ENGINEERING ECONOMICS


INTRODUCTION

Choice Amongst Alternatives
1) Why do it at all?
2) Why do it now?
3) Why do it this way?

Answers to 2) and 3) require an understanding of the principles of engineering economy - in particular an understanding of the time-value of money.

Recognising and Defining Alternatives

Even the most careful estimates of the monetary consequences of choosing different alternatives almost certainly will turn out to be incorrect.

It often is helpful to a decision maker to make use of secondary criteria that reflect in some way the lack of certainty associated with all estimates of the future.

The Need for a "Systems Viewpoint"

Often there are side-effects that tend to be disregarded when individual decisions are made. To consider such side-effects adequately, it may be necessary to examine the inter-relationships among a number of decisions before any of the individual decisions can be made.
Economics

Most problems in economy involve determining what is economical in the long run, i.e., over a considerable period of time. In such problems, it is necessary to recognise the time value of money; because of the existence of interest, a dollar now is worth more than the prospect of a dollar next year or at some later date.

Interest may be defined as money paid for the use of borrowed money. Broadly speaking, interest may be thought of as the return obtainable by the productive investment of capital.

The rate of interest is the ratio between the interest chargeable or payable at the end of a period of time, usually a year or less, and the money owed at the beginning of that period.

eg. $6 interest annually in debt of $100
interest rate -- 6/100 = .06 per annum
= 6% (p.a., understood)

Interest Formulae

Symbols

\[ i = \text{interest rate per interest period (usually per year, written as } pa \text{ from the Latin } per \text{ annum)} \]
\[ n = \text{number of interest periods (usually years)} \]
\[ P = \text{the present sum of money} \]
\[ F = \text{the sum of money at the end of } n \text{ periods, which is equivalent to } P \text{ at } i. \]
\[ A = \text{end of period payment in a uniform series for } n \text{ periods, entire series equivalent to } P \text{ at } i. \]

\[ F = P(1 + i)^n \quad (1.1) \]
\[ P = F / (1 + i)^n \quad (1.2) \]

\((1 + i)^n\) is called the ‘single payment compound amount factor’
\(1 / (1 + i)^n\) is called the ‘single payment present worth factor’

There are a number of other formulae which are frequently used which result from summation of a series of the same payments at regular intervals – in the same way as you make a regular loan repayment to a lending authority such as a bank. You might for example borrow a sum to buy a stereo system, which you pay off by a monthly
The repayment of $27, each month for three years. The converse is that you receive payments in a series like this and invest each payment at interest rate $i$ when it has been received. Because the interest accumulates in a compound fashion, with $n$ decreasing as we move through the series, the total value is a geometric series summation. The payment received at the end of the nth year attracts no interest, while the payment received at the end of the first year accumulates to $A(1+i)^{n-1}$.

\[ F = A[1 + (1+i) + (1+i)^2 + ... + (1+i)^{n-1}] \]  

The formula for this sum of a GP is derived by multiplying both sides of the equation by $(1+i)$ which results in:

\[ (1+i)F = A[(1+i) + (1+i)^2 + ... + (1+i)^n] \]  

If \((1.3)\) is subtracted from \((1.4)\) we get:

\[ iF = A[(1+i)^{n} - 1] \]  

and hence

\[ F = \frac{A[(1+i)^n - 1]}{i} \]  

The converse of this relationship is easily obtained by rearranging \((1.6)\)

\[ A = \frac{Fi}{[(1+i)^n - 1]} \]  

A series of payments designed to produce a future sum $F$ at the end of $n$ years in this way is called a *sinking fund*.

There are a couple of other useful formulae which can easily be derived by manipulating Equations \((1.1)\), \((1.2)\), \((1.6)\) and \((1.7)\).

By substituting for $F$ from \((1.1)\) into \((1.7)\) we get:

\[ A = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] \]  

In \((1.8)\) the bracketed expression is called the ‘capital recovery factor’ because it indicates the payments required to sum to the value of a current ‘capital’ sum.

The converse of this expression is called the ‘series present worth factor’ and is given by:

\[ P = A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] \]  

(1.9)
Examples

E.g. 1

A local government engineer designs a reinforced concrete culvert with an anticipated life of fifty years and cost of $4000. He is approached by a steel manufacturer who offers him a steel culvert costing $3000.

If the engineer estimates a design life of the steel culvert of twenty-five years, for a discount rate of 5%, which should he choose?

