Target Tracking in Wireless Sensor Networks Based on the Combination of KF and MLE Using Distance Measurements

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Abstract—A common technical difficulty in target tracking in a wireless sensor network is that individual homogeneous sensors only measure their distances to the target whereas the state of the target composes of its position and velocity in the Cartesian coordinates. That is, the senor measurements are nonlinear in the target state. Extended Kalman filtering is a commonly used method to deal with the nonlinearity, but this often leads to unsatisfactory or even unstable tracking performances. In this paper, we present a new target tracking approach which avoids the instability problem and offers superior tracking performances. We first propose an improved noise model which incorporates both additive noises and multiplicative noises in distance sensing. We then use a maximum likelihood estimator for prelocalization to remove the sensing nonlinearity before applying a standard Kalman filter. The advantages of the proposed approach are demonstrated via experimental and simulation results.

Index Terms—Target tracking, wireless sensor networks, maximum likelihood estimation, extended Kalman filtering.

1 INTRODUCTION

TECENT advances in micro-electro-mechanical systems K(MEMS), wireless communications and networking systems, and embedded microprocessor technologies have made possible the massive production of inexpensive and low-power sensors which are integrated with data collection, information processing, and wireless communication modules in miniature sizes. A wireless sensor network consists of a mesh of such small sensors, which are randomly and densely deployed in the surveillance area and form a multihop ad-hoc network system through wireless communication. These networked sensors are able to process sensed data locally and extract relevant information, to collaborate with other sensors on the applicationspecific task, and to provide the resultant information about the monitored events for a number of potential applications, ranging from battlefield monitoring and environmental surveillance to health care [3], [4]. Target tracking is one of the most fundamental tasks for wireless sensor networks [1], [2], [3], [4], [5], [6], [7], [8], [9].

Most proposed approaches to track and localize targets are based on different types of measurements, such as angle of arrival (AOA) [6], [10], [11], time of arrival (TOA) [12], time difference of arrival (TDOA) [7], [19] and received signal strength (RSS) [6], [18]. Some hybrid approaches of TOA, AOA, TDOA, and RSS have also been proposed for target tracking and localization [13], [14], [15], [16], [17].

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Manuscript received 15 Jan. 2010; revised 9 Jan. 2011; accepted 21 Jan. 2011; published online 17 Mar. 2011.

sensors which can detect the target measure their distances to the target and transmit the information to a leader (either a sensor or a separate computing unit). The leader estimates (updates) the current state of the target based on the received measurements and the target history, and reports the tracking results to the system's users. Since the measurements are usually nonlinear functions in the target state (which typically consists of the position and velocity of the target), the extended Kalman filters (EKFs) have been proposed in many papers for target tracking in wireless sensor networks [20], [21], [22], [23], [24], [25]. In [20] and [21], Brooks et al. described a self-organized distributed target tracking technique with sensor collaborations based on EKF algorithms. In [22], Kaplan presented a global sensor selection approach which was integrated with a decentralized bearings-only EKF tracker. Different sensor scheduling approaches proposed for target tracking in wireless sensor networks also employ EKF-based target state estimators, examples include the multistep adaptive sensor scheduling algorithm (MASS) [23], EKF-based adaptive sensor scheduling method [24] and EKF-based distributed adaptive multisensor scheduling scheme [25] for energy efficiency. The resulting corresponding covariance matrix of the state estimate error is further utilized to select the next tasking sensor (s) and/or the sampling interval for sensors.

In target tracking using distance measurements, the

The EKF algorithms are derived through first linearizing the nonlinear state and measurement equations around the latest state estimate and the predicted state, respectively, and then applying the standard Kalman filter [42], [28]. However, a significant drawback of the EKF algorithms is that the resulting state estimate may seriously diverge from the actual state [26] in many applications. In target tracking applications, target dynamics are usually linearly modeled in the Cartesian coordinates, while the measurements are nonlinear functions in the target state. In these cases, to

567

For information on obtaining reprints of this article, please send e-mail to: tmc@computer.org, and reference IEEECS Log Number TMC-2010-01-0024. Digital Object Identifier no. 10.1109/TMC.2011.59.

overcome the drawback of EKF, many measurement conversion methods have been proposed to transform the nonlinear measurement models into linear ones and estimate the covariances of the converted measurement noises before applying the standard Kalman filter; see a survey paper [27], a monograph [28], and a best linear unbiased estimation method [29]. Significantly improved accuracy and consistency have been reported.

