第47卷 第1期 山东大学学报(理学版) 2012年1月
Vol.47 No.1 Journal of Shandong University(Natural Science) Jan. 2012

文章编号:1671-9352(2012)01-0055-07 DOI:10.6040/j.issn.1671-9352.2012.01.011

Finite-horizon quantized estimation using sector bound approach

WEI Li[1], ZHANG Huan-shui[1*], FU Min-yue[2]
(1. School of Control Science and Engineering, Shandong University, Jinan 250061, Shandong, China;
2. School of Electrical Engineering and Computer Science, The University of Newcastle, NSW 2308, Callaghan, Australia)

Abstract: This paper is concerned with the finite-horizon filtering estimation problem by using the reductive information of the quantized innovations from the innovations. We consider the case where the quantizer is logarithmic, and an upper bound of the estimation error covariance is derived for all the quantized innovations. The calculation of the filter involves solving a Riccati equation related to the quantized innovations.
Key words: discrete-time system; logarithmic quantizer; quantized estimation; sector bound

基于有限时间的扇形界方法的量化估计

魏丽[1],张焕水[1*],付敏跃[2]
(1.山东大学控制科学与工程学院,山东 济南 250061;
2.纽卡斯尔大学电气工程与计算机科学学院,澳大利亚新南威尔士州纽卡斯尔市 2308)

摘要:研究了具有对数量化器的离散时间系统的有限时间量化估计问题。利用扇形界的方法给出了量化误差,进一步设计了有限时间的量化估计器,使得对于由所有的量化新息给出的量化估计误差都在一个有限界之内,并且使得这个界在范数意义下尽可能的小。最后通过求解一个与量化新息有关的黎卡提方程得到了量化估计器。
关键词:离散时间系统;对数量化器;量化估计;扇形界
中图分类号:TP13 文献标志码:A

0 Introduction

Recently, quantized control and quantized estimation problems have been investigated abundantly due to the development of networked control systems especially for industrial control and automation. Examples of quantized feedback control problem include [1]-[4] and references therein. For its applications in the bandwidth-constrained wireless sensor network, the coarsest quantizer that quadratically stabilizes a single input linear discrete-time invariant system is proven to be logarithmic in [1]. In [2] the logarithmic quantizer is considered, it is proved that the logarithmic quantizer performs better than the linear quantizer when it deals with the quantization error, for logarithmic quantizer gives a multiplicative quantization error which reduces as the measurement

Received date: 2011-05-12; **Online publishing date:** 2011-11-12 14:09
Online publishing address: http://www.cnki.net/kcms/detail/37.1389.N.20111112.1409.002.html
Foundation items: Supported by the Taishan Scholar Construction Engineering by Shandong Government; the National Natural Science Foundation for Distinguished Young Scholars of China (60825304); the Major State Basic Research Development Program of China (973 Program) (2009cb320600); Yangtse Rive Scholar Bonus Schemes (31400080963017)
Biography: WEI Li(1983-), Female, Ph.D student, Her current research interests mainly include quantized filtering and control, time delay systems and stochastic systems. Email: weili@mail.sdu.edu.cn
* ZHANG Huan-shui(1963-), Male, Professor, His interests include optimal estimation, robust filtering and control, time delay systems, singular systems, wireless communication, and signal processing. Email: hszhang@sdu.edu.cn

becomes small, while the quantization error of linear quantizer is addictive and grows linearly as the measurement becomes larger. What's more for the the logarithmic quantizer, the quantized feedback control problems can be converted into classical robust control problems with sector bound uncertainties [5] and so on. As in the classical control and estimation theory, state estimation plays an critical role in control theory due to the separation theory [8]. Though only a high resolution separation theorem holds, quantized estimation is also important to quantized feedback control problems [3]. In [9] and [10], the quantized estimation using quantized innovations is derived under the assumption that the conditional probability of the estimated state based on the innovations is the same as that based on the quantized innovations. As in [11], the infinite-level and finite-level quantizers are considered, but only the steady state quantized estimator is given. In this paper, we use the original innovations to recover the logarithmic quantized information to design a state estimator for a single output linear discrete-time invariant system, just as in [12] for the problem of control with communication constraints. [5] proposes a new quantization dependent Lyapunov function to study the problem of analysis and synthesis for quantized feedback control system, which leads to less conservative results. [6] investigates the quantized H_∞ control problem for discrete-time systems with random packet losses. In [7] the optimal tracking design for a linear system with a quantized control input is given. By using dynamic programming approach, the best attainable tracking performance is obtained, in terms of the space equation of given systems and the unique solution of the discrete-time algebraic Riccati equation.

