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A simple and efficient dynamic modelling method for compliant micropositioning mechanisms using flexure hinges

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ABSTRACT

In this paper we consider the dynamic modelling of compliant micropositioning mechanisms using flexure hinges. A simple modelling method is presented that is particularly useful for modelling parallel micropositioning mechanisms. This method is based upon linearisation of the geometric constraint equations of the compliant mechanism. This results in a linear kinematic model, a constant Jacobian and linear dynamic model. To demonstrate the computational simplicity of this methodology it is applied to a four-bar linkage using flexure hinges. Comparisons are made between the simple dynamic model and a complete non-linear model derived using the Lagrangian method. The investigation reveals that this new model is accurate yet computationally efficient and simple to use. The method is then further applied to a parallel 3-degree of freedom (dof) mechanism. It is shown that the method can be simply applied to this more complex parallel mechanism. A dynamic model of this mechanism is desired for use in optimal design and for controller design.

Keywords: Micropositioning, compliant mechanism, flexure hinge, linear dynamic model, parallel mechanism, four-bar linkage

1. INTRODUCTION

During the past two decades considerable research has been conducted to develop micromanipulators to be used for purposes such as biological cell manipulation in biotechnology or micro-component assembly in micro-technology. The majority of these micromanipulators are based on the use of the piezo-ceramic actuator (PZT) and the compliant mechanism. PZT actuators can provide near linear motion with resolution of nanometres or sub-nanometres. Compliant mechanisms, which move solely through deformation of flexures instead of bearings, provide smooth motion with no backlash or Coulomb friction. As there are no hard non-linearities in the compliant mechanism behaviour there are no physical limitations on the resolution of position control. Therefore a manipulator based on these components is able to provide ultra high-precision positioning.

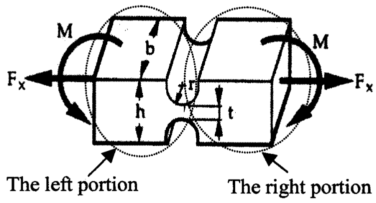

Figure 1 - Schematic of a flexure hinge

Flexure hinges, as shown in Figure 1, are a particular type of flexure that consists of a necked down section that deflects to provide a small range of near revolute motion. Mechanisms using this type of flexure hinges typically deflect only a

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very small amount and the angular rotation of the hinges is often less than 1 degree. These are commonly used due to their well-studied and predictable motion. Compliant mechanisms with flexure hinges are frequently modelled using the pseudo-rigid-body model (PRBM) approach. This method assumes that the flexure hinges behave as revolute joints with torsional springs attached while the thicker section of the mechanism behave as rigid links. Using this approach the mechanism can be modelled using well-established rigid-body modelling techniques.

Parallel micromanipulators are commonly used in micromanipulation due to the advantage of greater rigidity, which allows for more accurate motion and faster response. These attributes are particularly beneficial for ultra high precision positioning. In addition the actuators can be located in the base of the manipulator so that the link masses can be reduced. Kinematic and dynamic models have been derived for macro-scale parallel mechanisms with various topologies. However, these models are generally complex and non-linear. In particular the forward kinematics of a parallel mechanism is complicated due to the unknown relative motion of the unactuated joints. The motion of these joints can only be found by solving a set of simultaneous non-linear equations. This requires the use of a numerical iteration technique, which is computationally demanding. Therefore such models are not ideal for optimisation and of limited use for real-time control. However, in the case of micro-motion parallel manipulators it is possible to make linearising assumptions that allow for a simpler analysis. This may lead to the development of simple linear kinematic and dynamic models.

The method described in this paper is appropriate for mechanisms with angular displacement of less than 1 degree. This small angular displacement is generated in mechanisms whose movement range is small, say microns, compared to the size of its linkages, say millimetres. For this small angular motion it is appropriate to apply linearising small angle assumptions. It is then possible to apply simplified and unique approaches to derive the kinematic and dynamic models for such mechanisms. Such methodologies have not been well defined in the current literature. In this paper a method to derive a linear kinematic and dynamic model of closed loop mechanisms is presented. This method is based upon linearisation of the geometric constraint equations. Furthermore the geometry of the mechanism is defined in such a way as to further simplify the derivation. This method will be demonstrated using the 4 bar linkage as an example and comparisons made to a complete non-linear model. The method will then be further applied to a 3RRR* parallel compliant mechanism as shown in Figure 2.

