

## LOW COMPLEXITY MODELLING AND PARAMETER ESTIMATION IN COPPER BIOLEACHING PROCESSES

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**Abstract:** In heap bioleaching, metal is dissolved from ore by a percolating solution, catalysed by naturally occurring bacteria. Increased interest in the technology motivated the development of accurate models of such process in recent years. In contrast, the utilisation of such models for control has received limited attention, given their high mathematical complexity. This paper presents a low complexity model aimed as the basis for control design. The model parameters are estimated using least squares on data generated by a high complexity, experimentally validated model provided by BHP Billiton Innovation. Simulation results indicate significant potential of the approach.

**Keywords:** Process models, Reduced-order models, Control oriented models, Process parameter estimation, Recursive least squares

### 1. INTRODUCTION

Mineral leaching is a mining technology based on the dissolution of minerals by a percolating liquid. Although the leaching technology has been known for centuries, it has received greater attention in recent years as a potential lower-cost, more environmentally acceptable alternative to competing technologies such as pyrometallurgy (Barret *et al.*, 1993). In mineral leaching of copper ores, naturally occurring bacteria often act as a catalytic agent in the process, significantly enhancing the conversion of metal compounds into their water soluble forms. In such cases, we refer to the process as mineral *bioleaching*.

Despite the successful operation of stand-alone heap bioleaching facilities around the world, the technology has been characterised by lower than expected production rates and longer than expected start up times (Lizama, 2004). Thus, the interest of mining companies in developing control and optimisation strategies to improve the technology has increased, leading to greater research efforts to understand the mechanisms that make the process work.

In the last decade, many studies have appeared discussing mathematical modelling aspects of the process such as oxidation, bacterial growth, tempera-

ture, and chemical reaction dynamics (e.g., G.Pantelis and A.I.M.Ritchie, 1992; Dixon, 2000; Peterson and Dixon, 2003). However, although these studies can explain the sub-processes involved in bioleaching with great accuracy, the available models are characterised by a high mathematical complexity, which hinders the task of designing control strategies for the process.

This paper deals with the derivation of low complexity models for control and optimisation of copper heap bioleaching processes. A key motivation of our approach is a fact well known in the area of model identification for control: very often a relatively simple model suffices as the basis for successful control design for complex processes, as long as essential dynamic aspects of the process are captured by the model (Gevers, 1993).

This paper reports preliminary results towards the derivation of such a model. The model, partially derived on first principles, has a nonlinear structure, but is linear in its parameters, which are estimated using standard and recursive least squares using data generated by a high complexity model of a copper bioleaching process, provided by BHP Billiton Innovation. This high complexity model, which we refer to as BHPM, has in turn been validated against experimental data. The main components of the proposed

low complexity model (LCM) are the dynamics of concentrations, arising from the main chemical reactions, dynamics of temperature, and bacteria activity. Bacteria activity plays an important role in the dynamics of the process. We follow ideas suggested in the paper Dochain and Bastin (1984) for a simplified parametrisation of bacteria dynamics.

For the estimation of the model parameters, we first discretise the model using the Euler approximation. Parameter estimation is then run on this discretised version. We use a combination of standard least squares (LS) and recursive least squares (RLS) algorithms, and implement the latter in the form of a Kalman filter, to tackle the time-varying nature of part of the structure of the discretised model. Comparative simulation runs, using data generated by the BHPM, show a close matching of the LCM predictions, which indicates potential of the proposed model as a basis for the design of a control strategy for these processes.

## 2. THE PROCESS

In copper heap leaching, large heaps of up to several square kilometres by 6 to 20 metres height of crushed copper mine tailings are formed. A sulphuric acid solution, called *raffinate*, is sprinkled by means of an arrangement of drip lines at the top of the heap. As the solution percolates down through the heap, it becomes enriched by the copper dissolved from the heaped ore, forming the *pregnant leach solution* (PLS). The PLS is then collected at the base of the heap by an impervious liner and pumped to an electro-winning extraction plant, which produces cathodic copper. The residual solution is then recycled as raffinate to the top of the heap. The process is illustrated in Figure 1.