In this paper the quantized quadratic filter of logarithmic quantizer with guaranteed performance has been designed. The paper is organized as follows, section 1 puts forward some assumptions and formulates the quantized estimation problem. Section 2 presents the solution of the estimation problem using the quantized innovations. In section 3, an example is given to show the effectiveness of the proposed approach. Finally, section 4 ends this note with some conclusions.

Notations: Throughout this paper $E[\cdot]$ denotes the expectation and δ_{tl} denotes the Kronecker Delta.

1 Problem formulation

Consider the following linear discrete-time system:

$$x(t+1) = Ax(t) + Bw(t), \tag{1}$$

$$y(t) = Cx(t) + Dv(t), \tag{2}$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R$ is the observation, A, B, C, D are matrices with proper dimensions, $w(t) \in R^m$ and $v(t) \in R^k$ are noises satisfying:

for all integers t and $l \geqslant 0$,

$$\begin{aligned} &E[w(t)] = 0, E[w(t)w^{\mathrm{T}}(l)] = Q_w\delta_{tl}, \\ &E[v(t)] = 0, E[v(t)v^{\mathrm{T}}(l)] = Q_v\delta_{tl}, \\ &E[w(t)v^{\mathrm{T}}(l)] = 0. \end{aligned} \tag{3}$$

Assumption 1

(1) $E[x(0)x^{\mathrm{T}}(0)] = q_0$, *where* $q_0 = q_0^{\mathrm{T}} \geqslant 0$ *is a known matrix*,

(2) $rank[A\ BQ_w^{\frac{1}{2}}] = n.$ (4)

Consider the logarithmic quantizer with infinite levels as following:

$$U = \{\pm u_i : u_i = \rho^i u_0, i = 1, 2, \cdots\} \cup \{\pm u_0\} \cup \{0\},$$

$$0 < \rho < 1,\ u_0 > 0. \tag{5}$$

where ρ is called quantized density of the quantizer. As illustrated in [2], using the sector bound approach the quantization error satisfying:

$$y - Q(y) = \delta y,\ |\delta| \leqslant \Delta. \tag{6}$$

where

$$Q(y)=\begin{cases}u_i,\ if\ \frac{1}{1+\Delta}u_i<y\leqslant\frac{1}{1-\Delta}u_i,\ y>0;\\ 0,\ if\ y=0;\\ -Q(-y),\ if\ y<0.\end{cases} \tag{7}$$

with

$$\Delta=\frac{1-\rho}{1+\rho}. \tag{8}$$

Denote

$$\varepsilon(t)=y(t)-\hat{y}(t),$$
$$e(t)=x(t)-\hat{x}(t),$$

using the quantized innovations to design the filter, then the quantized innovation can be rewritten as:

$$z(t)=Q(\varepsilon(t))=\varepsilon(t)-\delta_t\varepsilon(t)=(C-\delta_tC)e(t)+(D-\delta_tD)v(t), \tag{9}$$

where $\|\delta_t\|\leqslant\Delta$, hence $\|\frac{\delta_t}{\Delta}|\leqslant1$.

The quantized filter design problem can be stated as: *Given the quantized innovations* $\{z(0),z(1),\cdots,z(t)\}$, *design a filter*

$$\hat{x}(t+1)=A\hat{x}(t)+K_tz(t), \tag{10}$$

where K_t *is a matrix to be determined in order that the variance of the estimation error is guaranteed, that is, there exist a sequence of positive-definite matrices* $M(t)=M(t)^{\mathrm{T}}\geqslant0$ $(0\leqslant t\leqslant N)$ *satisfying*:

$$E[(x(t)-\hat{x}(t))(x(t)-\hat{x}(t))^{\mathrm{T}}]\leqslant M(t),$$

and then minimize $M(t)$ *in the sense of norm.*

2 Quantized filter design

It is noted that (9) and (10) involve uncertainties, thus the accurate error covariance is hard to be determined. In this section, we try to find an upper bound for the time-varying estimation error covariance.