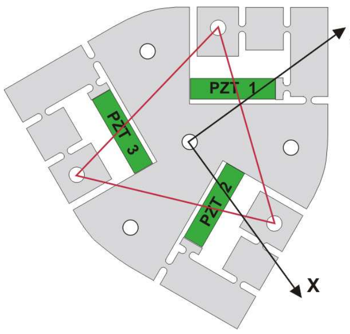

Figure 2 - Schematic of the 3RRR compliant mechanism and PZT actuators. The triangle represents the end-effector.

2. PREVIOUS WORK

In recent years there has been increasing interest in the use of parallel manipulators. Numerous macro-scale manipulators have been developed, for which kinematic and dynamic models have been developed to aid structure and controller design. The derivation of direct kinematic and dynamic models for these manipulators is non-trivial due to presence of unactuated joints. To determine the motions of these unactuated joints requires the solution of a set of simultaneous non-linear equations involving all loops of the manipulator. Ma and Angeles[1] developed an effective method to derive the direct kinematics and dynamics of a non-compliant 3RRR parallel manipulator. However the solution of this model requires numerical integration of non-linear equations, which is time consuming. Codourey[2] considered the efficient

* R: Revolute

dynamic modelling of parallel robots for the implementation of computed torque control. To develop a dynamic model that was simple enough for use in real-time control a number of model simplifications were made. Codourney and Burdet[3] proposed a body orientated method to derive a linear form of the dynamic model. Dynamic modelling methods developed for macro-scale manipulators could be applied to the case of micromotion systems. However due to the small-scale motion of these mechanisms it is more appropriate to apply a simplified linear analysis specific to this type of mechanism.

In the analysis of micro-motion mechanisms it is a common approach to model flexure hinges as revolute joints with torsional stiffness and the thicker segments of the structure as rigid links. Scire and Teague[4] first applied this approach in the analysis of a particular 1-dof micro-positioning stage. In their analysis they applied the classical analytical equations presented by Paros and Weisbord[5] to calculate the stiffness of flexure hinges. This modelling approach has been termed the pseudo-rigid-body-model and has since been widely applied to a range of micro-motion systems. Another common approach is to apply linearising small angle assumptions $\cos(\Delta\theta)\approx 1$, $\sin(\Delta\theta)\approx\Delta\theta$ in deriving the kinematics of compliant mechanisms. Based upon these two simplifying approaches kinematic and dynamic models have been derived for a range of micro-motion manipulators. Gao et al.[6, 7] applied the Lagrangian method to derive simple dynamic models for a 1-dof translator and a 1-dof stage, which both gave close predictions of the natural frequency. Yang et al.[8] also applied the Lagrangian method to derive the dynamic model of a 1-dof micropositioning stage, which also closely predicted the behaviour of the stage. Dynamic models of parallel multiple-dof mechanisms have been derived less frequently. Wang et al.[9] outlined a method to derive the kinematics and dynamics of a 6-dof micromanipulator that used a 3RRR stage. Their method used vector analysis and the Lagrangian method, resulting in a non-linear dynamic model. Zhang et al.[10] developed a constant Jacobian method for modelling the kinematics of a 3RRR compliant mechanism, which proved to be as accurate as a non-linear kinematic model. Further to this work Zou[11] applied the Lagrangian method to derive a dynamic model of the same compliant 3RRR mechanism. However the resulting model was non-linear, complex and unsuitable for optimisation or real-time control. Ryu et al.[12] also discussed the dynamic modelling of a 3RRR mechanism to be used for optimal design. They suggested application of the Lagrangian method but did discuss derivation of the stiffness and inertia from parametric values. As yet there has been no discussion in the literature of general methods to derive linear dynamic models of parallel compliant manipulators.

