Wireless Communications Systems with Spatial Diversity: A Volumetric Approach

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Abstract—This paper presents a new physical modelling approach for wireless systems with multiple antennas. The fundamental problem of modelling the communications channel if we are given an arbitrary spatial volume for transmitting, an arbitrary spatial volume for receiving, and a set of scattering bodies is studied. We show how to calculate the number of communication modes, both for direct (point-to-point) and for scattering environments. Our work explains the physical parameters which determine the channel model and its channel capacity.

I. INTRODUCTION

Scattering has been shown to improve capacity for wireless communications. More specifically, the capacity of a multi-antenna system with dense scattering is shown to grow proportionally with the minimum of the number of transmitters and receivers [1], [2]. This is the well-known linear growth property of MIMO systems.

Much of the work on multi-antenna systems assumes that the channel model is an identically distributed matrix [1], [2]. Recently some work has focussed on the physical modelling of multi-antenna channels. In [3], an experimental system was developed and tested for indoor environments to confirm the increase in channel capacity when multiple antennas are used. The work of [4] suggests that a limit to the linear capacity growth exists in outdoor environments, where the scattering is sparse.

This paper attempts to provide a more realistic physical model for multi-antenna systems. The fundamental question we ask is: how do we model the communications channel if we are given a spatial volume for transmitting, a spatial volume for receiving, and a set of scattering bodies? In addressing this question, we determine the number of communications modes (or degrees of freedom) for radio communications between two volumes. Surprisingly, the case without scattering bodies has already been studied in [5] (and its references) for optical communications in free space and complete solutions are given. It is interesting that the setting in [5] is very general and the results are equally applicable to radio communications.

We will first summarize the work of [5] in section II for the two body case. We then extend [5] in II-C to provide a numerical solution for channel modelling with arbitrary transmit and receive volumes. Section III provides a generalization of these results to include scattering bodies. Illustrative examples are used in section IV to compare direct transmission with transmission through a single scattering body as well as multiple scattering bodies.

II. MODELLING FOR TWO VOLUME COMMUNICATIONS

Consider wireless communications between two arbitrary volumes as depicted in figure 1, where $V_T$ is the transmitting body and $V_R$ is the receiving body. The centres of the two bodies are separated by a distance $r$. It is assumed that the dimensions of the two bodies are small compared to the distance $r$. We choose the coordinates $(x, y, z)$ such that the $z$-axis is along the distance $r$ and the origin is the centre of the transmitting body. For simplicity, we consider monochromatic signals, i.e., signals with a single frequency. The work in this paper can be easily generalized to the case where a finite frequency bandwidth is available.

It is shown in [5] that if $V_T$ and $V_R$ are hyper-rectangles with sides parallel to the $x, y, z$ axes and lengths $2\Delta x_T, 2\Delta y_T, 2\Delta z_T$ and $2\Delta x_R, 2\Delta y_R, 2\Delta z_R$, there is a simple expression for the transfer function from $V_T$ to $V_R$. Indeed, the eigenfunctions for $V_T$ (and those for $V_R$) can be expressed in terms of a focusing function and a set of prolate spheroidal functions. These eigenfunctions form a set of complete, orthonormal functions. Further, the corresponding singular values have roughly a constant value up to a critical number after which the singular values are negligible. This critical number is the number of communications modes between $V_T$ and $V_R$, given by

$$N_c = \frac{V_R V_T}{r^2 \lambda^2 (2\Delta z_T)(2\Delta z_R)} \quad (1)$$

where (with slight abuse of notation) $V_T$ and $V_R$ are the volumes of the two bodies and $\lambda$ is the wavelength of the transmit signal. Consequently, the transfer function between $V_T$ and $V_R$ is given by a diagonal matrix of a finite dimension, with the diagonal elements being the non-negligible singular values.

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Fig. 1. Two volume arrangement
A. Transfer Function

Let $\psi(\mathbf{r}_T)$ represent the source function at a point $\mathbf{r}_T \in V_T$ and $\phi(\mathbf{r}_R)$ be the received electromagnetic field at a point $\mathbf{r}_R \in V_R$. Then, we have

$$\phi(\mathbf{r}_R) = \int_{V_T} G(\mathbf{r}_R, \mathbf{r}_T) \psi(\mathbf{r}_T) d^3\mathbf{r}_T$$

(2)

where $G(\mathbf{r}_R, \mathbf{r}_T)$ is the (retarded) Green’s function defined by

$$G(\mathbf{r}_R, \mathbf{r}_T) = \frac{\exp(-ik|\mathbf{r}_R - \mathbf{r}_T|)}{4\pi|\mathbf{r}_R - \mathbf{r}_T|}$$

(3)

where $k = \sqrt{-1}$, $k = 2\pi/\lambda$, and $\lambda$ is the signal wavelength.