By (9) the filter can be written as:

$$\hat{x}(t+1)=A\hat{x}(t)+K_t[(C-\delta_tC)e(t)+(D-\delta_tD)v(t)]=$$
$$A\hat{x}(t)+(1-\delta_t)(K_tCe(t)+K_tDv(t)). \tag{11}$$

Define a new state vector and noise, let

$$\xi(t+1)\triangleq\begin{bmatrix}x(t+1)\\ \hat{x}(t+1)\end{bmatrix}, \tag{12}$$

$$\eta(t)\triangleq\begin{bmatrix}w(k)\\ v(k)\end{bmatrix}. \tag{13}$$

then by (1), (11) and (12), (13), the following auxiliary system holds:

$$\xi(t+1)=(\hat{A}_t+\hat{A}_{et})\xi(t)+\bar{B}\eta(t), \tag{14}$$

where

$$\hat{A}_t=\begin{bmatrix}A & 0\\ K_tC & A-K_tC\end{bmatrix}, \tag{15}$$

$$\bar{B}=\begin{bmatrix}B & 0\\ 0 & (1-\delta_t)K_tD\end{bmatrix}, \tag{16}$$

$$\hat{A}_{et}=\tilde{H}(t)F(t)\tilde{E}, \tag{17}$$

with

$$\tilde{H}(t)=\begin{bmatrix}0\\ \Delta K_tC\end{bmatrix},\ F(t)=\frac{\delta_t}{\Delta},\ \tilde{E}=[-I_n\quad I_n]. \tag{18}$$

It is straightforward to write the Lyapunov equation that governs the evolution of the covariance matrix of (12):

$$\tilde{\Sigma}_{t+1} = E[\xi(t+1)\xi^{\mathrm{T}}(t+1)] = (\hat{A}_t + \hat{A}_{et})\tilde{\Sigma}_t(\hat{A}_t + \hat{A}_{et})^{\mathrm{T}} + G_t, \tag{19}$$

where

$$G_t = \bar{B}E(\eta(t)\eta^{\mathrm{T}}(t))\bar{B}^{\mathrm{T}}.$$

According to (3), we have:

$$E[\eta(t)] = 0,\ E[\eta(t)\eta^{\mathrm{T}}(l)] = \begin{bmatrix} Q_w & 0 \\ 0 & Q_v \end{bmatrix}\delta_{tl}, \tag{20}$$

then G_t is calculated as:

$$G_t = \bar{B}E[\eta(t)\eta^{\mathrm{T}}(t)]\bar{B}^{\mathrm{T}} = \bar{B}\begin{bmatrix} Q_w & 0 \\ 0 & Q_v \end{bmatrix}\bar{B}^{\mathrm{T}} = \begin{bmatrix} BQ_wB^{\mathrm{T}} & 0 \\ 0 & (1-\delta_t)^2K_tDQ_vD^{\mathrm{T}}K_t^{\mathrm{T}} \end{bmatrix}. \tag{21}$$

Next, we want to find a series of positive-definite matrices $\Sigma_{t+1}(0 \leqslant t \leqslant N)$, satisfying:

$$\tilde{\Sigma}_{t+1} \leqslant \Sigma_{t+1}. \tag{22}$$

Lemma 1 [11] Given matrices A_t, H_t, E_t, *and* F_t *with compatible dimensions and* $F_tF_t^{\mathrm{T}} \leqslant I$. *Let* X_t *be a positive definite matrix and* $\alpha_t > 0$ *be an arbitrary constant such that* $\alpha_t^{-1}I - E_tX_tE_t^{\mathrm{T}} > 0$, *then the following inequality holds*:

$$(A_t + H_tF_tE_t)X_t(A_t + H_tF_tE_t)^{\mathrm{T}} \leqslant A_t(X_t^{-1} - \alpha_tE^{\mathrm{T}}(t)E_t)^{-1}A_t^{\mathrm{T}} + \alpha_t^{-1}H_tH_t^{\mathrm{T}}. \tag{23}$$

