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Finite-Horizon Quantized Estimation Using Sector Bound Approach

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***Abstract*— This paper is concerned with the finite-horizon filtering estimation problem by using the reductive information of the true quantized measurements from the measurements. We consider the case where quantizer is logarithmic, an upper bound of the estimation error covariance is derived for all the quantized measurements. The calculation of the filter involves solving two Riccati recursions related to the quantized measurements.**

***Index Terms*— Discrete-time system, logarithmic quantizer, quantized estimation, sector bound.**

I. Introduction

Recently, quantized control and quantized estimation problems have been investigated abundantly due to the development of networked control systems especially for industrial control and automation. Examples of quantized feedback control problem include [1][2][3][4] and references therein. For its applications in the bandwidth-constrained wireless sensor network, the coarsest quantizer that quadratically stabilized a single input linear discrete-time invariant system is proven to be logarithmic in [1]. In [2] the logarithmic quantizer is considered, it shows that the logarithmic quantizer performs better than the linear quantizer when it deals with the quantization error, for logarithmic quantizer gives a multiplicative quantization error which reduces as the measurements becomes small, while the quantization error of linear quantizer is addictive and grows linearly as the measurements becomes large. What's more for the the logarithmic quantizer, the quantized feedback control problems can be converted into classical robust control problems with sector bound uncertainties.

As in the classical control and estimation theory, state estimation plays an critical role to control theory due to the separation theory [11]. Though only a high resolution separation theorem holds [3], quantized estimation is also important to quantized feedback control problems [3]. In [8] and [9], the quantized estimation based on quantized innovation is derived based on the assumption that the conditional probability of the estimated state based on the innovations is the same as that based on the quantized innovations. As in [5], the infinite-level and finite-level quantizers are considered, but only the steady state estimator is given. In this paper, we use the original measurements to recover the logarithmic quantized information to design a state estimator for a single output linear discrete-time invariant system. In this paper infinite-level quantizer has been considered. The reminder of the paper is organized as follows. Section II formulates the quantized estimation problem. Section III presents the solution of the estimation problem using the quantized measurements. Finally, section IV draws some conclusions of this paper.

II. Problem Formulation

Consider the following linear discrete-time system:

$$x(t+1) = Ax(t) + Bw(t), \tag{1}$$

$$y(t) = Cx(t) + Dv(t), \tag{2}$$

where $x(t) \in R^n$ is the state, $y(t) \in R$ is the observation, B, D are matrices with proper dimensions, $w(t) \in R^m$ and $v(t) \in R^k$ are noises. We put forward the following assumptions:

Assumption 1: : For all integers t and $l \geq 0$,

$$\begin{aligned} &E[w(t)] = 0, E[w(t)w^T(l)] = Q_w\delta_{tl},\\ &E[v(t)] = 0, E[v(t)v^T(l)] = Q_v\delta_{tl},\\ &E[w(t)v^T(l)] = 0. \end{aligned} \tag{3}$$

where $E[\cdot]$ denotes the expectation and δ_{tl} denotes the Kronecker Delta.

Assumption 2: :

$$\begin{aligned} &(1)E[x(0)x^T(0)] = q_0, where\ q_0 = q_0^T \geq 0\ is\ a\\ &\quad known\ matrix,\\ &(2)rank[A\ BQ_w^{\frac{1}{2}}] = n, \end{aligned} \tag{4}$$

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We consider the logarithmic quantizer as following:

$$U = \{\pm u_i : u_i = \rho^i u_0, i = 1, 2, \cdots\} \bigcup \{\pm u_0\} \bigcup \{0\}, \quad with\ 0 < \rho < 1, u_0 > 0 \tag{5}$$

where ρ is called quantized density of the quantizer. As illustrated in [2], using the sector bound uncertainty we have

$$y - Q(y) = \delta y, \ |\delta| \leq \Delta. \tag{6}$$

with

$$Q(y) = \begin{cases} u_i, & if\ \frac{1}{1+\Delta} u_i < y \leq \frac{1}{1-\Delta} u_i, \ y > 0 \\ 0, & if\ y = 0 \\ -Q(-y), & if\ y < 0 \end{cases} \tag{7}$$

where

$$\Delta = \frac{1-\rho}{1+\rho}. \tag{8}$$

Then we can rewrite the quantized measurements as:

$$\begin{aligned} z(t) &= Q(y(t)) = y(t) + \delta_t y(t) \\ &= (C + \delta_t C)x(t) + (D + \delta_t D)v(t), \end{aligned} \tag{9}$$

where $\|\delta_t\| \leq \Delta$, hence $\|\frac{\delta_t}{\Delta}| \ \leq 1$.