In this paper, we only consider distance measurement for its simplicity, but our proposed target tracking approach can be extended to these different types of measurements. This paper is motivated by two problems. First, the measurement conversion method presented in [29] and those surveyed in [27] assume that there is only one sensor which is able to measure both the distance and bearing (and possibly other parameters) of the target, whereas in a wireless sensor network plenty of homogeneous sensors are deployed, and often only distance measurements are available for target tracking. For this reason, a new measurement conversion method needs to be developed. Second, for most EKF algorithms and measurement conversion methods, it is assumed that measurement noises are additive only with constant covariances. This assumption is valid only when the target is more or less stationary and there is a fixed set of sensors for target detecting and tracking. In most target tracking scenarios, the target is moving and the tasking set of sensors varies with the lapse of time. The implication of these is that the distance between each sensor and the target varies greatly as the target moves. As a consequence, the assumption of constant measurement noise covariances is grossly inaccurate because distance measurement errors typically grow as the distance increases.

To overcome these two problems mentioned above, we propose a new approach to target tracking in wireless sensor networks. This approach is simple, yet effective. First, we propose to use a new noise model for distance measurement to account for both additive noises and multiplicative noises. We also discuss how to numerically obtain the necessary noise statistics using the least square (LS) method. Second, we propose a new measurement conversion method using maximum likelihood estimation. This method is based on the triangulation idea, commonly used in global positioning systems (GPS), recently extended to sensor localization in wireless sensor networks. But here we apply maximum likelihood estimation to obtain a good linear estimate of the target position in the Cartesian coordinates and an approximate covariance matrix of the converted measurement noise. Finally, the converted measurement and noise covariance matrix are then used in a standard Kalman filter to update the target state estimate recursively. The key contribution of this paper is in its proper characterization of the measurement covariance (used in the Kalman filter) via a Gaussian approximation of the localization error.

We demonstrate via simulation and experimental results that the proposed approach performs significantly better than the EKF approach and the approach that is purely based only on the maximum likelihood estimation method. Real wireless sensor network systems with homogeneous or heterogeneous sensors, large, or small scales have been deployed in reality for energy efficient target tracking. VigilNet [31], implemented with 70 Mica2 motes, is designed to track moving targets in an energy-efficient and stealthy manner. Zhou et al. [30], consider the impact of radio irregularity on the design of media access control (MAC), routing, localization, and topology control protocols, in a real-time implementation of wireless sensor network testbed. Keally et al. [32] propose a multimodality event detection framework in heterogeneous sensor networks, which can activate the right sensors to accuracy detect event while significantly reducing energy consumption. Different issues, e.g., radio irregularity, the false alarm of sensors, transmission delay and packet loss, and energy limits, have complicated the implementation and design of real-time wireless sensor networks. In this paper, we only consider a small-scale WSN testbed consisting of four nodes, one base station connecting a PC, for easy implementation in a laboratory. Some of the different reality issues mentioned above also appeared in the implementation of the small scale testbed, e.g., transmission delay and packet loss. But these issues have not significantly affected on our proposed tracking approach. To validate our approach on a larger sensor network, simulations using 100 sensors have been performed.

We note that the maximum likelihood estimation methods are also popular for target or sensor localization in wireless sensor networks with different types of measurements; see, e.g., [33], [34], [35], [36], [37]. The localization problems based on maximum likelihood estimation are nonlinear optimization which is difficult to obtain a close form solution. Various numerical methods have also been proposed in these papers for the global optimization solution, including multiresolution search algorithm, and expectation-maximization (EM) like iterative algorithm [9], [33], [34], particle swarm optimization technique [35], etc. The iterative conjugate-gradient scheme [5], [6] and the Newton-Raphson iterative method [8] have also been applied to solve this nonlinear optimization problem. Unless the initialized value of the MLE is close to the correct solution, it is possible that these maximization search may not find the global maxima. Because our algorithm also utilizes Kalman predictor, much better initialization is provided.

The remainder of the paper is organized as follows: The target motion model, sensor measurement model, and problem formulation are given in Section 2. The proposed target tracking method is detailed in Section 3. Simulation and experimental results are reported in Section 4. Conclusions are reached in Section 5.

2 SYSTEM MODELS AND PROBLEM FORMULATION

For simplicity, we only consider the problem of tracking a single target moving in a two-dimensional field covered by a wireless sensor network. When the target moves through the monitored area, the sensors which have detected the target form a cluster [38], [39], [40] and one of them is selected to be the leader which serves as the center of signal and information processing. It is assumed that the leader knows the position of every sensor. The cluster members measure their distances to the target and transmit the measurements with other information, such as its identity (ID) and the corresponding time stamps, to the leader. It is assumed that there is no transmission delay or packet loss. After receiving all the measurements, the leader will compute an estimate of the state (position and velocity) of the target.