Using Lemma 1, the following inequality holds:

$$\tilde{\Sigma}_{t+1} = (\hat{A}_t + \hat{A}_{et})\tilde{\Sigma}_t(\hat{A}_t + \hat{A}_{et})^{\mathrm{T}} + G_t \leqslant \hat{A}_t[\tilde{\Sigma}_t^{-1} - \alpha_t\tilde{E}^{\mathrm{T}}\tilde{E}]^{-1}\hat{A}_t^{\mathrm{T}} + \alpha_t^{-1}\hat{H}(t)\hat{H}^{\mathrm{T}}(t) + G_t. \tag{24}$$

In order to deduce the matrices Σ_{t+1}, the following definition is necessary.

Introducing the following Riccati equation:

$$\bar{\Sigma}_{t+1} = \hat{A}_t[\bar{\Sigma}_t^{-1} - \alpha_t\tilde{E}^{\mathrm{T}}\tilde{E}]^{-1}\hat{A}_t^{\mathrm{T}} + \alpha_t^{-1}\hat{H}(t)\hat{H}^{\mathrm{T}}(t) + G_t, \tag{25}$$

where

$$\alpha_t^{-1}I - \tilde{E}\bar{\Sigma}_t\tilde{E}^{\mathrm{T}} > 0. \tag{26}$$

The design of the quantized quadratic filter (12) can be restated as: *If there exist a sequence of numbers* $\alpha_t > 0$, $\Sigma_t = \Sigma_t^{\mathrm{T}}$, $(0 \leqslant t \leqslant N)$ *satisfying the following Riccati equation*

$$\Sigma_{t+1} = \hat{A}_t[\Sigma_t^{-1} - \alpha_t\tilde{E}^{\mathrm{T}}\tilde{E}]^{-1}\hat{A}_t^{\mathrm{T}} + \alpha_t^{-1}\hat{H}(t)\hat{H}^{\mathrm{T}}(t) + J_t, \tag{27}$$

where

$$\alpha_t^{-1}I - \tilde{E}\Sigma_t\tilde{E}^{\mathrm{T}} > 0, \tag{28}$$

with

$$J'_t \triangleq \begin{bmatrix} BQ_wB^{\mathrm{T}} & 0 \\ 0 & K_t(D-\delta_tD)Q_v(D-\delta_tD)^{\mathrm{T}}K_t^{\mathrm{T}} \end{bmatrix},$$

subject to $|\delta_t| \leqslant \Delta$. *The following inequality holds*:

$$J'_t \leqslant J_t = \begin{bmatrix} BQ_wB^{\mathrm{T}} & 0 \\ 0 & (1+\Delta)^2K_tDQ_vD^{\mathrm{T}}K_t^{\mathrm{T}} \end{bmatrix}. \tag{29}$$

The next lemma gives the properties of the solutions of the equations (25) and (27) which is useful in deriving the upper bound of the quantized quadratic filter.

Lemma 2 *If the equations* (25) (27) *have solutions* $\bar{\Sigma}_t$ Σ_t, *trespectively with the initial states* $\bar{\Sigma}_0$ Σ_0, *satisfying* $\bar{\Sigma}_0 = \Sigma_0$, *then there holds*:

$$\bar{\Sigma}_t \leqslant \Sigma_t.$$

Proof For convenience of the proof, define the following operators:

$$h_t(\bar{\Sigma}_t) \triangleq \hat{A}_t\left[\bar{\Sigma}_t^{-1} - \alpha_t \tilde{E}^{\mathrm{T}}\tilde{E}\right]^{-1}\hat{A}_t^{\mathrm{T}} + \alpha_t^{-1}\hat{H}(t)\hat{H}^{\mathrm{T}}(t) + G_t,$$

$$S_t(\Sigma_t) \triangleq \hat{A}_t\left[\Sigma_t^{-1} - \alpha_t \tilde{E}^{\mathrm{T}}\tilde{E}\right]^{-1}\hat{A}_t^{\mathrm{T}} + \alpha_t^{-1}\hat{H}(t)\hat{H}^{\mathrm{T}}(t) + J_t,$$

