TUNING OF CONTROLLERS AND GENERALIZED HOLD FUNCTIONS IN SAMPLED-DATA SYSTEMS USING ITERATIVE FEEDBACK TUNING

Håkan Hjalmarsson * **Julio H. Braslavsky** **

* *Dept. of Signals, Sensors and Systems, The Royal Institute of Technology, S-100 44 Stockholm, Sweden*
** *Dept. of Electrical and Computer Eng., University of Newcastle, Callaghan, 2308, Australia*

Abstract: In this paper it is shown that Iterative Feedback Tuning can be applied to multi-input multi-output linear sampled-data systems where the sensed outputs may be sampled at different rates and where the controllers operate synchronously at one sampling rate which is a multiple of the sampling rates of the controlled variables. It is also shown that generalized sampled-data hold functions (GSDHF) can be tuned. By way of an example, it is shown that one case where GSDHF's may improve the performance significantly is when the continuous time system is minimum phase but the sampled counterpart is non-minimum phase.

Keywords: sampled data systems, sampled-data hold functions, tuning

1. INTRODUCTION

Iterative Feedback Tuning (IFT) is a direct method for controller tuning (Hjalmarsson *et al.*, 1994), (Hjalmarsson *et al.*, 1998). The characteristic feature of this tuning method is that the system itself is used instead of a model. A natural consequence of this is that IFT is robust to the underlying properties of the system. The method was originally derived for single-input single-output (SISO) linear time-invariant (LTI) systems, see (Hjalmarsson *et al.*, 1994), but the same method, with minor or no modifications at all, has later been shown to work for multi-input multi-output (MIMO) LTI systems, (Hjalmarsson and Birkeland, 1998), linear time-varying, in particular periodic, systems, (Hjalmarsson, 1995), as well as for many types of non-linear systems, (Hjalmarsson, 1998). Many applications, both laboratory scale and industry scale, exists, see (Hjalmarsson *et al.*, 1998). The origin of the idea behind IFT can be traced to the beginning of the 1960's (Narendra and Streeter, 1964) and is closely connected to model reference adaptive control.

In this paper we consider linear MIMO sampled data systems. It is shown how to take the intersample behaviour into account in IFT and also how to tune Generalized Sampled Data Hold Functions (GSDHFs).

The outline is as follows. In Section 2 we introduce the problem set-up and in Section 3 some basic properties of sampled data systems are reviewed. The control criterion which will be used is defined in Section 4 and the estimation of the gradient of this criterion w.r.t. controller parameters and GSDHF parameters is discussed in the following two sections. The IFT algorithm is summarized in Section 7 and some numerical examples are given in Section 8.

Notation: $|\cdot|$ is the vector 2-norm. $\mathrm{E}[\cdot]$ denotes expectation. Let $f(\rho)$: $\rho \in \mathbf{R}^{n_\rho} \to \mathbf{R}^n$ be differentiable. Then $\mathrm{d}f(\rho)/\mathrm{d}\rho$ is its derivative which has dimension $\mathbf{R}^{n_\rho}$ when $n=1$ and $\mathbf{R}^{n\times n_\rho}$ otherwise. Right-hand side superscripts are used for vector indices. q^{-1} denote the backward shift operator: $q^{-1}u_k = u_{k-1}$.

2. THE FEEDBACK SYSTEM

Consider the continuous time LTI MIMO system

$$\begin{pmatrix} y(t) \\ w(t) \end{pmatrix} = \begin{pmatrix} G_{yr} & G_{yv} & G_{yw} \\ G_{wr} & G_{wv} & G_{wu} \end{pmatrix} \begin{pmatrix} r(t) \\ v(t) \\ u(t) \end{pmatrix} \triangleq G \begin{pmatrix} r(t) \\ v(t) \\ u(t) \end{pmatrix} \tag{1}$$

where t represents the time, G is the generalized plant which contains the system that is to be controlled but also items that are part of the control design criterion such as frequency weighting filters etc. The signal $r(t) \in \mathbf{R}^{n_r}$ represents external, user selected, signals such as set-points, reference signals etc, $v(t) \in \mathbf{R}^{n_v}$ represents unmeasurable signals such as (process) disturbances and noise, $w(t) \in \mathbf{R}^{n_u}$ represents the sensed outputs and $u(t) \in \mathbf{R}^{n_u}$ represents the control signals. Furthermore $y(t) \in \mathbf{R}^{n_y}$ represents the variables that will be included in the control criterion. These signals can be filtered (frequency weighted) versions of measured signals in the system.