In order to calculate connection strengths between $\psi(\mathbf{r}_T)$ and $\phi(\mathbf{r}_R)$ we require a set of basis functions. Define $\{\alpha_{Ti}(\mathbf{r}_T), i = 1, \cdots, N_T\}$ as a finite set of orthonormal functions in $V_T$. Likewise, define $\{\alpha_{Rj}(\mathbf{r}_R), j = 1, \cdots, N_R\}$ to be a finite orthonormal set of functions in $V_R$. Then, given any transmitting function $\alpha_{Ti}(\mathbf{r}_T)$, the received signal is given by application of (2):

$$\phi_i(\mathbf{r}_R) = \int_{V_T} G(\mathbf{r}_R, \mathbf{r}_T) \alpha_{Ti}(\mathbf{r}_T) d^3\mathbf{r}_T$$

(4)

We may then decompose $\phi_i(\mathbf{r}_R)$ into components of $\alpha_{Rj}(\mathbf{r}_R)$. Define the projection of $\phi_i(\mathbf{r}_R)$ on $\alpha_{Rj}(\mathbf{r}_R)$ as:

$$\gamma_{ji} = \int_{V_R} \alpha_{Rj}^*(\mathbf{r}_R) \phi_i(\mathbf{r}_R) d^3\mathbf{r}_R$$

$$= \int_{V_R} \int_{V_T} \alpha_{Rj}^*(\mathbf{r}_R) G(\mathbf{r}_R, \mathbf{r}_T) \alpha_{Ti}(\mathbf{r}_T) d^3\mathbf{r}_T d^3\mathbf{r}_R$$

(5)

Then, we have

$$\phi_i(\mathbf{r}_R) = \sum_{j=1}^{N_R} \gamma_{ji} \alpha_{Rj}(\mathbf{r}_R) + \delta\phi_i(\mathbf{r}_R)$$

(6)

where $\delta\phi_i(\mathbf{r}_R)$ is a residual term orthogonal to all $\alpha_{Rj}(\mathbf{r}_R)$.

Take any transmitting signal as

$$\psi(\mathbf{r}_T) = \sum_{i=1}^{N_T} a_i \alpha_{Ti}(\mathbf{r}_T)$$

(7)

The received signal will be

$$\phi(\mathbf{r}_R) = \sum_{i=1}^{N_T} a_i \left( \sum_{j=1}^{N_R} \gamma_{ji} \alpha_{Rj}(\mathbf{r}_R) \right) + \delta\phi(\mathbf{r}_R)$$

$$= \sum_{j=1}^{N_R} b_j \alpha_{Rj}(\mathbf{r}_R) + \delta\phi(\mathbf{r}_R)$$

(8)

where $\delta\phi(\mathbf{r}_R)$ is a residual signal orthogonal to all $\alpha_{Rj}(\mathbf{r}_R)$. Let $a_i$ and $b_j$ be the coefficients of the projections of $\psi(\mathbf{r}_T)$ onto $\alpha_{Ti}(\mathbf{r}_T)$ and $\phi(\mathbf{r}_R)$ onto $\alpha_{Rj}(\mathbf{r}_R)$, respectively. Then, we have

$$b_j = \sum_{i=1}^{N_T} a_i \gamma_{ji} \quad \text{or} \quad b = \Gamma a$$

(9)

If $N_T = N_R \to \infty$ and the sets $\{\alpha_{Ti}(\mathbf{r}_T)\}$ and $\{\alpha_{Rj}(\mathbf{r}_R)\}$ are complete, then any transmitting signal can be expressed as in (8) without the residual term $\delta\phi(\mathbf{r}_R)$. In this case, (9) represents the true transfer function of the communications system, regardless of the choice of the basis functions $\{\alpha_{Ti}(\mathbf{r}_T)\}$ and $\{\alpha_{Rj}(\mathbf{r}_R)\}$.