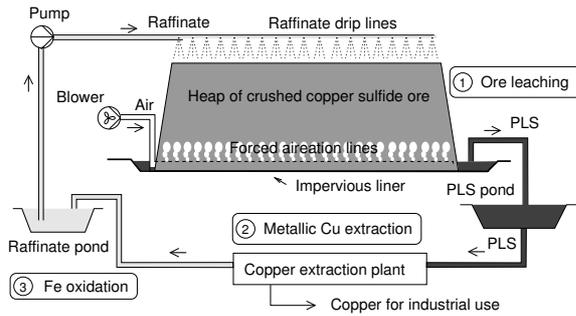
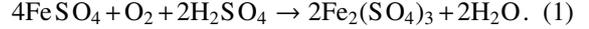


Fig. 1. Simplified copper heap leaching process

To tackle the complexity of this process, we start by considering a single geometric dimension of the process, namely, the vertical direction, assuming process homogeneity in every direction on the horizontal plane. Next, we identify three fundamental sub-process which are intercoupled. These sub-processes are: chemical reactions, temperature, and bacterial activity. We explain each of these sub-process separately, obtaining basic dynamic equations for each one.

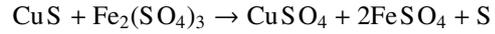
### 2.1 Chemical Reactions

One of the typical chemical reactions that take place in copper bioleaching processes is the conversion of iron sulphate compounds from ferrous ( $\text{FeSO}_4$ ) to ferric ( $\text{Fe}_2(\text{SO}_4)_3$ ) (Barret *et al.*, 1993)

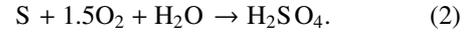


The reaction (1) is exothermic, and is typically catalysed by the bacteria *Thiobacillus ferrooxidans*, which uses the heat generated together with atmospheric oxygen and  $\text{CO}_2$  for growth. The oxygen and  $\text{CO}_2$  are supplied by blowing air through pipes at the bottom of the heap.

The  $\text{Fe}_2(\text{SO}_4)_3$  is then available to oxidise  $\text{Cu}_2\text{S}$  to create a  $\text{CuSO}_4$  solution, via two stages



Then, another bacteria, *Thiobacillus thiooxidans*, re-oxidises sulphur as follows:



We are interested in modelling concentrations of each component involved in this sequence of chemical reactions. To this end, we formulate mass balance equations (Luyben, 1990) and for simplicity, we initially discard water and oxygen concentrations.

The following state vector  $\mathbf{x}$  and input vector  $\mathbf{u}$  are defined as

$$\mathbf{x} = [x_1 \ x_2 \ \cdots \ x_{11}]^T, \quad \mathbf{u} = [u_1 \ u_2 \ \cdots \ u_5]^T$$

where  $x_1 = V$  denotes volume [lts];  $x_2 = C_{\text{FeSO}_4}$ , where  $C_{\text{FeSO}_4}$  denotes concentration in [mol/lts] of  $\text{FeSO}_4$ ; correspondingly,  $x_3 = C_{\text{H}_2\text{SO}_4}$ ,  $x_4 = C_{\text{Fe}_2(\text{SO}_4)_3}$ ,  $x_5 = C_{\text{Cu}_2\text{S}}$ ,  $x_6 = C_{\text{CuSO}_4}$ ,  $x_7 = C_{\text{CuS}}$ ,  $x_8 = C_{\text{S}}$ ,  $x_9 = T$ , average temperature in Kelvin degrees; and  $x_{10} = X_{th,f}$  and  $x_{11} = X_{th,t}$  are the bacteria populations [cells/lts] for *Thiobacillus ferrooxidans* and *Thiobacillus thiooxidans* respectively. In addition, the input vector  $\mathbf{u}$  is composed of  $u_1 = F_i$  the raffinate influx in [lts/h];  $u_2 = C_{\text{FeSO}_4}$ ,  $u_3 = C_{\text{H}_2\text{SO}_4}$ ,  $u_4 = C_{\text{Fe}_2(\text{SO}_4)_3}$ , denotes the input concentration of each compound; and  $u_5 = T_i$  is the input average temperature in Kelvin degrees.