The sensed outputs $w(t)$ are sampled

$$\overline{w}_k = w(kT) \tag{2}$$

and the sampled signals $\overline{w}_k$ are used to generate discrete-time control signals

$$\overline{u}_k = C(\rho)\overline{w}_k \tag{3}$$

where $C(\rho)$ is a diagonal $n_u \times n_u$ discrete-time transfer function matrix parameterized by the parameter vector $\rho = [\rho_1^T, \dots, \rho_{n_u}^T]^T$. The ith diagonal element of $C(\rho)$ is a SISO transfer function denoted by $C_i(\rho_i)$ and is parameterized by $\rho_i \in \mathbf{R}^{n_{\rho_i}}$. Notice that a diagonal C is not a restriction. Any MIMO LTI controller can be realized with the above structure (even though it may be nonminimal) by suitable definitions of $w(t)$, $u(t)$ and $C(\rho)$.

The discrete time control signals $\overline{u}_k$ are fed-back into the continuous time system through a Generalized Sampled-Data Hold Function (GSDHF) (Kabamba, 1987)

$$u(t) = \sum_{l=-\infty}^{\infty} h(t - lT, \eta)\overline{u}_k \tag{4}$$

where $h(t, \eta)$ is a diagonal matrix parameterized by some parameter vector $\eta = [\eta_1^T, \dots, \eta_{n_u}^T]^T$. The ith diagonal element of $h(t, \eta)$ is denoted by $h_i(t, \eta_i)$ and is parameterized by $\eta_i \in \mathbf{R}^{n_{\eta_i}}$. We will assume that the parametrization of the GSDHF's is made in terms of basis functions $p_i^l(t)$:

$$h_i(t, \eta_i) = \sum_{l=1}^{n_{\eta_i}} \eta_i^l p_i^l(t) \tag{5}$$

so that $\eta_i = [\eta_i^1, \dots, \eta_i^{n_{\eta_i}}]^T$.

The continuous time output signal vector $y(t) = [y^1(t), \dots, y^{n_y}(t)]^T$ is sampled. Each signal may be sampled at a different rate:

$$\overline{y}_k^l = y^l(kT_l) \tag{6}$$

The sampled signals $\overline{y}_k^l$ will be used in the control criterion.

The complete sampled-data feedback system is shown in Figure 1. The external, discrete time, signal $\overline{z} \in \mathbf{R}^{n_u}$ in this figure will be used later.

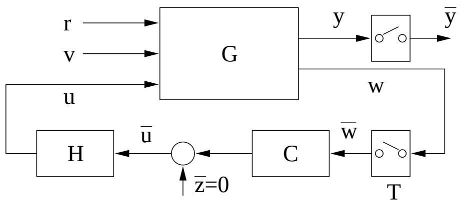

Fig. 1. Feedback system.

We will collect the parameter vectors ρ and η in one single vector $\theta = [\rho^T, \eta^T]^T$. Whenever signals are obtained from the closed loop system with the controller $C(\rho)$ and the generalized hold function $H(\eta)$ operating, we will indicate this by using θ as argument; thus, $y(\theta)$ will denote the output of the system (1) in feedback with the controller (3) and the GSDHF (4).

2.1 Signal assumptions

The following assumption will be used.

V1) In any experiment, the disturbance signal $v(t)$ is a bounded continuous time stochastic process with zero mean. The second order statistics of $v(t)$ are the same for all experiments. Sequences from different experiments are mutually independent. The signal $r(t)$ is bounded.

2.2 Sampling interval assumptions

Throughout the paper, it will be assumed that the sampling interval T of the sensed outputs w in (2) is a multiple of the sampling intervals T_l of the output signals y^l, i.e. there exists positive integers $L_1, \dots, L_{n_y}$ such that

$$T = L_l T_l \tag{7}$$

3. SAMPLED DATA SYSTEMS

A sampled data system is, due to the periodic nature of the sampling, a periodic linear time-varying system. The relation between the continuous time signal w and the sampled signal $\overline{w}$ is given in the Laplace domain by Poisson's summation formula

$$\overline{W}(e^{sT}) = \frac{1}{T} \sum_{l=-\infty}^{\infty} W\left(\frac{sT + i2\pi k}{T}\right) \tag{8}$$