B. Eigenfunctions

Because $r$ is large compared to $V_T$ and $V_R$, we have [5]:

$$|\mathbf{r}_R - \mathbf{r}_T| \approx r + z_R - z_T + \frac{(x_R - x_T)^2 + (y_R - y_T)^2}{2r}$$

(10)

This leads to a separation of $\psi(\mathbf{r}_T)$:

$$\psi(\mathbf{r}_T) = F_T(\mathbf{r}_T) \beta_T(x_T, y_T)$$

(11)

where $\beta_T(x_T, y_T)$ are new functions and $F_T(\cdot)$ is the so-called focusing function defined by

$$F_T(\mathbf{r}_T) = \exp \left( -ik(z_T - \frac{x_T^2}{2r} - \frac{y_T^2}{2r}) \right)$$

(12)

We can carry out a similar separation for the eigenfunctions $\phi_i(\mathbf{r}_R)$.

C. Numerical Solutions

To motivate our numerical solutions, we first consider the case where both $V_T$ and $V_R$ are hyper-rectangles with faces parallel to the $x, y, z$ axes. Suppose the dimensions of $V_T$ are given by $2\Delta x_T, 2\Delta y_T$ and $2\Delta z_T$, and similarly for $V_R$. This case is analysed in [5] and the solution is given as follows:

$$\beta_{T(m,n)}(x_T, y_T) = S_{0m}(c_x, \hat{x}_T) S_{0n}(c_y, \hat{y}_T)$$

(13)

for $m, n = 0, 1, \cdots$, where

$$c_x = \frac{k\Delta x_T}{r}, \quad \hat{x}_T = \frac{x_T}{\Delta x_T}$$

(14)

and similarly for $c_y$ and $\hat{y}_T$, and $S_{0m}(c, \xi)$ is the so-called (0, m)th angular prolate spheroidal function given by

$$v_m S_{0m}(c_x, \xi) = \int_{\xi}^{1} \frac{\sin(c(\xi - \xi'))}{\pi(\xi - \xi')} S_{0m}(c, \xi') d\xi'$$

(15)

where $v_m$ are the singular values with $1 > |v_1| > |v_2| > \cdots > 0$. Only a finite number of $v_i$ have non-negligible values. The functions $S_{0m}(c, \xi)$ are well studied. Computational methods are available in [6] and [7]. The functions $\beta_{T(m,n)}(\mathbf{r}_R)$ can be expressed in a similar fashion. To obtain $\beta_T(\mathbf{r}_T)$ from (13),
we simply note that the basis functions have been truncated to $N_{T_x} \times N_{T_y}$ non-negligible terms. Hence, we have

$$i \leftrightarrow mN_{T_y} + n, \ 0 \leq m < N_{T_x}, \ 0 \leq n < N_{T_y}$$

(16)

Now we return to the general case where $V_T$ and $V_R$ take arbitrary shapes. We project $V_T$ onto the $(x, y)$ plane to obtain a surface $S_T$. Then take $S_T$ to be the smallest bounding rectangle which is parallel to the $(x, y)$ axes. Let $2\Delta x_T$ and $2\Delta y_T$ be the lengths of $S_T$ in $x$ and $y$ directions, respectively. We do the same to $V_R$ and obtain $S_R, S_R, 2\Delta x_R$ and $2\Delta y_R$. Compute $\tilde{\beta}_T(x_T, y_T)$ as in (13) and similarly for $\tilde{\beta}_R(x_T, y_T)$ but for $(S_T, S_R)$.

Any function $\beta_T(x_T, y_T)$ in $S_T$ can be expressed as a linear combination of $\tilde{\beta}_T(x_T, y_T), i = 1, 2, \cdots$. That is, $\{\tilde{\beta}_T(x_T, y_T)\}$ is also complete in $S_T$ (but not necessarily orthogonal or orthonormal). To see this, we extend $\beta_T(x_T, y_T)$ to the whole set of $S_T$ by setting its value to zero outside of $S_T$. Then, this extended function can be expressed as a linear combination of $\tilde{\beta}_T(x_T, y_T)$ over $S_T$ due to the fact that $\{\tilde{\beta}_T(x_T, y_T)\}$ is a complete set in $S_T$. Obviously, the same combination still holds over $S_T$ because $S_T \subset S_T$. Hence, our claim is valid. A similar claim holds for $\beta_R(x_T, y_T)$.