The mass balance gives us the following equations

$$\dot{x}_1 = u_1 - F_o, \quad (3)$$

$$\dot{x}_2 = \frac{u_1}{x_1}(u_2 - x_2) - 4k'_1 x_2 x_3 + 2k_2 x_4 x_5 + 2k_3 x_4 x_7, \quad (4)$$

$$\dot{x}_3 = \frac{u_1}{x_1}(u_3 - x_3) - 2k'_1 x_2 x_3 + k'_4 x_8, \quad (5)$$

$$\dot{x}_4 = \frac{u_1}{x_1}(u_4 - x_4) + 2k'_1 x_2 x_3 - k_2 x_4 x_5 - k_3 x_4 x_7, \quad (6)$$

$$\dot{x}_5 = -\frac{u_1}{x_1} x_5 - k_2 x_4 x_5, \quad (7)$$

$$\dot{x}_6 = -\frac{u_1}{x_1} x_6 + k_2 x_4 x_5 + k_3 x_4 x_7, \quad (8)$$

$$\dot{x}_7 = -\frac{u_1}{x_1} x_7 + k_2 x_4 x_5 - k_3 x_4 x_7 \quad (9)$$

$$\dot{x}_8 = -\frac{u_1}{x_1} x_8 + k_3 x_4 x_7 - k'_4 x_8. \quad (10)$$

where  $F_o$  is the PLS out-flux in  $[lts/h]$ . As we have initially neglected water and oxygen concentrations in (1) and (2), the kinetics associated to each reaction must be changed. For example, for the first reaction given by (1), the reaction kinetic is denoted by  $k_1$ . However, as the oxygen has been discarded, then the concentration of this element is included in a *new* kinetic denoted  $k'_1$ . The same is then done for the other kinetics ( $k_2, k_3, k_4$ ) when oxygen or water concentrations were missing.

Although the kinetics of the chemical reactions in the same set of equations (4) to (10) theoretically should be the same, previous estimation tests have shown that the structure found is insufficient to describe the process. Thus, we consider these reactions to be decoupled, with their own dynamics.

Note that the modified parameters now might not have physical meaning, and even be negative. However, since our ultimate objective is to obtain a model for eventual control purposes, we sacrifice some of the physical meaning of these parameters in order to keep the model complexity as low as possible. Then, we include artificial terms in the equations to compensate for the missing dynamics. These terms are determined by iterative simulation runs until the predictions obtained are sufficiently close to the BHPM data. In summary, from the structure given by equations (3) to (10), we arrive at the following (*modified*) LCM

$$\dot{x}_1 = u_1 - F_o, \quad (11)$$

$$\dot{x}_2 = \frac{u_1}{x_1}(u_2 - x_2) - 4k'_{11}x_2x_3 + 2k_{31}x_4x_7, \quad (12)$$

$$\dot{x}_3 = \frac{u_1}{x_1}(u_3 - x_3) - 2k'_{12}x_2x_3 + k'_{41}x_8, \quad (13)$$

$$\dot{x}_4 = \frac{u_1}{x_1}(u_4 - x_4) + 2k'_{13}x_2x_3 - k_{21}x_4x_5, \quad (14)$$

$$\dot{x}_5 = -\frac{u_1}{x_1}x_5 - k_{23}x_4x_5 + k_{51}x_5x_6 + k_{61}x_6, \quad (15)$$

$$\dot{x}_6 = -\frac{u_1}{x_1}x_6 + k_{22}x_4x_5 + k_{62}x_6 + k_{71}x_7, \quad (16)$$

$$\dot{x}_7 = -\frac{u_1}{x_1}x_7 + k_{23}x_4x_5 - k_{32}x_4x_7 + k_{61}x_6 \quad (17)$$

$$\dot{x}_8 = -\frac{u_1}{x_1}x_8 + k_{72}x_7. \quad (18)$$

## 2.2 Temperature

Effects of temperature in this process has been extensively studied in Peterson and Dixon (2003) and Dixon (2000). Though these studies can accurately describe temperature aspects, the mathematics involved is still very complex. Our approach uses very simple natural rules from heat generation to obtain a parametrised model with parameters that can be estimated by a LS algorithm. Due to the approximations introduced, note that these parameters, again, might not have a direct physical meaning, and they might even turn out to be negative, for the model to fit properly.