This formula holds if G_{wr}, G_{wv} and G_{wu} are strictly proper. For the GSDHF (4) (H in Figure 1), the relation between the discrete time signal $\overline{u}$ and the continuous time signal $u(t)$ is given by

$$U(s) = H(s)\overline{U}(e^{sT}) \tag{9}$$

This gives the system equations

$$\overline{Y}^l(e^{sT_l}) = \frac{1}{T_l} \sum_{k=-\infty}^{\infty} Y^l\left(\frac{sT_l + i2\pi k}{T}\right) \tag{10}$$

$$Y(s) = G_{yr}(s)R(s) + G_{yv}(s)V(s) + G_{yu}(s)U(s) \tag{11}$$

$$W(s) = G_{wr}(s)R(s) + G_{wv}(s)V(s) + G_{wu}(s)U(s) \tag{12}$$

$$U(s) = H(s)\overline{U}(e^{sT}) \tag{13}$$

$$\overline{U}(e^{sT}) = C(e^{sT})\overline{W}(e^{sT}) + \overline{Z}(e^{sT}) \tag{14}$$

$$\overline{W}(e^{sT}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} W\left(\frac{sT + i2\pi k}{T}\right) \tag{15}$$

For LTI MIMO continuous or discrete time systems it holds that if all external signals are filtered through the same SISO filter, this filtering operation commutes with the closed-loop system, i.e. it can be done at the outputs instead. In the next lemma we show that this holds for sampled data systems if the only external signals are time-discrete. When the sampling intervals at the outputs are shorter than the sampling interval in the feedback system it is, however, necessary to up-sample the filter before the filtering at the output.

Lemma 3.1. Assume that the sampled data closed loop system (10) – (15) is internally stable and that $r \equiv 0$, $v \equiv 0$. Assume that the sampling intervals satisfies (7). Let $F(q)$ be a stable SISO LTI filter and $\{\overline{e}_k\}$, $\overline{e}_k \in \mathbf{R}^{n_u}$, any bounded time-discrete signal. Denote by $\overline{y}_k^l(F)$, the lth sampled output $\overline{y}_k^l$ when $\overline{z}_k = F(q)\overline{e}_k$ i.e. $F(q)$ operates on all elements of $\overline{e}_k$. Then it holds that

$$\overline{y}_k^l(F) = F(q^{T/T_l})\overline{y}_k^l(1)$$

if the filter transients are neglected.

4. THE CONTROL CRITERION

The control criterion is based on an experiment of some time duration T_{exp} which is assumed to be a multiple of the sampling interval T, i.e. $N = T_{exp}/T$ is a positive integer. During this period $N_l = T_{exp}/T_l$ samples are collected from the output $\overline{y}^l$. The control criterion is now defined as

$$J(\theta) = \sum_{l=1}^{n_y} \frac{1}{2N_l} \mathrm{E}\left[\sum_{k=1}^{N_l} \alpha_k^l \ \left|\overline{y}_k^l(\theta)\right|^2\right] \quad (16)$$

where $\mathrm{E}[\cdot]$ denotes expectation w.r.t. the disturbance v. For simplicity, the time weights $\{\alpha_k^l\}$ will be set to one in the following. The criterion (16) is very general and covers many H_2 type disturbance rejection and tracking problems.

One way of minimizing (16) is to do numerical minimization using e.g. a gradient based search:

$$\theta(i+1) = \theta(i) - \gamma_i\,[R(i)]^{-1}\,\frac{\mathrm{d}J(\theta(i))}{\mathrm{d}\theta}. \quad (17)$$

where, with some abuse of notation, $\frac{\mathrm{d}J(\theta(i))}{\mathrm{d}\theta}$ denotes $\frac{\mathrm{d}J(\theta)}{\mathrm{d}\theta}|_{\theta=\theta(i)}$. In (17), $R(i)$ is some appropriate symmetric positive definite matrix, typically an estimate of the Hessian of J, such as a Gauss-Newton approximation.

The gradient of $J(\theta)$ w.r.t. θ is given by

$$\frac{\mathrm{d}J(\theta)}{\mathrm{d}\theta} = \sum_{l=1}^{n_y} \frac{1}{N_l} \mathrm{E}\left[\sum_{k=1}^{N_l} \frac{\mathrm{d}\overline{y}_k^l(\theta)}{\mathrm{d}\theta}\ \overline{y}_k^l(\theta)\right] \quad (18)$$

In order to obtain this gradient, the signals $\overline{y}_k^l(\theta)$ and $\frac{\mathrm{d}\overline{y}_k^l(\theta)}{\mathrm{d}\theta}$ must be available. The former signals can be obtained by performing a closed loop experiment of duration T_{exp} where v and r act as external excitation signals and $\overline{z}$ is set to zero. We will denote this experiment the *normal experiment* and for the corresponding closed loop signals we will use a left-hand subscript 0, e.g. ${}_0\overline{y}_k^l(\theta)$ and ${}_0\overline{w}_k(\theta)$.