Her and Chang have developed a linear scheme for the displacement analysis of micropositioning stages[13]. All the geometrically constrained equations are linear and can be solved directly. The linear scheme can be generally applied to planar mechanisms and is demonstrated using single-loop and two-loop stage structures. The displacements of each flexure hinge in the structures, calculated using the linear scheme, are presented. The hinge displacement results are claimed to be accurate. However, the forward and inverse kinematics of the stage is not discussed and the method is not extended to dynamic modelling. In this paper a similar linear analysis approach will be applied to derive kinematic and dynamic models of a four-bar linkage and a parallel complaint mechanism.

3. LINEAR MODEL OF A FOUR-BAR LINKAGE

3.1 Kinematic Model

Compliant mechanisms designed for micromanipulation and micropositioning tasks commonly have structures consisting of closed kinematic chains. An example of such a structure is a four-bar linkage. A multi-dof parallel manipulator has multiple closed kinematic chains. The method discussed here is applicable to planar structures with one or more closed kinematic chains. The proposed method is first demonstrated by the example of a compliant four-bar linkage. The linkage shown in Figure 3 is a general four-bar linkage, which could have any geometry. It is used for illustrative purposes only and has no specific practical use. The flexure hinges are assumed to provide revolute motion only and therefore the movement of the mechanism is kinematically constrained. It is assumed that the angular displacement of any joint in this mechanism will be less than 1 degree.

This compliant mechanism can be modelled using a PRBM as shown in Figure 3. The flexure hinges are modelled as revolute joints with torsional stiffness. At this point we will only discuss the kinematic modelling and the stiffness will be ignored. The dynamic model will be discussed in the following section.

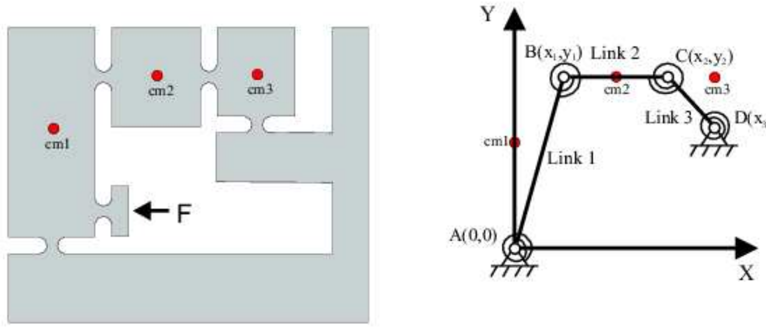

Figure 3 - A compliant four-bar linkage and its pseudo-rigid-body-model. The dots indicate the centre of mass of the links.

To begin, consider a single link rotating through a small angle, as shown in Figure 4. Point B is the free end of the link and is defined by its x and y coordinate as given in Eqs (1) and (2).

$$x_1 = R_1 \cos\theta_1 \tag{1}$$

$$y_1 = R_1 \sin\theta_1 \tag{2}$$

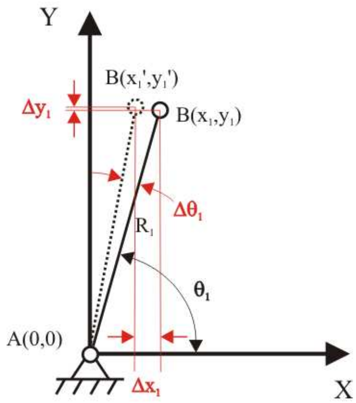

Figure 4 - A single link rotating through a small angle

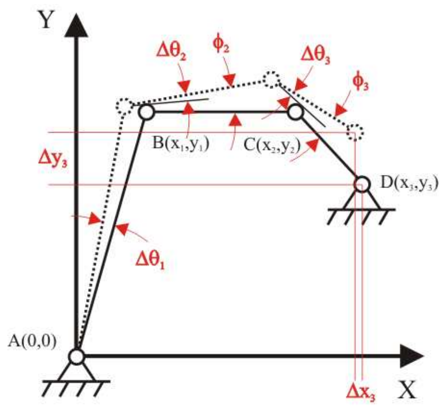

Figure 5 - A four-bar linkage with each joint rotating through a small angle

If link 1 is displaced by a small change in joint angle $\Delta\theta_1$ then the new coordinates are given by Eqs (3) and (4).

