -0.2 \end{bmatrix},\ C_2(1) = \begin{bmatrix} 1 & 0 \end{bmatrix} \\ &D_1(1) = 0.3,\quad D_2(1) = 0. \end{aligned}$$

2) Mode 2

$$A_1(2) = \begin{bmatrix} 1 & 1 \\ 0.2 & -1.5 \end{bmatrix},\quad A_2(2) = \begin{bmatrix} 1 & 2 \\ 0.25 & -1.6 \end{bmatrix}$$
$$F_1(2) = \begin{bmatrix} -0.5 \\ 0.3 \end{bmatrix},\quad F_2(2) = \begin{bmatrix} -0.5 \\ 0.22 \end{bmatrix}$$
$$H_1(2) = \begin{bmatrix} 0.3 & -0.1 \end{bmatrix},\quad H_2(2) = \begin{bmatrix} 0.15 & -0.3 \end{bmatrix}$$
$$C_1(2) = \begin{bmatrix} 0.2 & -0.2 \end{bmatrix},\quad C_2(2) = \begin{bmatrix} 0 & 1.5 \end{bmatrix}$$
$$D_1(2) = -0.2,\quad D_2(2) = 0.1.$$

The TP matrix is assumed to be time-varying in a polytope defined by its vertices

$$\Pi^1 = \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.5 \end{bmatrix},\quad \Pi^2 = \begin{bmatrix} 0.3 & 0.7 \\ 0.35 & 0.65 \end{bmatrix}.$$

The singular matrix is given by $E = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, setting $S(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $S(2) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, $G(1) = G(2) = I_4$, $c_1 = 0.1$, $d = 2$, $N = 7$, $\beta = 1.16$, $\kappa(1) = \kappa(2) = 0.1$. According to Theorem 3, solving the LMIs (32), (36)–(38), the optimal performance index, $\overline{\gamma}$, is calculated as $\overline{\gamma} = 1.2159$, the optimal value for c_2 is $c_2 = 2.5571$ and the associated fuzzy filters are given by

$$A_{f1}(1) = \begin{bmatrix} 0.0670 & 0.0116 \\ 0.0124 & 0.0660 \end{bmatrix},\quad B_{f1}(1) = \begin{bmatrix} 0.0405 \\ -0.0397 \end{bmatrix}$$
$$C_{f1}(1) = \begin{bmatrix} 0.0444 & 0.0048 \end{bmatrix}$$
$$A_{f2}(1) = \begin{bmatrix} 0.0779 & 0.0018 \\ 0.0001 & 0.0682 \end{bmatrix},\quad B_{f2}(1) = \begin{bmatrix} -0.2268 \\ 0.1909 \end{bmatrix}$$
$$C_{f2}(1) = \begin{bmatrix} -0.1228 & 0.0803 \end{bmatrix}$$
$$A_{f1}(2) = \begin{bmatrix} 0.0708 & 0.0077 \\ 0.0082 & 0.0701 \end{bmatrix},\quad B_{f1}(2) = \begin{bmatrix} 0.0364 \\ -0.1500 \end{bmatrix}$$
$$C_{f1}(2) = \begin{bmatrix} 0.0219 & -0.0152 \end{bmatrix}$$
$$A_{f2}(2) = \begin{bmatrix} 0.0693 & 0 \\ 0.0003 & 0.0792 \end{bmatrix},\quad B_{f2}(2) = \begin{bmatrix} 0.0711 \\ 0.0368 \end{bmatrix}$$
$$C_{f2}(2) = \begin{bmatrix} -0.0691 & -0.0368 \end{bmatrix}.$$

It should be noted that if we use the method given in [25] [i.e., restraining the structure of $U(r)$ and $V(r)$], it is obtained that the optimal performance index is $\overline{\gamma} = 4.0490$ and the optimal value for c_2 is $c_2 = 8.6315$. Thus, the methods proposed in this paper are less conservative.