Because this process is exothermic, temperature generated is strongly influenced by the chemical re-

actions. The basic temperature generation for an exothermic process (Luyben, 1990) is given by

$$\frac{dT}{dt} = \frac{F_i\rho_i}{V\rho}(T_i - T) + \frac{Q_{CR}}{\rho V\bar{C}_p}, \quad (19)$$

where  $\rho_i$  is the input density;  $\rho$  is the density; and  $\bar{C}_p$  is the heat capacity of the heap. Since we suppose that density is time-invariant, then  $\rho_i = \rho$ .

The term  $Q_{CR}$  depends on the chemical reactions. Although there are more than one expression which explains its dependency on the nature of the chemical reaction (Perry and Green, 1997), we consider them as parameters to be estimated.

The last term in (19),  $Q_{CR}/\rho V\bar{C}_p$  needs more explanation. The  $\bar{C}_p$  term is the heat capacity of the section of the heap considered and  $\rho V$  is the total mass. Since  $\bar{C}_p$  is unknown, we approximate it by an average heat capacity considering for that the individual heat capacities of the elements in the heap. That means

$$\rho V\bar{C}_p \approx \sum_k m_k\bar{C}_{p_k} = \sum_k \bar{C}_{p_k}(W_kVC_k) = V \sum_k \bar{C}_{p_k}C_kW_k,$$

where  $m_k$ ,  $\bar{C}_{p_k}$ ,  $C_k$  and  $W_k$  are the mass, heat capacity, concentration and molecular weight of compound  $k$ , respectively.

Since we have four chemical reactions, the heat generated by all of them is included in  $Q_{CR}$  as

$$Q_{CR} = \sum_j Q_{CR_j}, \quad j = 1, \dots, 4,$$

where the term  $Q_{CR_j}$  is as follows

$$Q_{CR_1} = \lambda_1 V k'_1(T) C_{FeSO_4} C_{H_2SO_4} = \gamma_1 x_1 x_2 x_3$$

$$Q_{CR_2} = \lambda_2 V k_2(T) C_{Cu_2S} C_{Fe_2(SO_4)_3} = \gamma_2 x_1 x_4 x_5$$

$$Q_{CR_3} = \lambda_3 V k_3(T) C_{CuS} C_{Fe_2(SO_4)_3} = \gamma_3 x_1 x_4 x_7$$

$$Q_{CR_4} = \lambda_4 V k'_4(T) C_S = \gamma_4 x_1 x_8.$$

The term  $\lambda_i$ , ( $i = 1, \dots, 4$ ) is a constant related to the nature of the reaction and  $k_i$  is the *modified* reaction rate coefficient for the reaction  $i$ . For simplicity, we group these parameters in  $\gamma_i = \lambda_i k_i$ .

## 2.3 Bacteria

The last fundamental element under consideration is the bacteria activity. A number of papers explain how to model bacteria activity in biological systems and why bacteria are an important factor in bioleaching (Hutchins *et al.*, 1986; Crundwell, 2001). However, there exist no unique expression because of the multiple factors which influence their growth and death. For example, in Haddadin *et al.* (1995) many models are mentioned to describe temporal evolution of the bacterial growth taking physical, chemical and electro-mechanical factors into account. In addition, the interactions between these factors must be considered in order to obtain a close prediction of bacteria activity.

A common structure to describe bacterial growth used in the literature is

$$\dot{X}(t) = \left\{ \mu(t) - \frac{F_i}{V} \right\} X(t), \quad (20)$$

where the parameter  $\mu$  may be given by different structures, depending on the modelling approach. Since the parameters involved can usually be estimated on-line, an exact expression for the factor  $\mu$  is not essential.