$$x_1' = R_1 \cos(\theta + \Delta\theta_1) = R_1(\cos\theta_1 \cos\Delta\theta_1 - \sin\theta_1 \sin\Delta\theta_1) \tag{3}$$

$$y_1' = R_1 \sin(\theta + \Delta\theta_1) = R_1(\sin\theta_1 \cos\Delta\theta_1 + \cos\theta_1 \sin\Delta\theta_1) \tag{4}$$

Using the small angle assumptions $\cos(\Delta\theta) \approx 1$, $\sin(\Delta\theta) \approx \Delta\theta$ and subtracting the original coordinate gives the change in coordinate to be Eqs (5) and (6).

$$\Delta x_1 = x_1' - x_1 \approx R_1[\cos\theta_1 - \sin\theta_1(\Delta\theta_1)] - R_1\cos\theta_1 = -(\Delta\theta_1)R_1\sin\theta_1 = -(\Delta\theta_1)(y_1 - 0) \tag{5}$$

$$\Delta y_1 = y_1' - y_1 \approx R_1[\sin\theta_1 + \cos\theta_1(\Delta\theta_1)] - R_1\sin\theta_1 = (\Delta\theta_1)R_1\cos\theta_1 = (\Delta\theta_1)(x_1 - 0) \tag{6}$$

It can be seen from Eqs (5) and (6) that the displacement of point B in the x-direction is given by the product of the change in angle and the initial distance in the y-direction between B and the centre of rotation of the link. Likewise the y displacement is the product of the change in angle and the x-distance between point B and centre of rotation. This basic approach can be further extended to analysis the case of multiple links.

Consider the four-bar linkage in Figure 5. It is momentarily assumed that point D, is detached from the ground. The total displacement of point D due to a change in angle of all joints is given by a sum of displacements, as given in Eqs (7) and (8), where the values of x_i and y_i (i=1,2,3) are the Cartesian co-ordinates of the joints. Point D is of course actually constrained and is attached to the ground, it therefore has zero displacement and so Eqs (7) and (8) equate to zero. These are the geometric constraint equations.

$$\Delta x_3 = -[(\Delta\theta_1)(y_3 - 0) + (\Delta\theta_2)(y_3 - y_1) + (\Delta\theta_3)(y_3 - y_2)] = 0 \tag{7}$$

$$\Delta y_3 = (\Delta\theta_1)(x_3 - 0) + (\Delta\theta_2)(x_3 - x_1) + (\Delta\theta_3)(x_3 - x_2) = 0 \tag{8}$$

We now have two linear kinematic constraint equations. The input to the four-bar linkage is a displacement of one link, say $\Delta\theta_1$, and we have two unknown displacements, $\Delta\theta_2$ and $\Delta\theta_3$. These two equations can be solved easily to provide $\Delta\theta_2$ and $\Delta\theta_3$ in terms of $\Delta\theta_1$.

The total change in angle of link 2 and 3 is given by Eqs (9) and (10) respectively.

$$\phi_2 = \Delta\theta_1 + \Delta\theta_2 = c_2\Delta\theta_1 \tag{9}$$

$$\phi_3 = \Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3 = c_3\Delta\theta_1 \tag{10}$$

Where c_2 and c_3 are constants. These are the kinematic equations describing the four–bar linkage. As the kinematics is described by constants the Jacobian, which is the time derivative of the kinematic equations, is described by the same constants, Eqs (11) and (12).

$$\dot{\phi}_2 = c_2\,\dot{\theta}_1 \tag{11}$$

$$\dot{\phi}_3 = c_3\,\dot{\theta}_1 \tag{12}$$

3.2 Dynamic Model

To derive the dynamic model the Lagrangian method can be applied. The Lagrangian equation is given in Eqn (13) below.

$$\frac{d}{dt}\frac{\partial K}{\partial\dot{\theta}_1} - \frac{\partial K}{\partial\theta_1} + \frac{\partial P}{\partial\theta_1} = \tau \tag{13}$$