For simulation, we choose the fuzzy weighting functions to be $h_1(x_2(k)) = [(1 + \sin(x_2(k)))/2]$, $h_2(x_2(k)) = [(1 - \sin(x_2(k)))/2]$, and the disturbance input as $\omega(k) = 0.5\exp(-0.1k)\sin(k)$. Fig. 2 shows the response of the filtering error system. From Figs. 2 and 3, it can be seen that system (9) is SSH_∞FTB with respect to $(0.1, 2.5571, I_4, 7, 2, 1.2159)$.

Remark 9: When setting system matrix $E = I$ and matrices $S(1) = S(2) = 0$, the problem studied in this paper reduces to finite-time H_∞ filtering problem for fuzzy NMJSs. In contrast to the method proposed in [38], where the optimal performance index $\overline{\gamma} = 8.1948$ and the optimal $c_2 = 17.3657$ are obtained, the optimal performance index $\overline{\gamma} = 5.6537$ and the optimal $c_2 = 12.0031$ are given in this paper. Further, if we set $\beta = 1$, the problem studied in this paper reduces

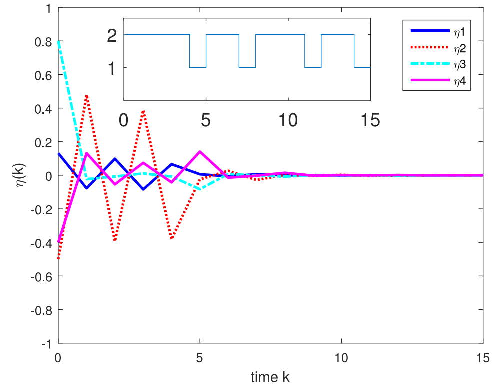

Fig. 2. State responses of the filtering error system (9).

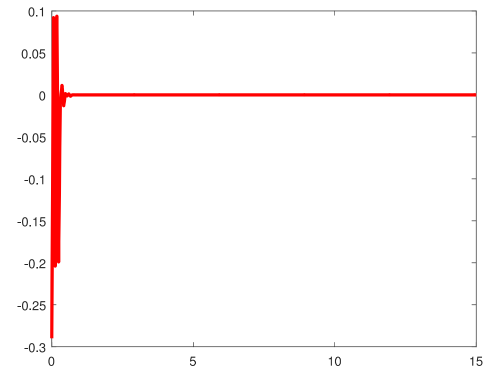

Fig. 3. Curve of the filtering error $e(k)$.

to the H_∞ filtering problem for fuzzy NMJSs. In this case, the optimal performance index $\overline{\gamma} = 2.7906$ and the optimal $c_2 = 6.0866$ are obtained in this paper. However, if we use the method proposed in [35] and [36], it is obtained that the optimal performance index is $\overline{\gamma} = 5.7211$ and the optimal value for c_2 is $c_2 = 12.3611$. Furthermore, the results given in [35], [36], and [38] are invalid for this example since the matrix E is singular. Based on the above discussion, the methods proposed in this paper are less conservative.

V. Conclusion

In this paper, the finite-time H_∞ filtering problem for a type of FSNMJSs has been investigated. Based on a stochastic Lyapunov functional and considering the time-varying transition probabilities inside a polytope, a sufficient condition on SSFTB for the underlying systems is presented. Then a novel sufficient condition on SSH_∞FTB of the filtering error system is obtained in the frame of strict LMIs. Meanwhile, the desired filter parameters are developed by solving a convex optimization problem. The new design methods in this paper improve some existing literatures by using the matrix inequality decoupling technique. In our further research, the extensions of the proposed approaches to the analysis and synthesis of FSNMJSs in the networked environment will be considered. In addition, applying the proposed theoretical results to bio-economic systems is also part of our future research efforts.

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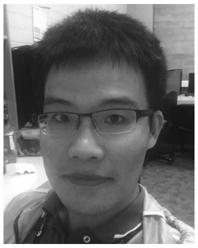

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