2.1 Target Motion Model

A target moving in a two-dimensional field is usually described by its position and velocity in the *X*-*Y* plane

$$x_k = \begin{bmatrix} x(k) & v_x(k) & y(k) & v_y(k) \end{bmatrix}^T$$

where (x(k), y(k)) are the position coordinates of the target along *X*- and *Y*-axes at time t_k , respectively, and $(v_x(k), v_y(k))$ are the velocities of the target along *X*- and *Y*-directions at time t_k , respectively. The following nearly-constant-velocity (CV) model [28] is adopted to represent the motion of the target

where

$$x_{k+1} = F_k x_k + G_k w_k, \tag{1}$$

$$F_{k} = \begin{bmatrix} 1 & \Delta t_{k} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t_{k} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ G_{k} = \begin{bmatrix} \frac{\Delta t_{k}^{2}}{2} & 0 \\ \Delta t_{k} & 0 \\ 0 & \frac{\Delta t_{k}^{2}}{2} \\ 0 & \Delta t_{k} \end{bmatrix}.$$

In the above, $\Delta t_k = t_{k+1} - t_k$ is the sampling time interval between two successive measurement times t_{k+1} and t_k , $w_k = [w_x w_y]^T$ is a white Gaussian noise sequence with zero mean and covariance matrix Q_w , and w_x and w_y correspond to noisy accelerations along the X- and Y-axes, respectively. In this paper, we assume that w_x is uncorrelated with w_y for simplicity, and Q_w is given by

$$Q_w = egin{bmatrix} \sigma^2_{w_x} & 0 \ 0 & \sigma^2_{w_y} \end{bmatrix}.$$

Remark 2.1. The above model for the moving target is a standard one studied in the literatures. It does not consider the case where the moving target follows a given trajectory, which happens when the target travels on a given road segment. But if such trajectory is available as in the case when a road map is available, the system model for the moving target can be easily modified and we expect that our approach is still applicable.

2.2 Measurement Model

We assume that all the sensors are of the same type and have the same noise statistics. Denote by $z_i(k)$ the distance measurement to the target obtained by sensor *i* at time t_k . To simplify our notation, the dependence on time t_k is suppressed in the sequel, e.g., $z_i(k)$ is simplified to be z_i .

Let r_i be the true distance between sensor i and the target, we have

$$r_i = \sqrt{(x - x^i)^2 + (y - y^i)^2},$$

where (x^i, y^i) is the known location of sensor *i*, and (x, y) is the unknown position of the target at time t_k . The measurement model we adopt is represented in the following form of additive and multiplicative noises

$$z_i = (1 + \gamma_i)r_i + n_i = r_i + u_i, \tag{2}$$

where n_i and γ_i are the additive and multiplicative Gaussian noises of sensor *i* with means μ_n and μ_{γ} and covariances σ_n^2 and $\sigma_{\gamma'}^2$ respectively. It is normally assumed

that these two types of noises are uncorrelated. The use of multiplicative noise is motivated by the fact that measurement error increases roughly linearly as a function of distance for many distance sensors. Indeed, relative errors are commonly used in accuracy specifications.

The total noise of sensor *i*, denoted by $u_i = n_i + r_i \cdot \gamma_i$, is also a Gaussian noise with mean $\mu_i = \mu_n + r_i \cdot \mu_\gamma$ and covariance $\sigma_i^2 = \sigma_n^2 + \sigma_\gamma^2 \cdot r_i^2$, which are dependent on the true distance r_i .

According to (2), the conditional probability density function (PDF) of the measurement z_i , given (x, y), is written as follows:

$$p(z_i|x,y) = \frac{1}{\sqrt{2\pi\sigma_i}} \exp\left\{-\frac{(z_i - r_i - \mu_i)^2}{2\sigma_i^2}\right\} = \frac{1}{\sqrt{2\pi\sigma_i}} \exp\left\{-\frac{[r_i - (z_i - \mu_i)]^2}{2\sigma_i^2}\right\}.$$
(3)

- **Remark 2.2.** We have assumed that the sensors are of the same type. Hence, their noise statistics are the same. Note that we have allowed multiplicative noises. Thus, different sensors will have different noise sizes depending on their distances to the target. Also note that our approach can be easily extended to the case where different noise statistics are used for different sensors.
- **Remark 2.3.** We have assumed that there are no packet loss and transmission delay when the measurements are sent to the processing center. These problems can be solved by combining the presented results and our previous works.