so by (25) (27) the following holds

$$\bar{\Sigma}_{t+1} = h_t(\bar{\Sigma}_t), \ \Sigma_{t+1} = S_t(\Sigma_t),$$

with the initial condition that

$$\Sigma_0 = \bar{\Sigma}_0,$$

we prove the lemma by induction. Obviously,

$$\bar{\Sigma}_0 \leqslant \Sigma_0.$$

Suppose

$$\bar{\Sigma}_t \leqslant \Sigma_t,$$

then $\bar{\Sigma}_{t+1} = h_t(\bar{\Sigma}_t) \leqslant S_t(\Sigma_t) = \Sigma_{t+1}$. The proof is completed here.

From lemma 2, the upper bound of the estimation error is given as follows:

Lemma 3 *For any t, the following inequality holds*:

$$E\left[e(t)\mathrm{e}^{\mathrm{T}}(t)\right] \leqslant [I \quad -I]\Sigma_t[I \quad -I]^{\mathrm{T}}, \tag{30}$$

where $e(t) = x(t) - \hat{x}(t)$ is the estimation error.

Proof According to lemma 2 with the fact that $\tilde{\Sigma}_{t+1} \leqslant \bar{\Sigma}_{t+1} \leqslant \Sigma_{t+1}$, the conclusion follows.

Let $P(t) \triangleq [I \quad 0]\Sigma_t[I \quad 0]^{\mathrm{T}}$, then by (27) the following recursions of $P(t)$ holds:

$$P(t+1) = AP(t)A^{\mathrm{T}} + AM(t)\left[\alpha_t^{-1}I - M(t)\right]^{-1} \times M(t)A^{\mathrm{T}} + BQ_wB^{\mathrm{T}}, \tag{31}$$

The initial condition of the equation (31) is

$$P(0) = q_0, \tag{32}$$

with $q_0 = E\left[x(0)x^{\mathrm{T}}(0)\right]$ and $M(0) = 0$. where

$$\alpha_t^{-1}I - M(t) > 0. \tag{33}$$

Lemma 4 *Under the assumption* 1, *for a given filter* (10) *and for some scalar $\alpha_t > 0$, the Riccati equation* (27) *has bounded solutions Σ_t over $[0, N]$ satisfying*:

$$\alpha_t^{-1}I - \tilde{E}\Sigma_t\tilde{E}^{\mathrm{T}} > 0,$$

then for the same Σ_t there exist bounded solutions $P(t)$ to the Riccati equation (31) *over $[0, N]$ satisfying*:

$$\alpha_t^{-1}I - P(t) > 0.$$

Proof According to the matrix inversion lemma, (27) can be converted into the following form:

$$\Sigma_{t+1} = \hat{A}_t\left[\Sigma_t + \Sigma_t\tilde{E}^{\mathrm{T}}(\alpha_t I - \tilde{E}\Sigma_t\tilde{E}^{\mathrm{T}})^{-1}\tilde{E}\Sigma_t\right]\hat{A}_t^{\mathrm{T}} + \alpha_t^{-1}\hat{H}(t)\hat{H}^{\mathrm{T}}(t) + J_t =$$

$$\hat{A}_t\Sigma_t\hat{A}_t^{\mathrm{T}} + \hat{A}_t\Sigma_t\tilde{E}^{\mathrm{T}}(\alpha_t I - \tilde{E}\Sigma_t\tilde{E}^{\mathrm{T}})^{-1}\tilde{E}\Sigma_t\hat{A}_t^{\mathrm{T}} + \alpha_t^{-1}\hat{H}(t)\hat{H}^{\mathrm{T}}(t) + J_t. \tag{34}$$

Pre-and post-multiplying the Riccati equation (34) by $[I \quad 0]$ and $[I \quad 0]^{\mathrm{T}}$, we obtain (31).

The next theorem presents a necessary and sufficient condition for the existence of a quantized quadratic filter with an optimized upper bound of the error variance.