Next, we transform the complete set $\{\tilde{\beta}_T(x_T, y_T)\}$ to another set $\{\tilde{\beta}_T(x_T, y_T)\}$ which is complete and orthonormal in $S_T$. This can be done, using the Gram-Schmidt process as follows: Take

$$\tilde{\beta}_T(x, y) = \frac{\tilde{\beta}_T(x_T, y_T)}{\|\tilde{\beta}_T(x_T, y_T)\|_{S_T}}$$

where

$$\|\tilde{\beta}_T(x_T, y_T)\|_{S_T} = \int_{S_T} \tilde{\beta}_T(x_T, y_T)\tilde{\beta}_T(x_T, y_T)dxdy_T$$

Then, for $i = 2, 3, \cdots, N$ we define:

$$\tilde{\beta}_T = \tilde{\beta}_T - \sum_{j=1}^{i-1} \langle \tilde{\beta}_T, \tilde{\beta}_T \rangle \tilde{\beta}_T$$

If the remaining function $\tilde{\beta}_T$ is zero, ignore it; otherwise, normalise it (in $S_T$) and we obtain $\tilde{\beta}_T(x_T, y_T)$. Transform the set $\{\tilde{\beta}_R(x_T, y_T)\}$ to $\{\tilde{\beta}_R(x_T, y_T)\}$ in a similar way.

Now define

$$\tilde{\psi}_i(r_T) = F_T(r_T)\tilde{\beta}_T(x_T, y_T)$$

and similarly for $\tilde{\phi}_i(r_R)$. We want to compute the transfer coefficient $\gamma_{ij}$ from $\tilde{\psi}_i(r_T)$ in $V_T$ to $\tilde{\phi}_j(r_R)$ in $V_R$.

Again, we use the approximation (10) and obtain

$$G(r_R, r_T) \approx \frac{\exp(-ikr)}{4\pi r} F_T(r_T) F_R(r_T)$$

$$\cdot \exp \left( \frac{ik}{r} (x_T x + y_T y) \right)$$

(19)

It follows that

$$\gamma_{ji} = \frac{\exp(-ikr)}{4\pi r} \int_{S_T} \tilde{\beta}_R(x_T, y_T)$$

$$\cdot \exp \left( \frac{ik}{r} (x_T x + y_T y) \right) \tilde{\beta}_T(x_T, y_T)dxdy_T$$

(20)

Since the integrand is independent of $z_T$ and $z_R$, we simplify (20) to:

$$\gamma_{ji} = \frac{\exp(-ikr)}{4\pi r} \int_{S_T} 2\Delta z_T(x_T, y_T)$$

$$\cdot f(x_T, y_T)dxdy_T$$

(21)

$$f(x_T, y_T) = \int_{S_T} 2\Delta z_T(x_T, y_T)$$

$$\cdot \exp \left( \frac{ik}{r} (x_T x + y_T y) \right) \tilde{\beta}_T(x_T, y_T)dxdy_T$$

(22)

where $2\Delta z_T(x_T, y_T)$ is the thickness of $V_T$ in the $z$ direction at $(x_T, y_T)$, and $2\Delta z_R(x_T, y_T)$ is similarly defined.

D. Simplification of Transfer Function

In practice, we only need to compute a finite number of terms of $\gamma_{ji}$, i.e., we obtain a truncated $\Gamma$. This raises the question of how to check if the numerical solution is sufficiently accurate. To answer this, we resort to the well-known sum rule [5]: Suppose $\{\tilde{\psi}_i(r_T)\}$ and $\{\tilde{\phi}_j(r_R)\}$ are any two sets of complete orthonormal basis functions for $V_T$ and $V_R$, respectively, then under the assumption that the dimensions of $V_T$ and $V_R$ are sufficiently small compared to the separating distance $r$, we have

$$\sum_{i,j} |\gamma_{ji}|^2 \approx \frac{V_T V_R}{(4\pi r)^2}$$

(23)

We also note a simple fact that if we truncate $\Gamma$, then the sum of $|\gamma_{ji}|^2$ always reduces.

Hence, if (23) holds sufficiently close, we know that the numerical solution is sufficiently accurate. If the sum of $|\gamma_{ji}|^2$ is too small, then we need to compute a larger $\Gamma$, that is, calculate a larger number of basis functions. As a rule of thumb, we may start the size of the truncated $\Gamma$ to be $N_c \times N_c$, where $N_c$ is the number of communicable modes for $(S_T, S_R)$.