2.3 Problem Formulation

We assume that there are n_k $(n_k \ge 3)$ sensors have detected the target at time t_k , and all the measurements (with the corresponding time stamps) are gathered at the leader. Let Z_k denote the measurements with the same time stamps from all the n_k sensors, that is, $Z_k = \{z_1(k), \ldots, z_{n_k}(k)\}$. The problem is for the leader to estimate the target state x_k , denoted by $\hat{x}_{k|k}$, given the measurements $\{Z_j\}$ from time 0 up to and including time k.

3 TARGET TRACKING ALGORITHM

In this section, we discuss our proposed target tracking algorithm in detail. We first explain how to establish the noise statistics for the sensors using the least square method. We then lay out the prelocalization algorithm using maximum likelihood estimation. The solution to the maximum likelihood estimation-based localization is also given by using a Newton iterative method. This will be followed by a Kalman filter for recursive estimation of the target state.

3.1 Noise Statistics Computation Using Least Squares Method

Recall the measurement model we have adopted in (2). We need to find a way to estimate the noise statistics (i.e., the means and variances of the additive and multiplicative noises) of the sensors.

We assume that all the sensors have independent and identically distributed (i.i.d.) multiplicative and additive noises. Then, the means and covariances of the additive noise and the multiplicative noise can be estimated through experiments as follows:

Suppose we take a sensor and run m tests with different distances between the sensor and the target. Denote the actual distance for the *i*th test to be r_i and we assume that r_i is known (measured through a different and accurate method). For each r_i , N measurement samples z_i^j , $j = 1, \ldots, N$, are collected, where N is a large number.

Take the empirical estimates of the mean and variance of the measurement samples for the *i*th test to be

$$\bar{\mu}_i = \frac{1}{N} \sum_{j=1}^N z_i^j, \ \bar{\sigma}_i^2 = \frac{1}{N-1} \sum_{j=1}^N (z_i^j - \bar{\mu}_i)^2,$$

respectively, i = 1, ..., m. Using the ergodicity of the stationary process z_i^j and the independence between the additive and multiplicative noises, we have, as $N \to \infty$

$$\bar{\mu}_i \to E\{z_i^j\} = r_i + r_i \mu_\gamma + \mu_n, \tag{4}$$

$$\bar{\sigma}_i^2 \to E\left\{\left(z_i^j - E\left\{z_i^j\right\}\right)^2\right\} = r_i^2 \sigma_\gamma^2 + \sigma_n^2.$$
(5)

Define approximation errors as

$$e_1(i) = \bar{\mu}_i - (r_i + r_i \mu_\gamma + \mu_n),$$

 $e_2(i) = \bar{\sigma}_i^2 - (r_i^2 \sigma_\gamma^2 + \sigma_n^2).$

We can use the least-squares method to determine the estimates of μ_{γ} , μ_n , σ_{γ}^2 , and σ_n^2 . That is, we minimize J_1 and J_2 defined in the following equations:

$$J_1 = \sum_{i=1}^m e_1^2(i), \ \ J_2 = \sum_{i=1}^m e_2^2(i)$$

The minimizations of J_1 and J_2 are given by the following necessary conditions:

$$\frac{\partial J_1}{\partial \mu_{\gamma}} = 0, \quad \frac{\partial J_1}{\partial \mu_n} = 0, \quad \frac{\partial J_2}{\partial \sigma_{\gamma}^2} = 0, \quad \frac{\partial J_2}{\partial \sigma_n^2} = 0.$$

Straightforward calculations lead to the following estimates:



3.2 Prelocalization Using Maximum Likelihood Estimation

We now discuss how to convert the nonlinear distance measurements into a linear model with respect to the position of the target in the Cartesian coordinates. We assume that the means and covariances of the multiplicative noises and the additive noises have been given. We further assume that the geometrical relation of the sensors is such that, if measurement noises are not present, the position of the target can be uniquely determined. Note that with three or more sensors, this is not a problem unless all the sensors are in a line, a case which can be easily discounted by a careful selection of sensors. We will call this measurement conversion process *prelocalization*.

Our method for prelocalization is based on maximum likelihood estimation. Assume that there are $n \ (n \ge 3)$ sensors which have detected the moving target at the same time t_k (which we will suppress in the following section for simplicity) and that the measurement noises of different sensors are mutually independent. Define $Z = \{z_i, i = 1, ..., n\}$. Denote by p(Z|x, y) the jointly conditional probability density function of Z, given (x, y).

The maximum likelihood estimation-based prelocalization is to seek the unknown target positions (x, y) such that p(Z|x, y) is maximized. Since the noises of individual sensors are mutually independent, we have the following equation:

$$p(Z|x,y) = \prod_{i=1}^{n} p(z_i|x,y),$$
(6)

where $p(z_i|x, y)$ is given by (3).