Theorem 1 *Under Assumption* 1, *there exists a quantized quadratic filter that minimizes the bound of the error variance if and only if for some $\alpha_t > 0$, the Riccati equation* (31) *has solutions $P(t) = P^{\mathrm{T}}(t) > 0$ over $[0, N]$. Under this condition, the quantized quadratic filter* (10) *with*

$$K_t = AM(t)Q(t)C^{\mathrm{T}}W^{-1}(t), \tag{35}$$

where

$$Q(t) = I + (\alpha_t^{-1}I - M(t))^{-1}M^{\mathrm{T}}(t), \tag{36}$$

$$M(t) = [I \quad -I]\Sigma_t[I \quad -I]^{\mathrm{T}}, \tag{37}$$

$$W(t) = (1+\Delta)^2 DQ_vD^{\mathrm{T}} + \Delta^2\alpha_t^{-1}CC^{\mathrm{T}} + CM(t)C^{\mathrm{T}} + CM(t)(\alpha_t^{-1}I - M(t))^{-1}M^{\mathrm{T}}(t)C^{\mathrm{T}}. \tag{38}$$

satisfies the optimized covariance bound is $M(t)$.

Proof Suppose there exists a quantized quadratic filter for the system. It follows from that there exists a bounded solutions $\Sigma_t \geqslant 0$ to (27) and such that $\alpha_t^{-1} I - E(t)\Sigma_t E^{\mathrm{T}}(t) > 0$ over $[0, N]$. Thus, from lemma 4 there exist bounded positive definite solutions $P(t)$ $M(t)$ to the Riccati equation (31) satisfying $\alpha_t^{-1} I - M(t) > 0$ over $[0, N]$. From lemma 3, we know that $E[e(t)e^{\mathrm{T}}(t)] \leqslant [I \quad -I]\Sigma_t \begin{bmatrix} I \\ -I \end{bmatrix}$. Next a necessary condition for the quantized quadratic filter with guaranteed cost on the variance of the estimation error is derived.

$$\begin{aligned} M(t+1) = {} & [I \quad -I]\Sigma_{t+1}[I \quad -I]^{\mathrm{T}} = (1+\Delta)^2 K_t D Q_v D^{\mathrm{T}} K_t^{\mathrm{T}} + \Delta^2 \alpha_t^{-1} K_t C C^{\mathrm{T}} K_t^{\mathrm{T}} + B Q_w B^{\mathrm{T}} + \\ & (A - K_t C) M(t) (A - K_t C)^{\mathrm{T}} + (A - K_t C) M(t) (\alpha_t^{-1} I - M(t))^{-1} M(t) (A - K_t C)^{\mathrm{T}} = \\ & K_t [(1+\Delta)^2 D Q_v D^{\mathrm{T}} + \Delta^2 \alpha_t^{-1} C C^{\mathrm{T}} + C M(t) C^{\mathrm{T}} + C M(t) (\alpha_t^{-1} I - M(t))^{-1} M^{\mathrm{T}}(t) C^{\mathrm{T}}] K_t^{\mathrm{T}} - \\ & A M(t) [I + (\alpha_t^{-1} I - M(t))^{-1} M^{\mathrm{T}}(t)] C^{\mathrm{T}} K_t^{\mathrm{T}} - K_t C [I + M(t)(\alpha_t^{-1} I - M(t))^{-1}] M^{\mathrm{T}}(t) A^{\mathrm{T}} + \\ & B Q_w B^{\mathrm{T}} + A M(t) A^{\mathrm{T}} + A M(t) (\alpha_t^{-1} I - M(t))^{-1} M^{\mathrm{T}}(t) A^{\mathrm{T}} = \\ & (K_t + K_*(t)) W(t) (K_t + K_*(t))^{\mathrm{T}} - A M(t) [I + (\alpha_t^{-1} I - M(t))^{-1} M^{\mathrm{T}}(t)] C^{\mathrm{T}} K_t^{\mathrm{T}} - \\ & K_t [C M(t)^{\mathrm{T}} A^{\mathrm{T}} - C M(t) (\alpha_t^{-1} I - M(t))^{-1} M^{\mathrm{T}}(t) A^{\mathrm{T}}] + \\ & B Q_w B^{\mathrm{T}} + A M(t) A^{\mathrm{T}} - K_t W(t) K_*^{\mathrm{T}}(t) + A M(t) (\alpha_t^{-1} I - M(t))^{-1} M^{\mathrm{T}}(t) A^{\mathrm{T}} - \\ & K_*(t) W(t) K_t^{\mathrm{T}} - K_*(t) W(t) K_*^{\mathrm{T}}(t), \end{aligned} \tag{39}$$