The potential energy of the mechanism is due to the stiffness of the flexure hinges. Gravity is ignored in this analysis as it is generally far less significant then the stiffness of the structure, while in addition these planar mechanisms commonly operate in the horizontal plane. The stiffness of the flexure hinges can be calculated using the equation presented by Paros-Weisbord[5] , Eqn (14) below.

$$K_b = \frac{2Ebt^{5/2}}{9\pi r^{1/2}} \tag{14}$$

The potential energy of the four-bar linkage is given by Eqn (15).

$$P = \frac{1}{2}\sum_{i=1}^{4} k_i \Delta\theta_i^2 = \frac{1}{2}\left[k_1\Delta\theta_1^2 + k_2\Delta\theta_2^2 + k_3\Delta\theta_3^2 + k_4\Delta\theta_4^2\right] \tag{15}$$

Where k_1, k_2, k_3 and k_4, are the stiffness of flexure hinges A, B, C and D respectively.

Using the kinematic Eqs (9) and (10) this can be written as a function of $\Delta\theta_1$, where all other values are constants and therefore the equation can be represented as in Eqn (16) where Ψ is a constant.

$$P = \frac{1}{2}[\Psi]\Delta\theta_1^2 \tag{16}$$

To calculate the stiffness term of the Lagrangian we take the partial derivative of P, which gives Eq (17).

$$\frac{\partial P}{\partial \theta_1} = [\Psi]\Delta\theta_1 \tag{17}$$

The kinetic energy of the mechanism is given by Eqn (18).

$$K = \sum_{i=1}^{3}\left[\frac{1}{2}m_i\left(V_{ix}^2 + V_{iy}^2\right) + \frac{1}{2}J_i\dot{\theta}_i^2\right] \tag{18}$$

Where

V = the velocity of the centre of mass of the links
m = the mass of the links
J = the moment of inertia of the links

V_x and V_y for each link can be determined using the Jacobian constants, Eqs (11) and (12), and the same approached used to generate the constraint equations, Eqs (7) and (8). These velocity terms can be substituted into the kinetic energy equation. The mass moment of inertia of each link, which is dependent on the shape of the link, can also be substituted into the kinetic energy equation.

The resulting equation can be written as a function of $\dot{\theta}_1$ where all other values are constants. The kinetic energy equation can thus be represented in the form of Eqn (19) where Ω is a constant.

$$K = \frac{1}{2}[\Omega]\dot{\theta}_1^2 \tag{19}$$

To obtain the Lagrangian terms we take two derivatives. The first is Eqn (20) below.

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{\theta}_1} = \frac{d}{dt}\frac{\partial\left(\frac{1}{2}[\Omega]\dot{\theta}_1^2\right)}{\partial \dot{\theta}_1} = \frac{d}{dt}[\Omega]\dot{\theta}_1 = [\Omega]\ddot{\theta}_1 \tag{20}$$

The second is Eqn (21) below.

$$\frac{\partial K}{\partial \theta_1} = \frac{\partial\left(\frac{1}{2}[\Omega]\dot{\theta}_1^2\right)}{\partial \theta_1} = 0 \tag{21}$$

Substituting Eqs (17), (20) and (21) into Eqn (13) gives the total dynamic model for the four-bar linkage as Eqn (22).

$$\Omega\ddot{\theta}_1 + \Psi\theta_1 = \tau \tag{22}$$

This is a constant, linear dynamic model.