There is a small technical difficulty in using (3). That is, σ_i and μ_i depend on the actual distance between the target and sensor *i*, r_i , which are unknown and are related to the unknown parameters (x, y). To get around this computation difficulty, we take the assumption that the difference between z_i and r_i is very small, therefore, σ_i^2 can be approximately replaced by $\sigma_{z_i}^2 = \sigma_n^2 + z_i^2 \cdot \sigma_\gamma^2$. Then, the probability density function (3) can be approximately rewritten as

$$p(z_i|x,y) \approx \frac{1}{\sqrt{2\pi\sigma_{z_i}}} \exp\left\{-\frac{\left[(1+\mu_{\gamma})r_i - \check{z}_i\right]^2}{2\sigma_{z_i}^2}\right\},$$
 (7)

where $\check{z}_i = z_i - \mu_n$ is the calibrated measurement.

- **Remark 3.1.** The assumption that z_i and r_i are close is used only for approximating the variance $\sigma_i^2 = \sigma_n^2 + r_i^2 \sigma_\gamma^2$ by $\sigma_n^2 + z_i^2 \sigma_\gamma^2$ which leads to (7). The induced approximation error in the variance above will not significantly affect the probability density function $p(z_i|x, y)$. In fact, it is well known that Kalman filtering is insensitive to small changes in the noise covariance.
- **Lemma 3.1.** Assuming that the approximation (7) is valid, then the maximum likelihood estimate of (x, y) is given by

$$(\bar{x}, \bar{y}) = \arg \min_{x,y} f(x, y), \tag{8}$$

where

$$f(x,y) = \sum_{i=1}^{n} \frac{\left[(1+\mu_{\gamma})r_i - \check{z}_i\right]^2}{2\sigma_{z_i}^2}.$$
(9)

Proof. It follows from (6) and (7) that

$$p(Z|x,y) = \frac{1}{(2\pi)^{\frac{n}{2}} \prod_{i=1}^{n} \sigma_{z_{i}}} \\ \cdot \exp\left\{-\sum_{i=1}^{n} \frac{\left[(1+\mu_{\gamma})r_{i}-\check{z}_{i}\right]^{2}}{2\sigma_{z_{i}}^{2}}\right\}.$$
(10)

Maximizing the above equation is identical to maximizing its log version:

$$-\sum_{i=1}^{n} \frac{\left[(1+\mu_{\gamma})r_{i}-\check{z}_{i}\right]^{2}}{2\sigma_{z_{i}}^{2}} - \frac{n}{2}\ln(2\pi) - \sum_{i=1}^{n}\ln\sigma_{z_{i}}$$

Ignoring the last two terms (which are constants), the maximum likelihood estimate is reduced to minimize the equation given by (9).

The minimization problem of (8) is numerically difficult because f(x, y) is nonlinear. We propose to solve this nonlinear optimization problem using the following Newton-Raphson iterative method [41]: For j = 0, 1, ..., compute

$$\begin{bmatrix} x^{(j+1)} \\ y^{(j+1)} \end{bmatrix} = \begin{bmatrix} x^{(j)} \\ y^{(j)} \end{bmatrix} - hH^{-1}(x^{(j)}, y^{(j)}) \cdot \nabla f^T(x^{(j)}, y^{(j)}), \quad (11)$$

until

$$|x^{(j+1)} - x^{(j)}| < \varepsilon, \quad |y^{(j+1)} - y^{(j)}| < \varepsilon,$$

where h is the step size (usually set to be 1),

$$\begin{aligned} \nabla f(x^{(j)}, y^{(j)}) &= \left[\frac{\partial f(x, y)}{\partial x} \frac{\partial f(x, y)}{\partial y} \right]_{x=x^{(j)}, y=y^{(j)}}, \\ H(x^{(j)}, y^{(j)}) &= \left[\frac{\partial^2 f(x, y)}{\partial x^2} \frac{\partial^2 f(x, y)}{\partial y \partial x} \right]_{x=x^{(j)}, y=y^{(j)}}, \end{aligned}$$

and $\varepsilon > 0$ is a prescribed threshold. The initial values

$$\begin{bmatrix} x^{(0)} \\ y^{(0)} \end{bmatrix} = \begin{bmatrix} \hat{x}^1_{k|k-1} \\ \hat{x}^3_{k|k-1} \end{bmatrix},$$

where $\hat{x}_{k|k-1}^1$ and $\hat{x}_{k|k-1}^3$ are the first and third elements of the one-step-ahead predicted state $\hat{x}_{k|k-1}$ of the Kalman filtering algorithm which is given in the next section.