The quantized filter design problem can be stated as: given the quantized measurements $\{z(0), z(1), \cdots, z(t)\}$, to design a filter

$$\widehat{x}(t+1) = A\widehat{x}(t) + K_t z(t), \tag{10}$$

where K_t is a time-varying matrix to be determined in order that the variance of the estimation error is guaranteed, that is, there exist a sequence of positive-definite matrices $M_t = M_t^T \geq 0$ $(0 \leq t \leq N)$ satisfying:

$$E\left[(x(t) - \widehat{x}(t))(x(t) - \widehat{x}(t))^T\right] \leq M_t,$$

and then minimize M_t.

III. Quantized filter design

It is noted that (9) and (11) involves uncertainties, and the accurate error covariance is impossible to be determined, so in this section we try to find an upper bound for the time-varying estimation error covariance. By (9) we have:

$$\begin{aligned} \widehat{x}(t+1) &= A\widehat{x}(t) + K_t[(C+\delta_t C)x(t) + (D+\delta_t D)v(t)] \\ &= A\widehat{x}(t) + K_t(C+\delta_t C)x(t) + K_t(D+\delta_t D)v(t) \end{aligned} \tag{11}$$

Define the following new state vector as:

$$\xi(t+1) \triangleq \begin{bmatrix} x(t+1) \\ \widehat{x}(t+1) \end{bmatrix}, \tag{12}$$

$$\eta(t) = \begin{bmatrix} w(k) \\ v(k) \end{bmatrix}, \tag{13}$$

So by (1) , (11) and (12), (13), we have the following auxiliary system:

$$\xi(t+1) = (\widehat{A}_t + \widehat{A}_{et})\xi(t) + \bar{B}\eta(t), \tag{14}$$

where

$$\widehat{A}_t = \begin{bmatrix} A & 0 \\ K_t C & A \end{bmatrix}, \tag{15}$$

$$\bar{B} = \begin{bmatrix} B & 0 \\ 0 & (1+\delta_t)K_t D \end{bmatrix}. \tag{16}$$

$$\widehat{A}_{et} = \widetilde{H}(t)F(t)\widetilde{E}, \tag{17}$$

while

$$\widetilde{H}(t) = \begin{bmatrix} 0 \\ \Delta K_t C \end{bmatrix}, \ F(t) = \frac{\delta_t}{\Delta}, \ \widetilde{E} = \begin{bmatrix} I_n & 0 \end{bmatrix}. \tag{18}$$

It is straightforward to write the Lyapunov equation that governs the evolution of the covariance matrix:

$$\begin{aligned} \widetilde{\Sigma}_{t+1} &= E[\xi(t+1)\xi^T(t+1)] \\ &= (\widehat{A}_t + \widehat{A}_{et})\widetilde{\Sigma}_t(\widehat{A}_t + \widehat{A}_{et})^T + G_t, \end{aligned} \tag{19}$$

where

$$G_t = \bar{B}E(\eta(t)\eta^T(t))\bar{B}^T.$$

Note Assumption 1, we have :

$$E[\eta(t)] = 0, \ E[\eta(t)\eta^T(l)] = \begin{bmatrix} Q_w & 0 \\ 0 & Q_v \end{bmatrix} \delta_{tl}, \tag{20}$$

then G_t is calculated as:

$$\begin{aligned} G_t &= \bar{B}E(\eta(t)\eta^T(t))\bar{B}^T \\ &= \bar{B}\begin{bmatrix} Q_w & 0 \\ 0 & Q_v \end{bmatrix}\bar{B}^T \\ &= \begin{bmatrix} BQ_wB^T & 0 \\ 0 & (1+\delta_t)^2 K_t D Q_v D^T K_t^T \end{bmatrix}. \end{aligned} \tag{21}$$

Next, we want to find a positive-definite matrix Σ_{t+1} satisfying:

$$\widetilde{\Sigma}_{t+1} \leq \Sigma_{t+1}. \tag{22}$$

Lemma 1: [13] Given matrices A, H, E, and F with compatible dimensions and $FF^T \leq I$. Let X be a positive definite matrix and $\alpha > 0$ be an arbitrary constant such that $\alpha^{-1}I - EXE^T > 0$, then we have :

$$\begin{aligned} & (A + HFE)X(A + HFE)^T \\ \leq\ & A(X^{-1} - \alpha E^T E)^{-1}A^T + \alpha^{-1}HH^T. \end{aligned} \tag{23}$$