We use the model given by (20) as the model proposed for the bacterial sub-process in bioleaching, including the two kinds of bacteria which are involved in the process. The final equations for bacteria are as follows

$$\dot{x}_{10}(t) = \left\{ \mu_1(t) - \frac{F_i}{V} \right\} x_{10}(t), \quad \dot{x}_{11}(t) = \left\{ \mu_2(t) - \frac{F_i}{V} \right\} x_{11}(t).$$

**2.3.1. Catalytic Effect of Bacteria** An important issue related with bacteria is how they affect the chemical reactions taking place in the process. Since it is known that bacteria catalyse the chemical reactions (Crundwell, 2001), we need to account for their effect in the equations (12) to (18). To this end, we add a new term directly related to the amount of bacteria to the out-coming mass balance for each compound involved in each chemical reaction. For example, take the first reaction given by (12). We propose a *new term* of the form

$$-4\bar{\mu}_1(t)X_{th,f},$$

where  $X_{th,f}$  is the biomass of bacteria *Thiobacillus ferrooxidans*;  $V$  is the Volume; and  $\bar{\mu}_1(t)$  is a factor to be estimated. The factor 4 is related to the stoichiometric equation for  $\text{FeSO}_4$  in the first reaction given by (1). If more bacteria are present, the effect of adding this new term will be to speed up the reaction. The *final* balance equation for  $\text{FeSO}_4$  would be

$$\dot{x}_2 = \frac{u_1}{x_1}(u_2 - x_2) - 4k'_{11}x_2x_3 + 2k_{31}x_4x_7 - 4\bar{\mu}_{11}x_{10}. \quad (21)$$

By the same reasoning, we obtain the following equations for the remaining reactions catalysed by bacteria

$$\dot{x}_3 = \frac{u_1}{x_1}(u_3 - x_3) - 2k'_{12}x_2x_3 + k'_{41}x_8 \quad (22)$$

$$- 2\bar{\mu}_{12}x_{10} + \bar{\mu}_{21}x_{11},$$

$$\dot{x}_4 = \frac{u_1}{x_1}(u_4 - x_4) + 2k'_{13}x_2x_3 - k_{21}x_4x_5 + 2\bar{\mu}_{13}x_{10}, \quad (23)$$

$$\dot{x}_8 = -\frac{u_1}{x_1}x_8 + k_{72}x_7 - \bar{\mu}_{22}x_{11}. \quad (24)$$

### 3. PARAMETER ESTIMATION

In the real-world process we do not have access to the variables  $x_5$ ,  $x_7$ ,  $x_8$  (solids in the heap). However, we need these variables in order to use (21), (22), (23) and (16) to estimate the parameters  $\{k'_{1i}\}_{i=1}^3$ ,  $\{k_{2i}\}_{i=1}^2$ ,  $k_{31}$ ,  $k'_{41}$ ,  $\{\bar{\mu}_{1i}\}_{i=1}^3$ ,  $\bar{\mu}_{21}$ ,  $k_{62}$  and  $k_{71}$ . Nevertheless, if we know  $k_{23}$ ,  $k_{51}$ ,  $k_{61}$ ,  $k_{32}$ ,  $k_{72}$  and  $\bar{\mu}_{22}$  we can compute  $x_5$ ,  $x_7$  and  $x_8$  on-line by solving (15), (17) and (24). Then suitable on-line algorithm can be used to *track* the remaining parameters (which are generally time-varying in nature) using the measurements obtained from the process. When we work with BHPM data,

we adopt the same strategy to verify whether the proposed algorithm would be suitable for a real-world implementation. The BHPM data set provides us with the simulated value of  $x_5$ ,  $x_7$  and  $x_8$ , which we use to estimate  $k_{23}$ ,  $k_{51}$ ,  $k_{61}$ ,  $k_{32}$ ,  $k_{72}$  and  $\bar{\mu}_{22}$ . These estimates are used in subsequent calculations. Therefore, our estimation procedure consists of three steps.

- (1) Estimate  $k_{23}$ ,  $k_{51}$ ,  $k_{61}$ ,  $k_{32}$ ,  $k_{72}$  and  $\bar{\mu}_{22}$  using the simulated values of  $x_5$ ,  $x_7$  and  $x_8$  in the BHPM data set. This is done by applying the multivariable LS algorithm to (15), (17) and (24).
- (2) Obtain estimates of  $x_5$ ,  $x_7$  and  $x_8$  by solving (15), (17) and (24), where the estimates of  $k_{23}$ ,  $k_{51}$ ,  $k_{61}$ ,  $k_{32}$ ,  $k_{72}$  and  $\bar{\mu}_{22}$  obtained in the previous step are used. We stress that this step will be necessary with real-world data.
- (3) Use a Kalman filter to estimate the time-varying parameters  $\{k'_{1i}\}_{i=1}^3$ ,  $\{k_{2i}\}_{i=1}^2$ ,  $k_{31}$ ,  $k'_{41}$ ,  $\{\bar{\mu}_{1i}\}_{i=1}^3$ ,  $\bar{\mu}_{21}$ ,  $k_{62}$  and  $k_{71}$  using (21), (22), (23) and (16).