The partial derivatives of a signal in the closed loop system w.r.t. ρ and η will be treated below.

5. GRADIENT W.R.T. CONTROLLER

In this section we will se how $\frac{\partial \overline{y}_k^l(\rho)}{\partial\rho}$ can be generated. Recall that the controller blocks C_i are assumed to be independently parameterized: $C_i = C(\rho_i)$. Differentiation of (10) – (15) w.r.t. the mth element ρ_i^m in ρ_i gives

$$\frac{\partial}{\partial\rho_i^m}\overline{Y}^l(e^{sT_l}) = \frac{1}{T_l}\sum_{k=-\infty}^{\infty}\frac{\partial}{\partial\rho_i^m}Y^l\left(\frac{sT_l + i2\pi k}{T}\right) \quad (19)$$

$$\frac{\partial}{\partial\rho_i^m}Y(s) = G_{yu}(s)\ \frac{\partial}{\partial\rho_i^m}U(s) \quad (20)$$

$$\frac{\partial}{\partial\rho_i^m}W(s) = G_{wu}(s)\ \frac{\partial}{\partial\rho_i^m}U(s) \quad (21)$$

$$\frac{\partial}{\partial\rho_i^m}U(s) = H(s)\ \frac{\partial}{\partial\rho_i^m}\overline{U}(e^{sT}) \quad (22)$$

$$\frac{\partial}{\partial\rho_i^m}\overline{U}(e^{sT}) = \left(0,\ldots,\frac{\partial}{\partial\rho_i^m}C_i(e^{sT})\ \overline{W}^i(e^{sT}),0,\ldots\right)^T + C(e^{sT})\ \frac{\partial}{\partial\rho_i^m}\overline{W}(e^{sT}) \quad (23)$$

$$\frac{\partial}{\partial\rho_i^m}\overline{W}(e^{sT}) = \frac{1}{T}\sum_{k=-\infty}^{\infty}\frac{\partial}{\partial\rho_i^m}W\left(\frac{sT + i2\pi k}{T}\right) \quad (24)$$

Consider an experiment where

$$r \equiv 0, \quad v \equiv 0 \quad (25)$$

and $\bar{z} = \left(0,\ldots,0,\ {}_0\overline{w}^i,0,\ldots,0\right)^T$ where ${}_0\overline{w}^i$ is the ith element of the signal vector $\overline{w}$ obtained in a normal experiment. For reasons that will become clear below we will denote this experiment, the *gradient experiment w.r.t.* C_i. The corresponding signals in the closed loop will be denoted by left-hand subscripts such as e.g. ${}_i\overline{y}_k^l(\rho)$. Define

$$\widehat{\frac{\partial}{\partial\rho_i^m}\overline{y}_k^l}(\theta) = \frac{\partial}{\partial\rho_i^m}C_i(q^{T/T_l})\ \ {}_i\overline{y}_k^l(\theta) \quad (26)$$

Now Lemma 3.1 shows that

$$\frac{\partial}{\partial\rho_i^m}\overline{y}_k^l(\theta) = \widehat{\frac{\partial}{\partial\rho_i^m}\overline{y}_k^l}(\theta) \quad (27)$$

Hence the gradient w.r.t. all parameters in ρ_i can be obtained from the gradient experiment w.r.t. C_i. Furthermore, by performing gradient experiments w.r.t. to all controller blocks C_i, $i = 1,\ldots,n_u$ (as many experiments as there are controller blocks), the gradients for all elements in ρ can be generated.

Let us now examine whether it is possible to realize the gradient experiments or not. To set $r \equiv 0$ is normally not a problem but v can not be influenced. However, the sampled data system is linear and the contribution from $\overline{z}$ and v will be additive. Now under assumption V1), the contribution from v in e.g. ${}_i\overline{y}_k^l(\rho)$ will be independent of ${}_0\overline{y}_k^l(\rho)$. This means that the gradient estimate

$$\widehat{\frac{\partial}{\partial\rho}J}(\theta) = \sum_{l=1}^{n_y}\frac{1}{N_l}\mathrm{E}\left[\sum_{k=1}^{N_l}\widehat{\frac{\partial}{\partial\rho}\overline{y}_k^l}(\theta)\ {}_0\overline{y}_k^l(\theta)\right] \quad (28)$$

where $\widehat{\frac{\partial}{\partial\rho}\overline{y}_k^l}(\theta)$ is constructed by stacking the $\widehat{\frac{\partial}{\partial\rho_i^m}\overline{y}_k^l}(\theta)$ on top of each other in a vector, is an *unbiased* estimate of $\frac{\partial}{\partial\rho}J(\theta)$. For details of this we refer to (Hjalmarsson *et al.*, 1998).