Although it is theoretically possible for the iterative method not to converge to a global minimum, experimental and simulation results show that this is rarely a problem. Typically, only a few iterations are sufficient to get the global minimum. The nice convergence is partly assisted by the fact that the initial estimate of the iterative process comes from the Kalman predictor which is typically good, provided that the sampling time interval is not too long.

3.3 Kalman Filtering

Once the prelocalization is done, we only need to consider the new converted measurement $\bar{z}_k = [\bar{x}(k) \ \bar{y}(k)]^T$, which has the following linear representation in the target state:

$$\bar{z}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x_k + v_k = Cx_k + v_k,$$
(12)

where v_k is the converted measurement noise.

It remains to specify the statistics for noise v_k before the converted measurement \bar{z}_k can be used in Kalman filtering.

Using Bayes rule, we have the following posterior probability distribution of (x, y):

$$p(x, y|Z) = \frac{p(Z|x, y)p_a(x, y)}{p(Z)}$$

where $p_a(x, y)$ is the prior probability density function of (x, y) known to the sensing nodes. Since we will do Kalman prediction later, $p_a(x, y)$ does not include any prior knowledge on the target position. For this reason, we assume that there is no a priori, knowledge where the target is, as far as the sensing nodes are concerned. In other words, $p_a(x, y)$ is assumed to be uniform (i.e., constant) within the monitored area. In view of this, p(x, y|Z) can represented as

$$p(x, y|Z) = \alpha p(Z|x, y),$$

with some α independent of (x, y). Recall from (9) and (10) that

$$p(x,y|Z) = \frac{\alpha}{(2\pi)^{\frac{n}{2}} \prod_{i=1}^{n} \sigma_{z_i}} \exp(-f(x,y)),$$

and note that, after prelocalization, we have the following approximation equation:

$$f(x,y) \approx f(\bar{x},\bar{y}) + \frac{1}{2} \begin{bmatrix} x - \bar{x} \\ y - \bar{y} \end{bmatrix}^T H(\bar{x},\bar{y}) \begin{bmatrix} x - \bar{x} \\ y - \bar{y} \end{bmatrix},$$
(13)

where $H(\bar{x}, \bar{y})$ is the Hessian matrix given by

$$H(\bar{x},\bar{y}) = \begin{bmatrix} \frac{\partial^2 f(x,y)}{\partial x^2} & \frac{\partial^2 f(x,y)}{\partial x \partial y} \\ \frac{\partial^2 f(x,y)}{\partial y \partial x} & \frac{\partial^2 f(x,y)}{\partial y^2} \end{bmatrix}_{x=\bar{x},y=\bar{y}} .$$
(14)

In (13), we have neglected the higher order terms. Using the above approximation, we have

$$p(x,y|Z) \approx \beta \exp\left(-\frac{1}{2} \begin{bmatrix} x - \bar{x} \\ y - \bar{y} \end{bmatrix}^T \quad H(\bar{x},\bar{y}) \begin{bmatrix} x - \bar{x} \\ y - \bar{y} \end{bmatrix}\right),$$

for some constant β . That is, v_k is approximately a Gaussian distribution with zero mean and covariance matrix

$$R_k = H^{-1}(\bar{x}(k), \bar{y}(k)).$$
(15)

The analysis above is summarized in the following lemma.

Lemma 3.2. Assuming that the prior probability density function for the position of the target, $p_a(x(k), y(k))$, is uniform, the converted measurement equation after prelocalization, \bar{z}_k , can be approximated by (12) and the associated converted noise v_k is Gaussian white with zero mean and covariance matrix R_k given in (15).

Our final step is to utilize the Kalman filtering algorithm to update the target state using the converted measurements and associated noises. Its expression is standard [42] and given below

$$\hat{x}_{k+1|k} = F_k \hat{x}_{k|k},\tag{16}$$

$$P_{k+1|k} = F_k P_{k|k} F_K^T + G_k Q_w G_k^T, (17)$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}(\bar{z}_{k+1} - C\hat{x}_{k+1|k}), \qquad (18)$$

$$P_{k+1|k+1} = P_{k+1|k} - K_{k+1}S_{k+1}K_{k+1}^T,$$
(19)

Fig. 1. Wireless sensor network testbed consisting of four sensor nodes, a base-station connecting to a PC, and a robot carrying a node as the moving target.

$$S_{k+1} = CP_{k+1|k}C^T + R_{k+1}, (20)$$

$$K_{k+1} = P_{k+1|k} C^T S_{k+1}^{-1}.$$
 (21)

The initial estimates are given as $\hat{x}_{0|0} = \hat{x}_0$ and $P_{0|0} = P_0$ for some large positive definite P_0 .