So by lemma 1, we have:

$$\begin{aligned} \widetilde{\Sigma}_{t+1} &= (\widehat{A}_t + \widehat{A}_{et})\widetilde{\Sigma}_t(\widehat{A}_t + \widehat{A}_{et})^T + G_t \\ &\leq \hat{A}_t[\widetilde{\Sigma}_t^{-1} - \alpha_t \widetilde{E}^T\widetilde{E}]^{-1}\hat{A}_t^T + \alpha_t^{-1}\widehat{H}(t)\widehat{H}^T(t) + G_t, \end{aligned} \tag{24}$$

Definition 1: we define the following Riccati equation

$$\overline{\Sigma}_{t+1} = \hat{A}_t[\overline{\Sigma}_t^{-1} - \alpha_t\widetilde{E}^T\widetilde{E}]^{-1}\hat{A}_t^T + \alpha_t^{-1}\widehat{H}(t)\widehat{H}^T(t) + G_t, \tag{25}$$

where

$$\alpha_t^{-1}I - \tilde{E}\overline{\Sigma}_t\tilde{E}^T > 0. \tag{26}$$

Definition 2: The filter (10) is called quantized quadratic filter, if there exist a sequence of $\alpha_t > 0$, $\Sigma_t = \Sigma_t^T$, $(0 \leq t \leq N)$ satisfying the following Riccati equation

$$\Sigma_{t+1} = \hat{A}_t[\Sigma_t^{-1} - \alpha_t \widetilde{E}^T \widetilde{E}]^{-1} \hat{A}_t^T + \alpha_t^{-1} \widehat{H}(t) \widehat{H}^T(t) + J_t, \quad (27)$$

where

$$\alpha_t^{-1} I - \widetilde{E} \Sigma_t \widetilde{E}^T > 0, \quad (28)$$

with

$$G_t \triangleq \begin{bmatrix} BQ_w B^T & 0 \\ 0 & K_t(D + \delta_t D) Q_v (D + \delta_t D)^T K_t^T \end{bmatrix},$$

subject to $|\delta_t| \leq \Delta$,so we have

$$G_t \leq J_t = \begin{bmatrix} BQ_w B^T & 0 \\ 0 & (1+\Delta)^2 K_t D Q_v D^T K_t^T \end{bmatrix}, \quad (29)$$

Lemma 2: If the equations (27) (25) have solutions

$$\Sigma_t, \overline{\Sigma}_t$$

respectively with the initial condition satisfying:

$$\Sigma_0 = \overline{\Sigma}_0,$$

then we have

$$\overline{\Sigma}_t \leq \Sigma_t. \quad (30)$$

Proof: For convenience we define the following equations:

$$S_t(\Sigma_t) \triangleq \hat{A}_t[\Sigma_t^{-1} - \alpha_t \widetilde{E}^T \widetilde{E}]^{-1} \hat{A}_t^T + \alpha_t^{-1} \widehat{H}(t) \widehat{H}^T(t) + J_t,$$

$$h_t(\overline{\Sigma}_t) \triangleq \hat{A}_t[\overline{\Sigma}_t^{-1} - \alpha_t \widetilde{E}^T \widetilde{E}]^{-1} \hat{A}_t^T + \alpha_t^{-1} \widehat{H}(t) \widehat{H}^T(t) + G_t,$$

So by (27) (25) we have

$$\Sigma_{t+1} = S_t(\Sigma_t), \; \overline{\Sigma}_{t+1} = h_t(\overline{\Sigma}_t)$$

with the initial condition that

$$\Sigma_0 = \overline{\Sigma}_0,$$

we prove the lemma by induction. Obviously,

$$\overline{\Sigma}_0 \leq \Sigma_0,$$

Suppose

$$\overline{\Sigma}_t \leq \Sigma_t,$$

then $\overline{\Sigma}_{t+1} = h_t(\overline{\Sigma}_t) \leq S_t(\Sigma_t) = \Sigma_{t+1}$. The proof is completed here.