<table>
<thead>
<tr>
<th>R.C.</th>
<th>Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.W. = $4000</td>
<td>P.W. = $3000 + $3000 ( \frac{(1.05)^{25}}{0.05} ) = $3885.9</td>
</tr>
</tbody>
</table>

Steel Culvert has lowest present worth, choose it.

E.g. 2

A dam is to be built for irrigation purposes in a major irrigation area. Two sizes of dam are considered, the larger of which would cost $20 million and the smaller $10 million. It is estimated that the annual irrigation returns from the larger dam would be $1.5 million and from the smaller dam $0.8 million. At a discount rate of 6% p.a., which dam should be chosen? What is the benefit cost ratio if the duration of the project is forty years?

<table>
<thead>
<tr>
<th>Capital Cost</th>
<th>Large Present Worth</th>
<th>Small P. Worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20 million</td>
<td>$2.57 million</td>
<td>$2.04 million</td>
</tr>
<tr>
<td>$10 million</td>
<td>1.13</td>
<td>1.20</td>
</tr>
</tbody>
</table>

\[ \begin{align*}
\text{Returns} & = \frac{(1.06)^n - 1}{0.06(1.06)^n} = 22.57 \\
\text{B-C} & = 2.57 \times \frac{(0.06(1.06)^n)}{1.06^n} = 12.04 \\
\text{B/C} & = 1.13 \\
\end{align*} \]
Comparisons Using Annual Payments

E.g. 3
A materials handling operation in a concrete plant is at present carried out by hand. The total annual cost for this labour is $19,500. Machinery could be installed which would reduce the labour costs substantially. The equipment costs $45000 and would reduce labour costs to $7,000 but additional insurance and tax payments of about $6,300 per year would have to be made. If the life of the plant is 10 years and interest rates are about 8.5%, assuming the new machinery would have a salvage value of $15,000 at the end of ten years, should it be installed?

<table>
<thead>
<tr>
<th>Present Costs</th>
<th>With New Machinery</th>
</tr>
</thead>
<tbody>
<tr>
<td>$19,500</td>
<td>CRF (8.5%, 10 years) = .1524</td>
</tr>
<tr>
<td>45,000 x .1524</td>
<td>= 6858</td>
</tr>
<tr>
<td>Labour</td>
<td>7000</td>
</tr>
<tr>
<td>Tax</td>
<td>6300</td>
</tr>
<tr>
<td></td>
<td>20158</td>
</tr>
<tr>
<td>Salvage (Sinking fund 8.5%, 10 yrs. =.0674)</td>
<td>-1011</td>
</tr>
<tr>
<td></td>
<td>19,147</td>
</tr>
</tbody>
</table>

E.g. 4
Solve example I by comparing equivalent series of uniform payments

<table>
<thead>
<tr>
<th>R.C.</th>
<th>Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4000 x .05478 = $219.12</td>
<td>$3000 x .07095 = $212.85</td>
</tr>
</tbody>
</table>
Homework examples:

A dam is to be built for irrigation purposes in a major irrigation area. Two sizes of dam are considered, the larger of which would cost $20 million and the smaller $10 million. It is estimated that the annual irrigation returns from the larger dam would be $1.5 million and from the smaller dam $0.8 million. At a discount rate of 6% p.a., which dam should be chosen? What is the benefit cost ratio if the duration of the project is forty years? Use annualized values.

<table>
<thead>
<tr>
<th></th>
<th>Large Annual</th>
<th>Small Annual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>1.329231</td>
<td>0.664615</td>
</tr>
<tr>
<td>Returns (B)</td>
<td>1.5</td>
<td>0.8</td>
</tr>
<tr>
<td>B/C</td>
<td>1.128472</td>
<td>1.203704</td>
</tr>
</tbody>
</table>

E.g. 3
A materials handling operation in a concrete plant is at present carried out by hand. The total annual cost for this labour is $19,500. Machinery could be installed which would reduce the labour costs substantially. The equipment costs $45000 and would reduce labour costs to $7,000 but additional insurance and tax payments of about $6,300 per year would have to be made. If the life of the plant is 10 years and interest rates are about 8.5%, assuming the new machinery would have a salvage value of $15,000 at the end of ten years, should it be installed? Use present worth.

Present worth of labour cost is: $127946.3

Present worth of machinery labour and tax is: $45929.4 + $41336.49 = $87265.93

Present worth of machinery salvage is: $6634.281

Total present worth costs of machinery+labour + tax - salvage: $125631.6

Machinery is cheaper – note that you can convert this figure back to $19,147 directly using \( A=f(P) \), where \( f \) is a capital recovery factor.