6. GRADIENT W.R.T. GSDHF

In this section we will show how an unbiased estimate of the partial derivative of J w.r.t. the parameters η in the GSDHFs can be computed. We start with the system equations (10) – (15) but this time we differentiate w.r.t. the mth element η_i^m in η_i. This gives

$$\frac{\partial}{\partial \eta_i^m}\overline{Y}^l(e^{sT_l}) = \frac{1}{T_l}\sum_{k=-\infty}^{\infty}\frac{\partial}{\partial \eta_i^m}Y^l\left(\frac{sT_l+i2\pi k}{T}\right) \quad (29)$$

$$\frac{\partial}{\partial \eta_i^m}Y(s) = G_{yu}(s)\,\frac{\partial}{\partial \eta_i^m}U(s) \quad (30)$$

$$\frac{\partial}{\partial \eta_i^m}W(s) = G_{wu}(s)\,\frac{\partial}{\partial \eta_i^m}U(s) \quad (31)$$

$$\frac{\partial}{\partial \eta_i^m}U(s) = \left(0,\dots,P_i^m(s)\;\overline{U}^i(e^{sT}),0,\dots\right)^T + H(s)\,\frac{\partial}{\partial \eta_i^m}\overline{U}(e^{sT}) \quad (32)$$

$$\frac{\partial}{\partial \eta_i^m}\overline{U}(e^{sT}) = C(e^{sT})\,\frac{\partial}{\partial \eta_i^m}\overline{W}(e^{sT}) \quad (33)$$

$$\frac{\partial}{\partial \eta_i^m}\overline{W}(e^{sT}) = \frac{1}{T}\sum_{k=-\infty}^{\infty}\frac{\partial}{\partial \eta_i^m}W\left(\frac{sT+i2\pi k}{T}\right) \quad (34)$$

Equations (29) – (34) show that the gradient w.r.t. η_i^m can be obtained by performing an experiment where $r \equiv 0$, $v \equiv 0$, $\overline{z} \equiv 0$ but where the ith control signal $u^i(t)$ contains an externally injected signal

$$\sum_{k=-\infty}^{\infty} h_i^m(t-kT)\;{}_0\overline{u}_k^i$$

where ${}_0\overline{u}_k$ is from the normal experiment described in Section 4. We will denote this experiment, the *gradient experiment w.r.t.* η_i^m. This means that the continuous time control signals are given by

$$ {}_i^m u^i(t) = \sum_{k=-\infty}^{\infty} h_i(t-kT)\;{}_i^m\overline{u}_k^i + \sum_{k=-\infty}^{\infty} h_i^m(t-kT)\;{}_0\overline{u}_k^i$$

$$ {}_i^m u^j(t) = \sum_{k=-\infty}^{\infty} h_j(t-kT)\;{}_i^m\overline{u}_k^j, \quad j \neq i$$

where we have used left-hand side subscript i and superscript m to denote signals in the experiment that generates the gradient w.r.t. η_i^m.

We define

$$\widehat{\frac{\partial}{\partial \eta_i^m}\overline{y}_k^l}(\theta) = {}_i^m\overline{y}_k^l(\theta) \quad (35)$$

As noted in the previous section, it is not possible to generate noise free experiments so the disturbance ${}_i^m v$ in the gradient experiment will perturb the gradients. However, using the same arguments as in Section 5, the noise contribution will be additive and the gradient estimate

$$\widehat{\frac{\partial}{\partial \eta}J}(\theta) = \sum_{l=1}^{n_y}\frac{1}{N_l}\mathrm{E}\left[\sum_{k=1}^{N_l}\widehat{\frac{\partial}{\partial \eta}\overline{y}_k^l}(\theta)\;\overline{y}_k^l(\theta)\right] \quad (36)$$

where $\widehat{\frac{\partial}{\partial \eta}\overline{y}_k^l}(\theta)$ is constructed by stacking the $\widehat{\frac{\partial}{\partial \eta_i^m}\overline{y}_k^l}(\theta)$ on top of each other in a vector, is an unbiased estimate of $\frac{\partial}{\partial \eta}J(\theta)$.