∇

Lemma 3: For any t, we have the following inequality holds:

$$E[e(t)e^T(t)] \leq \begin{bmatrix} I & -I \end{bmatrix} \Sigma_t \begin{bmatrix} I & -I \end{bmatrix}^T, \quad (31)$$

where $e(t) = x(t) - \hat{x}(t)$ *is the estimation error.*

Proof: This lemma can be easily deduced by lemma 2 from the fact that $\widetilde{\Sigma}_{t+1} \leq \overline{\Sigma}_{t+1} \leq \Sigma_{t+1}$. ∇

Let

$$P(t) \triangleq \begin{bmatrix} I & 0 \end{bmatrix} \Sigma_t \begin{bmatrix} I & 0 \end{bmatrix}^T,$$

then by Definition 2 we have

$$\begin{aligned} P(t+1) &= AP(t)A^T + AP(t)[\alpha_t^{-1} I - P(t)]^{-1} \\ &\quad \times P(t)A^T + BQ_w B^T, \end{aligned} \quad (32)$$

where

$$\alpha_t^{-1} I - P(t) > 0. \quad (33)$$

The initial condition of the equation (32) is

$$P(0) = q_0. \quad (34)$$

with $q_0 = E[x(0)x^T(0)]$ ∇

Remark 1: It can be easily known that the upper Riccati equation is similar to that defined in [7].

Lemma 4: Under the assumption 2, for a given filter (10) and for some scalar $\alpha_t > 0$, the Riccati equation (27) has bounded solutions Σ_t over $[0, N]$ satisfying:

$$\alpha_t^{-1} I - \widetilde{E} \Sigma_t \widetilde{E}^T > 0,$$

then for the same Σ_t there exist bounded solutions $P(t)$ to the Riccati equation (32) over $[0, N]$ satisfying:

$$\alpha_t^{-1} I - P(t) > 0.$$

Proof: Applying the matrix inversion lemma, (27) can be converted into the following form:

$$\begin{aligned} \Sigma_{t+1} &= \hat{A}_t[\Sigma_t + \Sigma_t \widetilde{E}^T (\alpha_t I - \widetilde{E} \Sigma_t \widetilde{E}^T)^{-1} \widetilde{E} \Sigma_t] \hat{A}_t^T \\ &\quad + \alpha_t^{-1} \widehat{H}(t) \widehat{H}^T(t) + J_t \\ &= \hat{A}_t \Sigma_t \hat{A}_t^T + \hat{A}_t \Sigma_t \widetilde{E}^T (\alpha_t I - \widetilde{E} \Sigma_t \widetilde{E}^T)^{-1} \widetilde{E} \Sigma_t \hat{A}_t^T \\ &\quad + \alpha_t^{-1} \widehat{H}(t) \widehat{H}^T(t) + J_t. \end{aligned} \quad (35)$$

By multiplying $[I \; 0]$ to the left side and multiplying $[I \; 0]^T$ to the right side of the Riccati equation (35), we obtain (32). ∇

Our next theorem presents a necessary and sufficient condition for the existence of a quantized quadratic filter with an optimized upper bound of the error variance.

Theorem 1: Under the Assumption 1 and Assumption 2, there exists a quantized quadratic filter that minimizes the bound of the error variance if and only if for some $\alpha_t > 0$, there exist solutions $P(t) = P^T(t) > 0$ over $[0, N]$ to the Riccati equation (32). Under this condition, a quantized quadratic filter with an optimized upper bound of error covariance is given by

$$\widehat{x}(t+1) = A\widehat{x}(t) + K_t z(t), \quad (36)$$

where

$$K_t = AM(t)Q(t)C^T W^{-1}(t), \quad (37)$$

$$M(t) = \begin{bmatrix} I & -I \end{bmatrix} \Sigma_t \begin{bmatrix} I & -I \end{bmatrix}^T, \quad (38)$$

$$Q(t) = I + (\alpha_t^{-1} I - P(t))^{-1} P^T(t), \quad (39)$$

and

$$\begin{aligned} W(t) &= (1+\Delta)^2 D Q_v D^T + \Delta^2 \alpha_t^{-1} C C^T + C P(t) C^T \\ &\quad + C P(t)(\alpha_t^{-1} I - P(t))^{-1} P^T(t) C^T. \end{aligned} \quad (40)$$

Moreover the optimized covariance bound is $M(t)$.