#### 3.1 Step 1: Least Squares Estimation

We estimate parameters using the data collected from the BHPM. Note that the BHPM gives access to variables which can not be measured in a real process due to technical or economical reasons. We apply multivariable LS to the set of equations given by (15), (17) and (24), as well as to the temperature equation (19) to estimate  $\{\gamma_i\}_{i=1}^4$ .

To illustrate the procedure consider the discretised version of (15), (17) and (24). Let us define

$$z_j = \begin{bmatrix} x_{5_{j+1}} - x_{5_j} + \frac{u_1}{x_1}x_{5_j} \\ x_{7_{j+1}} - x_{7_j} + \frac{u_1}{x_1}x_{7_j} \\ x_{8_{j+1}} - x_{8_j} + \frac{u_1}{x_1}x_{8_j} \end{bmatrix}; \quad \theta = \begin{bmatrix} k_{23} \\ k_{51} \\ k_{61} \\ k_{32} \\ k_{72} \\ \bar{\mu}_{22} \end{bmatrix},$$

$$\phi_j = \begin{bmatrix} -x_{4_j}x_{5_j} & x_{5_j}x_{6_j} & x_{6_j} & 0 & 0 & 0 \\ x_{4_j}x_{5_j} & 0 & x_{6_j} & -x_{4_j}x_{7_j} & 0 & 0 \\ 0 & 0 & 0 & 0 & x_{7_j} & -x_{11_j} \end{bmatrix},$$

$$Z = [z_1 \cdots z_n]^T; \quad \Phi = [\phi_1^T \cdots \phi_n^T]^T,$$

where  $n$  is the number of samples ( $n = 4000$ ). Then we get

$$Z = \Phi\theta,$$

and the least-squares estimate of  $\theta$  is given by  $\hat{\theta} = [\Phi^T\Phi]^{-1}\Phi^TZ$ . Direct implementation of this expression may lead to numerical problems due to a badly scaled  $\Phi$ . Hence it is recommended to compute  $\hat{\theta}$  using a numerically robust procedure (Ljung, 1999, pp.383).

A comparison between CuS (BHPM) and predicted (LCM) CuS is shown in Figure 2 (top). Similarly, a comparison between temperature (BHPM) and predicted temperature (LCM) is shown in Figure 2 (bottom).

We can observe in both plots that close behaviours can be predicted reasonably. As our model was developed

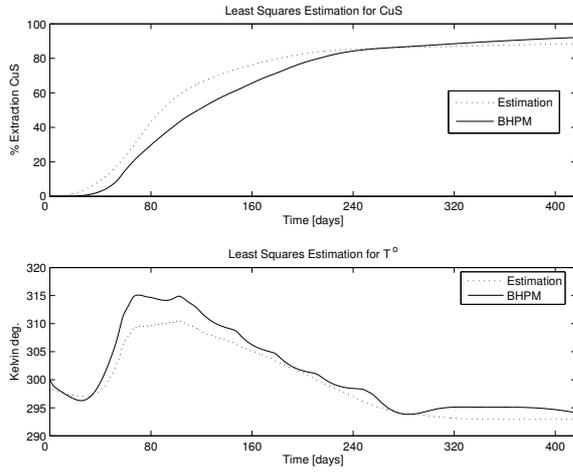


Fig. 2. Estimation of Temperature and % CuS extraction using multivariable LS.

for control purposes, then we might deal with non-exact models as in this case.

### 3.2 Step 3: Recursive Algorithms

In this section we present a brief description of the step-3 in our algorithm.

We assume that the following concentrations can be measured in the PLS, which is a reasonable assumption in a first approach,

- (i)  $\text{FeSO}_4$  concentration in [mols/lts]
- (ii)  $\text{H}_2\text{SO}_4$  concentration in [mols/lts]
- (iii)  $\text{Fe}_2(\text{SO}_4)_3$  concentration in [mols/lts]
- (iv)  $\text{CuSO}_4$  concentration in [mols/lts]
- (v)  $X_{th,f}$  and  $X_{th,t}$  concentration in [cells/lts].