7. THE IFT ALGORITHM

An estimate, $\widehat{\frac{\mathrm{d}}{\mathrm{d}\theta}J}(\theta)$, of the total gradient $\frac{\mathrm{d}}{\mathrm{d}\theta}J(\theta)$ can be obtained by stacking $\widehat{\frac{\partial}{\partial \rho}J}(\theta)$, see (28) and $\widehat{\frac{\partial}{\partial \eta}J}(\theta)$, see (36), on top of each other in a vector. The IFT algorithm can be summarized as follows: At iteration i perform a number of closed loop experiments on the plant with the *same* controller parameters $\rho(i)$ and parameters $\eta(i)$ in the GSDHFs such that gradient estimates (28) and (36) which are unbiased can be formed. Then update the controller parameters according to

$$\theta(i+1) = \theta(i) - \gamma_i\,[R(i)]^{-1}\,\widehat{\frac{\mathrm{d}}{\mathrm{d}\theta}J}(\theta(i)) \quad (37)$$

where γ_i is the step-size and where $R(i)$ is a symmetric positive definite matrix.

8. NUMERICAL ILLUSTRATIONS

Example 8.1. The system in this example is non-minimum phase and is given by

$$G(s) = \frac{s-5}{s^2+4s+3}$$

This system has previously been used in (Er and Anderson, 1994). The system is sampled with $T = 0.04$ and its sampled counterpart is

$$G_d = \frac{0.0331q^{-1} - 0.0405q^{-2}}{1 - 1.8477q^{-1} + 0.8521q^{-2}}$$

The output of the system is corrupted by a disturbance, which at the sampling instances is given by

$$\overline{v}_k = \frac{1}{1 - 1.8477q^{-1} + 0.8521q^{-2}}\overline{e}_k$$

where $\overline{e}_k$ is zero mean white noise with unit variance.

The controller is given by

$$C(\rho) = 1 + \rho^1 q^{-1} + \rho^2 q^{-2} + \rho^3 q^{-3}$$

and the GSDHF is given by

$$h(t) = \eta^1 p^1(t) + \eta^2 p^2(t)$$

where the basis functions are rectangular pulses

$$p^i(t) = \begin{cases} 1 & (i-1)*T/2 \le t < i\,T/2 \\ 0 & \text{otherwise} \end{cases}$$

A minimum variance control criterion

$$J(\theta) = \mathrm{E}[|\overline{y}_k(\theta)|^2]$$

is used. The optimal LQG controller has a cost of $J_{LQG} = 63.7$. This is thus the lowest cost that can be obtained using conventional discrete time control with this sampling interval.

The experiment length was taken as $T_{exp} = 200$ s. In the gradient experiments, the excitation signals were up-scaled a factor 10 to improve the signal to noise ratio. Initially, the controller was given by

$$C(\rho) = 1 - 0.9q^{-1} + 0q^{-2} + 0q^{-3}$$

and a zero order hold function ($\eta^1 = \eta^2 = 1$) was used. This configuration gave a cost of $J_0 = 353$.

After 9 iterations the cost had decreased to $J_9 = 11$, the corresponding controller was

$$C(\rho) = 1-1709.515q^{-1}+2105.200q^{-2}-440.947q^{-3}$$

and the GSDHF parameters were $\eta^1 = 2.248$ and $\eta^2 = -2.003$. The cost function is shown in Figure 2. The improvement in performance is quite striking. Notice especially that the performance is significantly improved over the LQG controller using a zero-order hold function. However, it has been pointed out (Freudenberg *et al.*, 1997) that for non-minimum phase systems the intersample behaviour when using GSDHFs can be bad. This is also what happens here; If the sampling interval with which the output y is sampled in the criterion is changed to 0.004 while keeping the same sampling interval $T = 0.04$ for the feedback system as above, the cost function becomes $J = 1373$. This problem can also be illustrated using fundamental sensitivity functions which govern the first harmonic of the frequency response of a sampled-data system. Such functions embody intersample information and can be used to quickly analyze robustness and sensitivity properties of the SD system in continuous-time, (Freudenberg *et al.*, 1997) and (Freudenberg *et al.*, 1995). In the upper part of Figure 4 such a sensitivity function is shown for the closed loop system with the final controller in the loop. It is seen that there is a frequency band where the sensitivity function is very large which is not the case with the corresponding discrete time sensitivity function, shown in the lower figure.