4 **EXPERIMENTAL RESULTS**

In this section, we use simulation and experimental results to compare the proposed approach with the extended Kalman filtering approach for target tracking. We will also compare our approach with the approach that is purely based only on the maximum likelihood estimation method (i.e., without Kalman filtering). To this end, a small wireless sensor network testbed is set up using off-the-self products from Crossbow.

4.1 Testbed Setup

Our testbed is shown in Fig. 1, which consists of four Cricket nodes [43] and a base station. The base station is built up using an MIB510CA serial interface board and a Cricket node. The base station serves as the information processing center which receives measurements from the four Cricket nodes and transmits the tracking results to a computer. An Amigo robot carrying a Cricket node is used as the moving target.

Cricket [43] nodes make use of the time difference of arrival of a radio frequency (RF) signal and an ultrasonic pulse to evaluate the distance between nodes. The target node S_0 anchored on the robot periodically broadcasts its identity and time stamp information through the RF channel and transmits an ultrasonic pulse at the same time. Other sensor nodes within the radio range of S_0 listen to the RF signal, and upon receiving the first few bits of the signal, start the ultrasound detector to detect the corresponding ultrasonic pulse. When detected, the distance to the robot is computed by counting the time difference of arrival between the RF signal and the ultrasonic pulse. The sensor nodes then transmit the distances and other information, including their ID and the time stamps received from S_0 , to the base station for data processing.

The monitored field covered by the testbed is $2 \text{ m} \times$ 2 m with the coordinate from (0,0) to (2,2). The four sensor nodes are fixed at the four corners of the square field. The detection range of every sensor node is greater than $2\sqrt{2}$ m, so the full area can be covered by these four sensors. The based station has the knowledge of the positions of the four sensor nodes and estimates the target state only when it receives three or more distances with the same time stamps.

4.2 Experimental Results

We first run many offline experiments to test the sensors' noise statistics using the least-squares method discussed in Section 3. The mean and covariance of the additive noise are found to be $\mu_n = -0.0386$ and $\sigma_n^2 = 7.97 \times 10^{-5}$, respectively, and the mean and covariance of the multiplicative noise are given by $\mu_{\gamma} = 0.0174$ and $\sigma_{\gamma}^2 = 2.916 \times 10^{-4}$, respectively. This shows that the multiplicative noise is not negligible in comparison with the additive noise.

The target node broadcasts once every 0.2 seconds approximately. The robot first moves along a circle centered at (1.00 m, 0.65 m) with radius 0.35 m, then moves along another circle centered at (1.00 m, 1.35 m) with the same radius. The initial position of the robot is (1.00 m, 1.00 m), and the angular velocity is programmed to be -0.122 rad/s when moving along the first circle and 0.122 rad/s when moving along the second circle. For this motion, the process noise w_k corresponds to the variable acceleration of the target and can be approximated by a white Gaussian sequence with zero mean and covariance matrix of $Q_w = \begin{bmatrix} 0.0027 & 0.0 \\ 0.0 & 0.0027 \end{bmatrix}$.

In the experiments the initial state estimate and the corresponding covariance matrix for the proposed target tracking approach and the extended Kalman filtering approach are chosen to be

$$\hat{x}_{0|0} = \begin{bmatrix} 1.0 & 0.0428 & 1.0 & 0.0 \end{bmatrix}^T; P_{0|0} = 0.01 \cdot I_4,$$

where I_n is an $n \times n$ identity matrix. But the variance for the distance measurement noises used in the extended Kalman filtering approach also depend on the true distance from the target to sensor i, r_i as well. In these experiments and the following simulations, the variance R_k of the measurement noises is approximately computed as

$$R_k = R_n + R_\gamma * z_i(k)^2,$$

where $z_i(k)$ is the distance measurement of sensor *i* at time instant t_k .

The target tracking results are shown in Figs. 2, 3, and 4. The green curves are the programmed target trajectories of the moving target, and the red ones are the estimated target trajectories using different approaches. The results demonstrate that the proposed target tracking method is more accurate than the other two methods.

4.3 Simulation Results

Since the tracking errors are random in nature and it is not practical to repeat the experiments many times, we try to compare our proposed approach with the extended Kalman filtering approach and the maximum likelihood estimation approach for target tracking again using Monte Carlo simulation.