Parameters  $\{k'_{1i}\}_{i=1}^3$ ,  $\{k_{2i}\}_{i=1}^2$ ,  $k_{31}$ ,  $k'_{41}$ ,  $\{\bar{\mu}_{1i}\}_{i=1}^3$ ,  $\bar{\mu}_{21}$ ,  $k_{62}$  and  $k_{71}$  using (21), (22), (23) and (16) are considered *time-varying*. We update the estimates thereof in order to use a Kalman filter. This enables us to improve the process-model significantly because the recent state and updated process parameters are used to adjust the model.

A popular approach to describe a process with slow time-varying parameter  $\beta$  is the *random-walk* model (Ljung, 1999, pp.367)

$$\beta_{k+1} = \beta_k + w_k, \quad y_k = \psi_k \beta_k + v_k, \quad (25)$$

where  $w_k$  and  $v_k$  are zero-mean white noise sequences defined by

$$E\{w_k^2\} = R_1, \quad E\{v_k^2\} = R_2, \quad E\{v_k w_k\} = 0.$$

It is assumed the value of the parameter at sampling instant  $k$  is  $\beta_k$ . The variables  $y_k$  and  $\psi_k$  are assumed to be known. By using Kalman filter we can obtain the estimate  $\hat{\beta}_k$  and the associated covariance matrix  $P_k$  recursively as Ljung (1999, pp.369)

$$\begin{aligned} \hat{\beta}_{k+1} &= \hat{\beta}_k + P_k \psi_k^T [\psi_k P_k \psi_k^T + R_2]^{-1} (y_k - \psi_k \hat{\beta}_k), \\ P_{k+1} &= P_k - P_k \psi_k^T [\psi_k P_k \psi_k^T + R_2]^{-1} \psi_k P_k + R_1. \end{aligned}$$

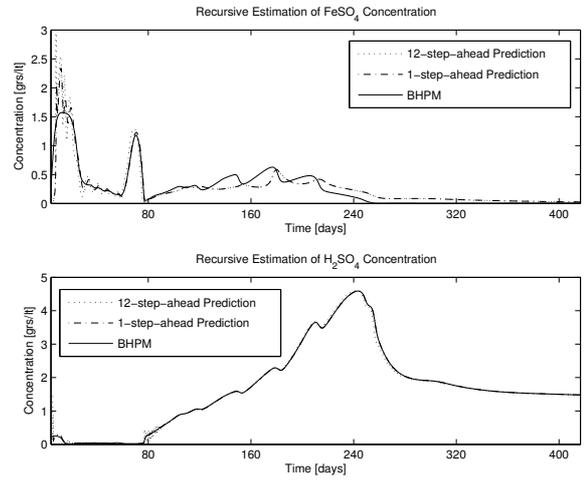


Fig. 3. Recursive 1-step-ahead and 12-step-ahead Prediction using Kalman filtering for  $\text{FeSO}_4$  and  $\text{H}_2\text{SO}_4$ .

The algorithm needs to be initialised with some initial guess  $\hat{\beta}_0$  and the covariance of the guess  $P_0$ . If the initial guess is accurate enough,  $P_0$  should be small. In the above  $R_1$  and  $R_2$  are generally unknown, and must be chosen by the user. Depending on the values of  $R_1$  and  $R_2$  we get several special cases. For example if  $R_1 = 0$  and  $R_2 = I$  we get the recursive least-squares (RLS) algorithm (note that we use  $I$  to denote an identity matrix). This is used when time-invariant parameters are estimated. If  $R_1 = 0$  and  $R_2 < I$  then we get RLS with a *forgetting factor*. In this way, old observations are less weighted than the new ones.