In general, $\tilde{\psi}_i(r_T)$ and $\tilde{\phi}_j(r_R)$ are not the eigenfunctions for the transfer matrix between $V_T$ and $V_R$. In order to obtain a minimal representation of $\Gamma$, we simply need to carry out a singular value decomposition, i.e., we form

$$\Gamma = U^* \Lambda V$$

(24)

where $U$ and $V$ are unitary matrices and $\Lambda$ is a diagonal matrix containing the singular values of $\Gamma$ in a descending order.
III. MODELLING WITH SCATTERING BODIES

To motivate our discussion, we first consider the following thought experiment: A two volume communications system has an infinite plane $P_z$ placed at point $z$ between $V_T$ and $V_R$ with an arbitrary normal direction $n$. The plane absorbs all incident energy except for a hole $S(z,n)$ in the middle. We ask how large must this hole be so that communications between the two volumes is not changed?

If $\lambda$ is sufficiently small, then $S(z,n)$ is the minimum area containing all the intersection points of $P_z$ and $\ell(r_T,r_R)$, where $\ell(r_T,r_R)$ is the line linking $r_T \in V_T$ and $r_R \in V_R$. If $\lambda$ is not sufficiently small, $S(z,n)$ needs to be enlarged by several wavelengths in each direction so that diffractions on the edges will be negligible. We will call $S(z,n)$ the viewing area (with respect to $P_z$). The viewing area is the minimum area such that for every point in $V_T$, we may draw a line to every point in $V_R$, and vice versa. We note that the viewing area is a minimum if $P_z$ is normal to the $z$ direction and is enlarged as $P_z$ is tilted.

A. Single Scatterer

Now we consider the scenario where there is a single scatterer $S$, as depicted in figure 2. For simplicity, we assume that $S$ is a purely reflective plane, i.e., there is no penetration of electric field. The reflection angle $\theta_r$ is the same as the incident angle $\theta_i$, but the reflective electric field involves some possible loss which is represented by:

$$
\eta(\theta_i) = \exp \left\{ \left( \frac{\pi \sigma_h \cos \theta_i}{\lambda} \right)^2 \right\} J_0 \left\{ 8 \left( \frac{\pi \sigma_h \cos \theta_i}{\lambda} \right)^2 \right\}
$$

where $\sigma_h$ is the standard deviation of the surface height, and $J_0(\cdot)$ is the modified Bessel function of order zero (see [8]). For a perfectly flat plane, $\sigma_h = 0$ and $\eta(\theta_i) = 1$.

We first note that the normal angle $n$ of the scatterer needs to be such that the reflected signals are directed at $V_R$ so that signal transmission from $V_T$ to $V_R$ is possible. To determine the reflected electrical field at $V_R$, we can simply mirror-image $V_R$ with respect to the scatterer (as shown by $V'_R$ in figure 2). Note that the scatterer can be viewed as the “hole” through which communications between $V_T$ and mirror-imaged $V_R$ takes place. We note that the area of $S$ must be larger than the corresponding viewing area between $V_T$ and $V'_R$ in order for $V_T$ to fully communicate with $V_R$. We will call this viewing area a reflective viewing area. Once this condition holds, the transfer function between $V_T$ and $V_R$ can be computed using the result in Section II. There are, however, three changes to note:

(i) The effective propagation distance increases to $r_s = r_1 + r_2$;
(ii) $V'_R$ (not $V_R$) is used for computing the singular values and eigenfunctions of the channel model;
(iii) The computed eigenfunctions for $V'_R$ need to be mirrored back so that they become the eigenfunctions for $V_R$.

Note that the same result can be obtained by mirror-imaging $V_T$ due to the symmetry between transmission and reception.

Let us summarise the assumptions for the scatterer:

A1. It is a purely reflective plane with gain (or loss) given by (25);
A2. The distance $r_1$ is sufficiently large compared to the size of the scatterer so that for any point in $V_T$, the incident angle across the whole scatterer is roughly constant. A similar condition holds for $r_2$.
A3. The area of the scatterer covers its reflective viewing area.

B. Multiple Scatterers

Each scatterer $S(k), k = 1, \cdots, K$ provides a communications path. The transfer function $\Gamma(k)$ for each path can be computed using the method in Section III.A. One may think that we can simply sum up all the $\Gamma(k)$ to obtain the overall transfer function $\Gamma$. This is incorrect because each $\Gamma(k)$ uses different eigenfunctions.