where $K_*(t) = -A M(t) Q(t) C^{\mathrm{T}} W^{-1}(t)$. It can be seen that chosen $K_t = -K_*(t)$, then $M(t+1)$ will be minimized. Under this condition

$$\begin{aligned} M(t+1) = {} & B Q_w B^{\mathrm{T}} + A M(t) A^{\mathrm{T}} + A M(t)(\alpha_t^{-1} I - M(t))^{-1} M^{\mathrm{T}}(t) A^{\mathrm{T}} - \\ & A M(t) Q(t) C^{\mathrm{T}} W^{-1}(t) C Q^{\mathrm{T}}(t) M(t) A^{\mathrm{T}}, \end{aligned} \tag{40}$$

where the initial value of $M(t)$ is $M(0) = 0$.

By Lemma 4 there exists a bounded solution $P(t) = P^{\mathrm{T}}(t) > 0$, let

$$\Sigma_t = \begin{bmatrix} P(t) & P(t) - M(t) \\ P(t) - M(t) & P(t) - M(t) \end{bmatrix}.$$

Then by some manipulations, it follows that

$$\begin{aligned} & A_t [\Sigma_t + \Sigma_t \tilde{E}^{\mathrm{T}} (\alpha_t I - \tilde{E}\Sigma_t \tilde{E}^{\mathrm{T}})^{-1} \tilde{E}\Sigma_t] \hat{A}_t^{\mathrm{T}} + \alpha_t^{-1} \hat{H}(t) \hat{H}^{\mathrm{T}}(t) + J_t = \\ & \hat{A}_t \Sigma_t \hat{A}_k^{\mathrm{T}} + \hat{A}_t \Sigma_t \tilde{E}^{\mathrm{T}} (\alpha_t I - \tilde{E}\Sigma_t \tilde{E}^{\mathrm{T}})^{-1} \tilde{E}\Sigma_t \hat{A}_t^{\mathrm{T}} + \alpha_t^{-1} \hat{H}(t) \hat{H}^{\mathrm{T}}(t) + J_t. \end{aligned}$$

which equals to Σ_{t+1}. By (27), we know (10) with the optimal gain (35) is quantized quadratic filter with an upper bound of error covariances $M(t)$. The proof is completed here.

Remark 1 *It can be seen that $M(t)$ also depends on the time-varying parameter α_t, so in order that $M(t)$ is minimized in the sense of matrix norm, we can applying a similar technique as in [13] to find the best α_t.*

3 Simulation

In this part, an example is given to verify the obtained theoretical results.

Consider the following system: $A = \begin{bmatrix} 0.2 & 1 \\ 0 & 1.05 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $D = 1$, the statistical properties of the noises $w(t)$ $v(t)$ are $Q_w = 0.04$, $Q_v = 0.01$, the quantization density is taken as $\rho = 0.25$, $\Delta = 0.6$, α_t is chosen to be a constant with $\alpha = 0.2$, the initial state of the system is $x_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$, using the approach discussed in the paper, we get the gain with the error covariance as $M(t)$, what we want to show is that the covariance is guaranteed with the upper bound $W(t)$, that is the $W(t) \leqslant M(t)$. The following two figures show the eigenvalues of $M(t) - W(t)$, from which it is easily to know that the $M(t) - W(t)$ is semi-positive definite which is equivalent to $W(t) \leqslant M(t)$.