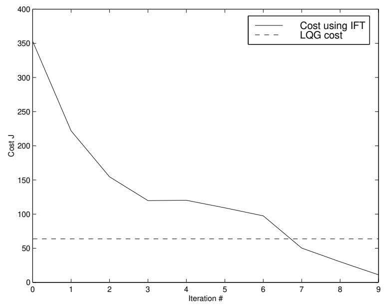

Fig. 2. Evolution of cost function in Example 8.1.

With this set-up, where the criterion is based on a rapidly sampled output, the improvement using GSDHFs compared to a fixed zero-order hold is very small. In the next example we will see a case where a GSDHF is useful. ■

Example 8.2. In this example, we use the same problem set-up as in Example 8.1 including the up-scaling of the excitation signals in the gradient experiments. The continuous time system is, however, now minimum-phase and given by $G(s) = \frac{10000}{(s+3)^3}$ The sampling time is $T = 0.1$ which gives the discrete-time counterpart

$$G_d(q) = \frac{1.333q^{-1} + 4.265q^{-2} + 0.85q^{-3}}{1 - 2.222q^{-1} + 1.646q^{-2} - 0.4066q^{-3}}$$

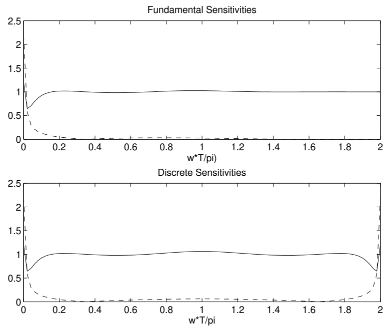

Fig. 3. Dashed line: Complementary sensitivity function. Solid line: Sensitivity function. Example 8.1 with the initial controller. Upper graph: Continuous time. Lower graph: Discrete time.

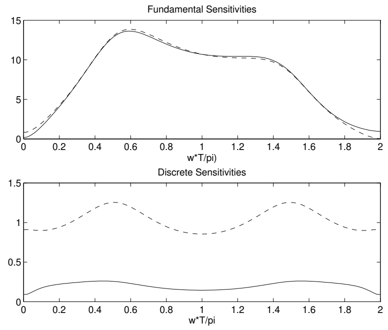

Fig. 4. Sensitivity functions in Example 8.1 with the final controller. For legends see Figure 3.

The disturbance is given by

$$\overline{v}_k = \frac{1}{1 - 2.222q^{-1} + 1.646q^{-2} - 0.4066q^{-3}} \overline{e}_k$$

where $\overline{e}_k$ is zero mean white noise with unit variance. If the control was done in continuous time, the minimum cost function would be $1/2$ since the variance of the driving noise is 1.

However, even though the continuous time system is minimum-phase, the discretized system has a non-minimum phase zero at -2.9858. This zero imposes a design limitation and the cost for conventional LQG (or minimum variance) control is $J_{LQG} = 1.43$. The structure of the controller that will be tuned is

$$C(\rho) = 1 + \rho_1 q^{-1} + \rho_2 q^{-2} \qquad (38)$$

The GSDHF uses 3 rectangular basis functions

$$h(t) = \eta^1 p^1(t) + \eta^2 p^2(t) + \eta^3 p^3(t)$$

where $p^i(t) = \begin{cases} 1 & (i-1)T/3 \le t < i\,T/3 \\ 0 & \text{otherwise} \end{cases}$

Initially, the controller was given by

$$C(\rho) = 1 - 0.9q^{-1} + 1q^{-2}$$

and a zero order hold function ($\eta^1 = \eta^2 = \eta^3 = 1$) was used. The experiment length was $T_{exp} = 100$ s. This configuration gave a cost of $J_0 = 15.9$. After 12 iterations the cost was down at $J_{12} = 0.54$, the controller was

$$C(\rho) = 1 - 0.448q^{-1} + 0.313q^{-2}$$

and the GSDHF parameters were $\eta^1 = 3.873$, $\eta^2 = -3.433$ and $\eta^3 = 0.023$. The evolution of the cost function is shown in Figure 6. Clearly in the last iterations, the performance is very close to the ideal continuous time controller performance and significantly better than the conventional discrete time LQG controller even though this controller is not included in the used controller structure.