Proof: " $\Rightarrow$ " :

$$\begin{aligned} & M(t+1) \\ = & \begin{bmatrix} I & -I \end{bmatrix} \Sigma_{t+1} \begin{bmatrix} I & -I \end{bmatrix}^T \\ = & (1+\Delta)^2 K_t D Q_v D^T K_t^T + \Delta^2 \alpha_t^{-1} K_t C C^T K_t^T \\ & + K_t C P(t)(\alpha_t^{-1} I - P(t))^{-1} P^T(t) C^T K_t^T \\ & + K_t C P(t) C^T K_t^T - A M(t) C^T K_t^T \\ & - K_t C P(t)(\alpha(t)^{-1} I - P(t))^{-1} M^T(t) A^T \\ & - A M(t)(\alpha_t^{-1} I - P(t))^{-1} P(t)^T C^T K_t^T \\ & - K_t C M^T(t) A^T + B Q_v B^T + A M(t) A^T \\ & + A M(t)(\alpha_t^{-1} I - P(t))^{-1} M(t)^T A^T \\ = & K_t [(1+\Delta)^2 D Q_v D^T + \Delta^2 \alpha_t^{-1} C C^T + C P(t) C^T \\ & + C P(t)(\alpha_t^{-1} I - P(t))^{-1} P^T(t) C^T] K_t^T \\ & - A M(t)[I + (\alpha_t^{-1} I - P(t))^{-1} P^T(t)] C^T K_t^T \\ & - K_t C[M^T(t) + P(t)(\alpha_t^{-1} I - P(t))^{-1} M^T(t)] A^T \\ & + B Q_w B^T + A M(t) A^T \\ & + A M(t)(\alpha_t^{-1} I - P(t))^{-1} M^T(t) A^T \\ = & (K_t + K_*(t)) W(t) (K_t + K_*(t))^T \\ & - A M(t)[I + (\alpha_t^{-1} I - P(t))^{-1} P^T(t)] C^T K_t^T \\ & - K_t [C M(t)^T A^T - C P(t)(\alpha_t^{-1} I - P(t))^{-1} M^T(t) A^T] \\ & + B Q_w B^T + A M(t) A^T - K_t W(t) K_*^T(t) \\ & + A M(t)(\alpha_t^{-1} I - P(t))^{-1} M^T(t) A^T \\ & - K_*(t) W(t) K_t^T - K^*(t) W(t) K_*^T(t), \end{aligned} \quad (41)$$

where $K_*(t) = -A M(t) Q(t) C^T W^{-1}(t)$ it can be seen that if we choose $K_t = -K_*(t)$, then $M(t+1)$ will be minimized. In this condition we have

$$\begin{aligned} M(t+1) &= B Q_w B^T + A M(t) A^T \\ &\quad + A M(t)(\alpha_t^{-1} I - P(t))^{-1} M^T(t) A^T \\ &\quad - A M(t) Q(t) C^T [R_{\alpha_t} + C P(t) Q(t) C^T]^{-1} \\ &\quad \times C Q^T(t) M^T(t) A^T, \end{aligned} \quad (42)$$

where

$$R_{\alpha_t} = (1+\Delta)^2 D Q_v D^T + \Delta^2 \alpha_t^{-1} C C^T. \quad (43)$$

" $\Leftarrow$ ": By Lemma 4 we know that there exists a bounded solution $P_t = P_t^T > 0$, let

$$\Sigma_t = \begin{bmatrix} P(t) & P(t) - M(t) \\ P(t) - M(t) & P(t) - M(t) \end{bmatrix}$$

Then by some straight manipulations, it follows that

$$\begin{aligned} \Sigma_{t+1} &= \hat{A}_t [\Sigma_t + \Sigma_t \widetilde{E}^T (\alpha_t I - \widetilde{E} \Sigma_t \widetilde{E}^T)^{-1} \widetilde{E} \Sigma_t] \hat{A}_t^T \\ &\quad + \alpha_t^{-1} \widehat{H}(t) \widehat{H}^T(t) + J_t \\ &= \hat{A}_t \Sigma_t \hat{A}_k^T + \hat{A}_t \Sigma_t \widetilde{E}^T (\alpha_t I - \widetilde{E} \Sigma_t \widetilde{E}^T)^{-1} \widetilde{E} \Sigma_t \hat{A}_t^T \\ &\quad + \alpha_t^{-1} \widehat{H}(t) \widehat{H}^T(t) + J_t. \end{aligned}$$

By Definition 2, we know (36) is quantized quadratic filter with an upper bound of error covariances $M(t)$. The proof is completed here. ∇

Remark 2: It can be seen that $M(t)$ also depends on α_t. So in order that $M(t)$ is minimized in the sense of matrix norm, we can applying a similar technique as in [6] to find the best α_t.

IV. CONCLUSIONS

In this paper, we use the sector bound approach to treat the quantization error caused by the logarithmic quantizer. We give an algorithm which only involves solving two related Riccati equations to guarantee that the covariances of the estimation error of the finite-horizon estimator is bounded. This approach can also be extended to systems with norm-bounded parameter uncertainties and missing measurements.

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