**3.2.1. Application of the algorithm** The algorithm described in the previous section is applied to the discretised version of the set of equations given by (21), (22), (23) and (16). This discretised version is obtained using the Euler approximation with  $\Delta = 4$  [hrs] (= 1/6 [days]). For example, for the equation (21) we have

$$\begin{aligned} x_{2k+1} &= x_{2k} + \Delta \frac{u_{1k}}{x_{1k}} (u_{2k} - x_{2k}) - 4\Delta k'_{11} x_{2k} x_{3k} \\ &\quad + 2\Delta k_{31} x_{4k} x_{7k} - 4\Delta \bar{\mu}_{11} x_{10k}. \end{aligned} \quad (26)$$

In this case we construct  $\beta$  and  $\psi_k$  as

$$\begin{aligned} \beta &= [k'_{11} \quad k_{31} \quad \bar{\mu}_{11}]^T, \\ \psi_k &= \Delta [-4x_{2k} x_{3k} \quad 2x_{4k} x_{7k} \quad -4x_{10k}], \\ y_k &= x_{2k+1} - x_{2k} - \Delta \frac{u_{1k}}{x_{1k}} (u_{2k} - x_{2k}). \end{aligned}$$

We choose an initial value  $P_0 = I_{3 \times 3}$  and set  $R_2 = 10^2$ , and  $R_1 = 10^{-2}$ . Similar procedures are carried out for the parameter estimation in equations (22), (23) and (16) and for bacteria equations.

We show in Figure 3 the tracking of  $\text{FeSO}_4$  and  $\text{H}_2\text{SO}_4$  concentrations in the PLS. For the first one, the tracking has some difficulties even adjusting the

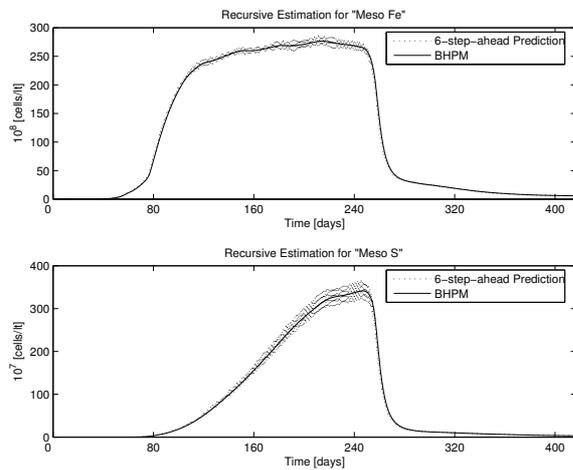


Fig. 4. Recursive 6-step-ahead Prediction using Kalman filtering for Bacteria.

matrices  $R_1$  and  $R_2$  to different values. We attribute these difficulties to a lack of sufficient model structure. Thus, some modifications in the LCM may be required in the future. However, for  $H_2SO_4$  concentration, we can observe in Figure 3 (bottom) almost perfect tracking, which means that in this case the LCM structure has captured the main dynamic features for this variable.

Regarding bacteria activity, we can observe from Figure 4 that they can be followed without difficulties, then the model structure for bacteria in the bioleaching process seems to have been captured adequately. Notice, however, that these predictions are obtained assuming daily access to bacteria measurements, which in a real-world application may be difficult to implement.

#### 4. CONCLUSIONS AND FUTURE RESEARCH

A way to estimate parameters in a model of a bioleaching process have been proposed. We have derived a very simple model from natural laws which is simpler compared with standard models in the literature. Although this simple model is nonlinear, it has the advantage of to be *linear* on its parameters, which is useful for estimation purposes.

The model initially obtained on first principles lack some dynamic structure to generate reasonable predictions for the BHPM data. Hence, some ad-hoc modifications were introduced to obtain the proposed LCM, based on trial-error method. However, a more formal way to deal with lack of structure is given in (Lindskog, 1996) and one of our future research topic will be given in this direction.

The LCM parameters were estimated using data generated by the experimentally validated, high complexity model BHPM. Simulations indicate that in spite of the simplicity of the proposed LCM, the data generated by the BHPM could be adjusted adequately. These observations show that the LCM could be used

as a basis for future control design stage, which is an undeveloped field of study in bioleaching.

Our approach has still the disadvantage of being dependent on a number of measurements, possibly inaccessible in a real-world process. Future research efforts will be concentrated on developing state estimation algorithms using the low complexity structure proposed in this paper. Possible improvements will also be explored following the line of work done by (Lindskog, 1996).

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