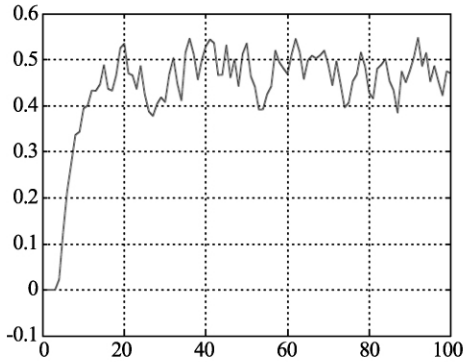

Fig. 1 The first set of eigenvalues of the 2 × 2 matrix $M(t) - W(t)$

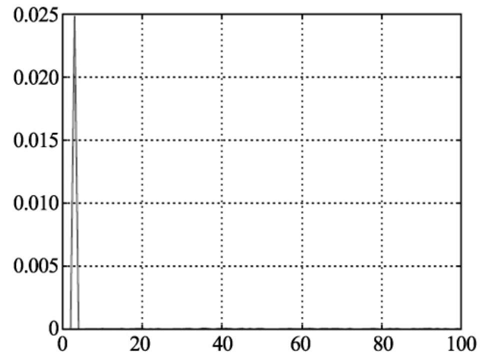

Fig. 2 The second set of eigenvalues of the 2 × 2 matrix $M(t) - W(t)$

4 Conclusion

In this paper, the sector bound approach is used to characterize the quantization error caused by a logarithmic quantizer. An algorithm which only involves solving two related Riccati equations has been given to guarantee that the covariances of the quantized estimation error of the finite-horizon estimator is bounded. This approach can also be extended to systems with norm-bounded parameter uncertainties and missing measurements.

References:

[1] ELIA N, MITTER S. Stabilization of linear systems withlimited information [J]. IEEE Transactions on Automatic Control, 2001, 46(9): 1384-1400.

[2] FU M, XIE L. The sector bound approach to quantizedfeedback control [J]. IEEE Transactions on Automatic Control, 2005, 50(11): 1698-1711.

[3] FU M. Linear quadratic gaussian control with quantized feedback [C] / / Proceeding American Control Conference. Washington: IEEE Computer Society, 2009: 2172-2177.

[4] FU M, XIE L. Finite-level quantized feedback controlfor linear systems [J]. IEEE Transactions on Automatic Control, 2009, 54(5): 1165-1170.

[5] GAO H, CHEN T. A new approach to quantizedfeedback control systems [J]. Automatica, 2008, 44(2): 534-542.

[6] CHE Weiwei, YANG Guanghong. Quantized dynamic output feedback H controller design [C] // Proceedings of the 26th Chinese Control Conference. Washington: IEEE Computer Society, 2007: 1318-1323.

[7] QI T, SU W. Tracking performance limitation of a linear MISO unstable system with quantized control signals [C] // Proceedings of the 10th International Conference on Control, Automation, Robotics and Vision. Washington: IEEE Computer Society, 2008: 1424-1429.

[8] ANDERSON B, MOORE J B. Optimal control: linear quadratic methods [M]. Englewood Cliffs, NJ: Prentice-Hall, 1990.

[9] RIBEIRO A, GIANNAKIS G, ROUMELIOTIS S. SOI-KF: distributed kalman filteringwith low-cost communications using the Sign of Innovations [J]. IEEE Transactions on Signal Processing, 2006, 54(12): 4782-4795.

[10] YOU K, XIE L, SUN S, et al. Multiple-level quantized innovation kalman filter [C] // Proceedings of the 17th International Federation of Automatic Control. [S. l.]: [s. n.], 2008: 1420-1425.

[11] FU M, SOUZA C E. State estimation for linear discrete-time systems using quantized measurements [J]. Automatica, 2009, 45(12): 2937-2945.

[12] TATIKONDA S, SAHAI A, MITTER S K. Control of LQG systems under communication constraints [C] // Proceedings of the 37th IEEE Conference on Control Decision Systems IEEE. Washington: IEEE Computer Society, 1998: 1165-1170.

[13] WANG Z, ZHU J, UNBEHAUEN H. Robust filter design with time-varying parameter uncertainty and error variance constraints [J]. International Journal of Control, 1999, 72(1): 30-38.

[14] ZHU X, SOH Y, XIE L. Design and analysis of discrete-time robust kalman filters [J]. Automatica, 2002, 38(6): 1069-1077.

[15] FU M, SOUZA CE, LUO Z. Finite horizon robust kalman filter design [J]. IEEE Transactions on Signal Processing, 2001, 49(9): 2103-2112.

(编辑: 许力琴)