3.3 Comparison of linear model to complete non-linear model

A complete general non-linear model was derived for the four-bar linkage using the Lagrangian method. The details of this model are given in Zhang[14]. This has the form given in Eqn (23).

$$M(\theta_1)\ddot{\theta}_1 + C(\theta_1,\dot{\theta}_1)\dot{\theta}_1 + K(\theta_1) = Q \tag{23}$$

Working Model software was used to verify this model and gave an identical response. The linear model was far simpler than the general non-linear model, and has the form given in Eqn (24).

$$M_{lin}\Delta\ddot{\theta}_1 + K_{lin}\Delta\theta_1 = Q \tag{24}$$

To compare the models a step input torque was applied to link 1, which caused an oscillation of the four-bar linkage. The step input was such that the angular displacement of link 1 was less then 1 degree. The inertia, M, corriolis/centripetal, C, and stiffness, K, terms of the non-linear model are functions of position and velocity. For the range of motion these terms were compared to the constant linear-model inertia, M_{lin}, and stiffness, K_{lin}. A number of four-bar models with different geometric parameters were investigated. An example of the geometry considered is L_1=50mm, L_2 = 70mm, L_3=50mm, L_4=60mm, m_1=10g, m_2=500g, m_3=10g, hinge stiffness of $k_1=k_2=k_3=k_4$=40Nm/rad, and initial position, θ_1= 90. For this mechanism a step torque of 1.2Nm was applied. This excited an oscillation with amplitude of $\Delta\theta_1$ = 0.97degrees. The resulting dynamic model terms are shown in Table 1.

$\Delta\theta_1$	**0**	**0.97**
M	0.0014	0.0014
M_{lin}	0.0014	0.0014
C	1.22E-04	1.23E-04
$K(\theta)$	0	2.396
$K_{lin}\Delta\theta$	0	2.398

Table 1 - Comparison of dynamic model terms when $\Delta\theta_1 = 0$ and 0.97 degrees

From these results it is apparent that; the inertia terms are identical and the non-linear term does not vary significantly for this small-scale motion; the C term is insignificant; and the stiffness terms are nearly identical.

Four-bar models with a range of different geometry's were investigated to ensure the results were general. Regardless of the geometry the linear model gave almost identical results to the complete model.

4. MULTI-DOF PARALLEL MECHANISM

The method outlined above will now be applied to a practical 3-dof parallel compliant mechanism. The 3RRR mechanism is shown again in Figure 7 with its PRBM. Considering the 3RRR mechanism it can be seen that there are three closed loops. However only two of these need be considered in order to determine the kinematics of the mechanism. Three inputs must be defined to determine the position of the mechanism. The inputs to the mechanism are the rotations of links AB, given by $\Delta\theta_{A1}$, $\Delta\theta_{A2}$ and $\Delta\theta_{A3}$. The geometry of the mechanism is simply described using the Cartesian coordinates of the joints. These coordinate values are then used directly in the constraint equations to give the distance between the end-points and the centres of rotation. The points describing the joint locations can be given using a Cartesian reference frame of any orientation. The solution for the unknown $\Delta\theta$ will be the same.