In the simulations, a wireless network consisting of 100 sensor nodes is deployed to monitor road segments from GoogleMap as shown in Fig. 5. The sensors cover the





Fig. 2. Target tracking using proposed method: The red line is the estimated trajectory using the proposed method, the green line is the programmed trajectory, and the four blue circles are sensor nodes.

road segments from point *A* to C and D through B with a width of 4 m. Every sensor measures its distance to the target and transmits the measurement to a fusion center (or a leader) if the distance is less than or equals to 3 m, that is, the detecting range of the sensor is 3 m. We assume that the sensors' noise biases are removed (which does not affect the tracking performance). Therefore, the additive noise is Gaussian and white with zero mean and covariance of 0.001, and the multiplicative noise is Gaussian and white with zero mean and covariance of 0.001 for every sensor. The sensors detect the target every 0.1 seconds. The target moves along the road with initial state given as $x_0 = [0.0 \ 1.0 \ 2.0 \ 0.3]^T$. The covariance matrix



Fig. 3. Target tracking using extended Kalman filtering: The red line is the estimated trajectory using EKF, the green line is the programmed trajectory, and the four blue circles are sensor nodes.



Fig. 4. Target tracking using maximum likelihood estimation only: The red line is the estimated trajectory using MLE only, the green line is the programmed trajectory, and the four blue circles are sensor nodes.

 Q_w of the target noisy acceleration is given by $Q_w = \text{diag}\{0.025, 0.01\}$ when the target moves from A to B or C, and $Q_w = \text{diag}\{0.01, 0.025\}$ when moves from B to C.

The same initial state estimate, $\hat{x}_{0|0}$, and its corresponding error covariance matrix, $P_{0|0}$, are used for the proposed tracking approach and the extended Kalman filtering method, given as $x_{0|0} = x_0 + 0.1 * \operatorname{randn}(4, 1)$ and $P_{0|0} = 0.01I_4$. The covariance matrix Q_w of the process noise used in the two target tracking approaches is given as $Q_w = 0.025I_2$. The number of runs used in the Monte Carlo simulation is 100.

The simulation results are shown in Figs. 2, 3, 4, and 5. When the target moves from A to B then to D, the true trajectory and the estimated one using the three tracking approaches are illustrated in Fig. 6, and the root-mean-square errors (RMSEs) of the three tracking approaches are compared in Fig. 7. The RMSEs of the position estimate improves approximately by 15.12 percent using the proposed method compared to the extended Kalman filtering



Fig. 5. Road segment in GoogleMap.



Fig. 6. The true and estimated trajectories of the target when it moves from A to B, then to D.

method, and 17.19 percent compared to the maximum likelihood estimation method alone.

When the target moves from A to C through B, the true trajectory and the estimated ones using the three approaches are illustrated in Fig. 8 and the RMSEs of the three tracking approaches are compared in Fig. 9. The RMSE of the position estimate improves approximately by 14.87 percent using the proposed method compared to the extended Kalman filtering method, and 31.19 percent compared to the maximum likelihood estimation method alone.

We note that the proposed method gives a better estimate than the extended Kalman filter-based method and the maximum likelihood estimation-based method for most of the tracking process. And also the extended Kalman filter-based method is more accurate than the maximum likelihood estimation-based method for most of the time. This is because the maximum likelihood estimation-based method only utilizes the sensing information but the two other methods further utilize model information of the target. When the target turns, the original model is not accurate. In this case, the extended Kalman filterbased method and our proposed method are getting less accurate than the maximum likelihood estimation-based method, but our proposed method is still more accurate than the extended Kalman filter-based method. If we have



Fig. 7. RMSE of the three tracking approach when the target moves from A to B, then to D.



Fig. 8. The true and estimated trajectories of the target when it moves from A to C through B.

an accurate model to describe the turns of the target, we expect that our method can achieve more accurately than the maximum likelihood estimation-based method. Furthermore, if the position estimation is the primary interest (rather than the whole state), it may be advantageous to use the maximum likelihood estimator initially and switch to the proposed estimator later when the target turns, which can be detected using the methods for target maneuver detection, see [44], [45] for details.

5 CONCLUSION

In this paper, we have presented a new approach for target tracking in a wireless sensor network by combining maximum likelihood estimation and Kalman filtering using the distance measurements. The maximum likelihood estimator is used for prelocalization of the target and measurement conversion to remove the measurement nonlinearity. The converted measurement and its associated noise statistics are then used in a standard Kalman filter for recursive update of the target state. The proposed approach is very simple and yet effective. Simulation and experimental results have shown that the proposed approach improve the tracking accuracy compared to the commonly used extended Kalman filtering approach.



Fig. 9. RMSE of the three tracking approaches when the target moves from A to C through B.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation for Distinguished Young Scholars of China (No. 60825304), the National Basic Research Development Program of China (973 Program) (No. 2009cb320600), and the open project of State Key Laboratory of Industrial Control Technology (ICT1006).

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