To obtain the correct $\Gamma$, we denote by $\{\psi_i^{(k)}(r_T)\}$ and $\{\phi_i^{(k)}(r_R)\}$ the two sets of eigenfunctions corresponding to $\Gamma(k)$ and let the dimension of $\Gamma(k)$ be $N_T^{(k)} \times N_R^{(k)}$. Then we have the following mapping:

$$
\{\psi_i^{(k)}(r_T)\} \xrightarrow{\Gamma(k)} \{\phi_i^{(k)}(r_R)\}
$$

Let $\{\psi_i(r_T)\}$ and $\{\phi_i(r_R)\}$ be any sets of complete, orthonormal basis functions for $V_T$ and $V_R$, respectively. We can project each $\psi_i^{(k)}(r_T)$ onto $\psi_j(r_T), j = 1, 2 \cdots$ to obtain the following:

$$
\psi_i^{(k)}(r_T) = \sum_{j=1}^{\infty} \gamma_{ij}^{(k)} \psi_j(r_T)
$$

where $\gamma_{ij}^{(k)}$ are projection coefficients, or in a vector form:

$$
\psi^{(k)}(r_T) = \Gamma^{(k)} \phi^{(k)}(r_T)
$$

A similar projection can be obtained on the receiver side:

$$
\phi(r_R) = \Gamma^{(k)} \phi^{(k)}(r_R) + \delta \phi^{(k)}(r_R)
$$

where $\delta \phi^{(k)}(r_R)$ is the residual term orthogonal to all $\phi_j^{(k)}(r_R)$. 
With the projections of (28) and (29) the overall transfer function is given by

$$
\Gamma = \sum_{k=1}^{K} \Gamma_{\phi}^{(k)} \Gamma_{\psi}^{(k)}
$$

(30)

The only difficulty is that there is no sum rule to help us determine the numerical accuracy of the computation. This means that we may have to use large sizes for $\Gamma_{\phi}^{(k)}$ and $\Gamma_{\psi}^{(k)}$ to ensure numerical accuracy.

The growth in channel capacity can be explained intuitively as follows. Assume that the scatterers are local to either $V_T$ or $V_R$. We can expect that the strengths of different paths are similar, but their eigenfunctions may be very different: For different scatterers different mirror-imaged bodies are obtained and used for computing the eigenfunctions. Because the effective propagation distances are roughly the same for different scatterers and mirror-imaging does not change the volume size, the sum rule ensures each path has roughly the same sum of gains (disregarding the reflection loss), although the number of communications modes may vary.

Each new scatterer $S^{(k)}$ brings a new pair of basis function sets $\psi^{(k)}(r_T)$ and $\phi^{(k)}(r_R)$. The components of $\psi^{(k)}(r_T)$ and $\phi^{(k)}(r_R)$ which are orthogonal to existing eigenfunctions will bring in new modes of communications. The remaining components will contribute to the gain. We may then interpret “rich” scattering as the case where scatterers introduce new communication modes, and “pin hole” where scattering increases channel gain.

IV. SIMULATION

We examined the case for 3GHz, $\lambda = 0.1m$. The channel was modelled where both $V_T$ and $V_R$ were hyper-rectangles with side-lengths of $2\Delta x_T = 9\lambda$, $2\Delta y_T = 27\lambda$, $2\Delta z_T = 9\lambda$, $2\Delta x_R = 18\lambda$, $2\Delta y_R = 9\lambda$, $2\Delta z_R = 9\lambda$, and separated by $r = 81\lambda$ as described in [5]. For this case the number of communications modes for direct transmission (1) is $N_c = 6$.

Scatterers were modelled as small randomly arranged planes with a gain $\eta = 1$. We have used [9] for the numerical computation of (13). In figure 3 we have plotted the normalised squared singular values, and in figure 4 we have shown the unnormalised squared singular values.

The direct (no scatter) transmission case is shown solid. The largest un-normalized singular value is $\sigma_1 \approx 0.0069$. The circled line shows a non-line of sight transmission via single scatterer, placed at a point between $V_T$ and $V_R$. As can be seen, the overall $N_c$ value is significantly less than for the direct case. This represents the “pin-hole” channel model. For the multiple scattering case, we have used $K = 10$ scatterers, placed randomly within $2m$ of $V_T$. We have considered two cases: the non-line of sight, transmission via scattering, shown with asterisks, and the case where a line of sight path was also present.

The channel improvement due to scattering can be seen in figure 4 where the modes produced by the scatterers are orthogonal to the direct case.

V. CONCLUSIONS

We have presented a numerical approach for the computation of communications modes between two volumes with scatterers. This has been used to provide insight into the characteristics of dense or “rich” scattering and sparse scattering in terms of the contribution to communications modes.

REFERENCES