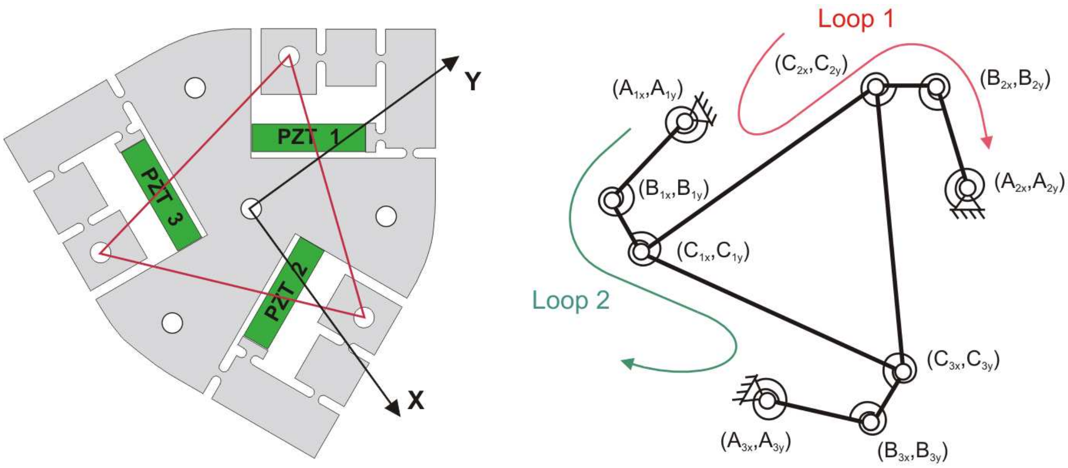

Figure 6 - Schematic of the 3RRR compliant mechanism and it's PRBM

4.1 Kinematic Model

Like the case of the four-bar linkage there are two linear constraint equations for each loop and so we have four constraint equations in total. There are four unknown link angles, $\Delta\theta_{B1}$, $\Delta\theta_{B2}$, $\Delta\theta_{B3}$ and $\Delta\theta_{C1}$. Therefore the four linear equations can be solved simultaneously to give the four unknowns. Each link displacement is given as a function of the 3 input rotations $\Delta\theta_{A1}$, $\Delta\theta_{A2}$ and $\Delta\theta_{A3}$. These are in turn each a function of the PZT displacements, L_1, L_2 and L_3 respectively.

The resulting kinematics is described by a 3x3 matrix of constants, which is multiplied by the input PZT displacement to give the end-effector motion, as given in Eqn (25).

$$\begin{bmatrix} x_{end-effector} \\ y_{end-effector} \\ \phi_{end-effector} \end{bmatrix} = \begin{bmatrix} c1x & c2x & c3x \\ c1y & c2y & c3y \\ c1\phi & c2\phi & c3\phi \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} \tag{25}$$

The Jacobian is given by the same constant matrix.

The geometric parameters of a particular 3RRR mechanism were substituted into the general kinematic model resulting in the kinematic matrix in Eqn (26) below.

$$\begin{bmatrix} x_{end-effector} \\ y_{end-effector} \\ \phi_{end-effector} \end{bmatrix} = \begin{bmatrix} 1.905 & 1.315 & -3.220 \\ 2.618 & -2.960 & 0.341 \\ -59.96 & -59.96 & -59.96 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} \tag{26}$$

Zhang et al.[10] and Yong et al.[15] have derived kinematic models for this particular geometry of 3RRR mechanism. They also based their models upon the PRBM, but used different methods to derive the kinematics. All three kinematic models agree.

4.2 Dynamic Model

The dynamic model is derived using the same approach as described in the four-bar case, using the Lagrangian. However in this case partial derivatives are taken of kinetic and potential energy with respect to 3 PZT displacements/velocities, L_1, L_2 and L_3 / $\dot{L}_1$, $\dot{L}_2$, and $\dot{L}_3$. There are also 3 input forces Q_1, Q_2 and Q_3, giving the Lagrangian in Eqn (27).

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{L}_i} - \frac{\partial K}{\partial L_i} + \frac{\partial P}{\partial L_i} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} \quad , \text{ i=1,2,3} \tag{27}$$

The final dynamic model derived for the 3RRR mechanism has the form of Eqn (28).

$$\begin{bmatrix} 2\psi & \sigma & \sigma \\ \sigma & 2\psi & \sigma \\ \sigma & \sigma & 2\psi \end{bmatrix} \begin{bmatrix} \ddot{L}_1 \\ \ddot{L}_2 \\ \ddot{L}_3 \end{bmatrix} + \begin{bmatrix} 2\alpha & \beta & \beta \\ \beta & 2\alpha & \beta \\ \beta & \beta & 2\alpha \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} \tag{28}$$

Where ψ, σ, α and β are constants.

5. CONCLUSIONS AND FUTURE WORK

This paper presents a simple method to derive a linear dynamic model for parallel mechanisms undergoing small angular displacement. The method is based upon the use of a pseudo-rigid-body model representation of a complaint mechanism with flexure hinges. Using the example of a four-bar linkage it was demonstrated that the linear model gives an almost indistinguishable result from a complete non-linear model. The modelling method was then applied to a 3RRR parallel compliant mechanism. The forward kinematics of the parallel mechanism can be derived without needing to solve simultaneous non-linear equations, while the constraint equations are easily formulated. The resulting kinematics is the same as derived by other researchers considering the same mechanism. The linear dynamic model was also derived for the 3RRR compliant mechanism. This model now needs to be experimentally verified. The dynamic model may then be used in structure/controller optimisation.

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