In this case, the continuous time system is minimum-phase and one should not expect any severe intersample behaviour problem. However, Figure 5, shows that the fundamental sensitivity function (upper figure) has a high peak. Also, if the sampling interval with which the output y is sampled in the criterion is changed to 0.01 while keeping the same sampling interval $T = 0.1$ as before for the feedback system, the cost function becomes higher $J = 0.9$. An increase compared to the cost at the sampling intervals but still better than the cost of 1.43 for conventional LQG (minimum variance) control. Tuning the controller and

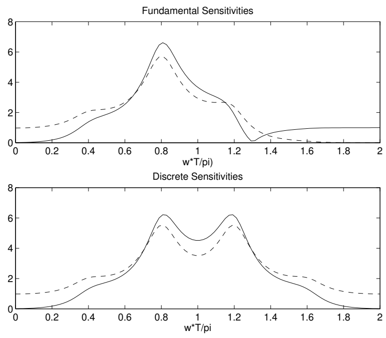

Fig. 5. Sensitivity function in Example 8.2 with the controller after 12 iterations. For legends see Figure 3.

the GSDHF in this new-set up result in a cost of $J = 0.56$ after 3 additional iterations. The controller is then given by

$$C(\rho) = 1 + 0.922q^{-1} + 0.305q^{-2}$$

and the GSDHF parameters are $\eta^1 = 2.585$, $\eta^2 = -2.151$ and $\eta^3 = 0.009$. Figure 7 shows that now the fundamental sensitivity function behaves well and is also very close to the discrete time counterpart. Therefore the performance is good and the intersample behaviour does not cause any problems. From this we conclude that GSDHF can indeed be useful when the sampling introduces non-minimum phase zeros but that it is important to take the intersample behaviour into account in the tuning. ∎

9. CONCLUSIONS

In this contribution we have shown how one can tune sampled-data controllers and Generalized Sampled-Data Hold Functions in a model-free way. The method was tested on two examples and it was pointed out that systems where the

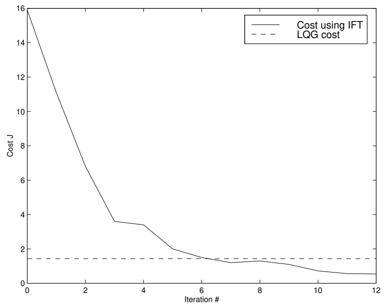

Fig. 6. Evolution of cost function in Example 8.2.

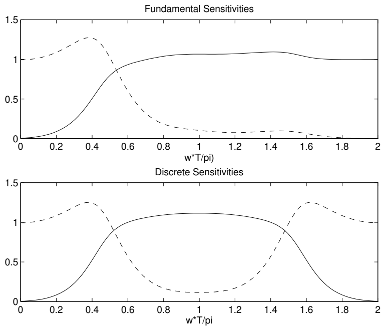

Fig. 7. Sensitivity function in Example 8.2 with the controller after 15 iterations. For legends see Figure 3.

sampling introduces non-minimum phase zeros is a potential application for the method.

10. REFERENCES

Er, M.J. and B.D.O. Anderson (1994). Discrete-time loop transfer recovery via genearlized sampled-data hold functions based compensator. *Int. J. Robust Nonlinear Contr.* **4**(6), 741–756.

Freudenberg, J.S., R.H. Middleton and J.H. Braslavsky (1995). Inherent design limitations for linear sampled-data feedback systems. *International Journal of Control* **61**(6), 1387–1421.

Freudenberg, J.S., R.H. Middleton and J.H. Braslavsky (1997). Robustness of zero shifting via generalized sampled-data hold functions. *IEEE Trans. Automatic Control* **42**, 1681–1692.

Hjalmarsson, H. (1995). Model-free tuning of controllers: Experience with time-varying linear systems. In: *Proceedings of European Control Conference*. Rome, Italy. pp. 2869–2874.

Hjalmarsson, H. (1998). Control of nonlinear systems using Iterative Feedback Tuning. In: *Proc. 1998 American Control Conference*. Philadelphia. pp. 2083–2087.

Hjalmarsson, H. and T. Birkeland (1998). Iterative feedback tuning of linear time-invariant MIMO systems. In: *CDC-98*. Tampa.

Hjalmarsson, H., M. Gevers and O. Lequin (1998). Iterative feedback tuning: theory and applications. *IEEE Control Systems Magazine* **18**(4), 26–41.

Hjalmarsson, H., S. Gunnarsson and M. Gevers (1994). A convergent iterative restricted complexity control design scheme. In: *Proc. 33rd IEEE CDC*. Orlando, Florida. pp. 1735–1740.

Kabamba, P.T. (1987). Control of linear systems using generalized sampled-data hold functions. *IEEE Trans. on Automatic Control* pp. 772–783.

Narendra, K. S. and D.N. Streeter (1964). An adaptive procedure for controlling undefined linear processes. *IEEE Trans. Automatic Control* pp. 545–548.