Proceedings of the 34th Chinese Control Conference
July 28-30, 2015, Hangzhou, China

Consensusability of Multi-Agent Systems with Constant Communication Delay

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Abstract: This paper focus on the consensusability of high-order multi-agent systems (MASs) with uniform constant communication delay in an undirected network. We show that N agents achieve consensus if and only if $N-1$ time-delay subsystems associated with the eigenvalues of the Laplacian matrix are simultaneously asymptotically stable. By employing a linear matrix inequality (LMI) method, we present a sufficient condition for MASs to reach consensus. Also we consider the consensus condition of first-order integrator system by using frequency domain analysis.

Key Words: Multi-agent systems, Consensusability, Linear matrix inequality, Frequency domain

1 Introduction

Distributed coordination control of multi-agent networks is a basic problem, and one of the most fundamental problems is consensus control. Consensus control problem has been widely studied due to its relevance in distributed computation, biological group behaviors such as swarms and flocks ([3]), etc. The key of consensus control is to design an appropriate consensus protocol using locally exchanged information such that all the agents in a network agree upon certain quantities of common interest.

The seminal work [1] solved the consensus problem and average-consensus problem of first-order integrator networks with and without time delay by using algebraic graph theory and frequency domain analysis. In [2], consensus protocols were designed for both the first-order integral linear multi-agent systems (MASs) and discrete-time MASs. In [6] Ma et.al. considered the consensusability of linear MASs without delay and showed that the consensusability of MASs depends on the dynamic structure of each agent and the communication network topology among agents. Reference [7] studied the consensus conditions of first-order integrator systems under both directed and undirected communication network topologies. Consensus using quantized information has also been considered in [10].

All of the existing works on the consensus problem focus on first-order integrator networks or networks without delay [8]-[9]. However, in practical engineering applications, time delay is always inevitable in the process of information exchange between agents. As we all know that the frequency domain analysis is effective for stability analysis of linear time-invariant systems. But this method is only applicable to some special systems with time delay due to the fact that the corresponding characteristic equation becomes a transcendental function. The linear matrix inequality method has been used for stability analysis of general linear systems with time delay [4]-[5].

In this paper, we consider the consensus condition for a class of general continuous-time MASs in an undirected network with N nodes. Since the presence of communication delay, our consensus protocol contains a delay in the relative state information. By introducing some linear transformation, we will prove that the consensuability of MASs with N agents is equivalent to that $N-1$ time-delay subsystems associated with the eigenvalues of Laplacian matrix are simultaneously asymptotically stable. And there is a common gain matrix K in all of the $N-1$ time-delay subsystems, so the key of consensusability is the existence of the common K.

We consider state feedback for a linear time-invariant system with input time-delay. By constructing a common Lyapunov function, we obtain a sufficient condition in the form of linear matrix inequality (LMI) for the stability of one such system with the same state and input matrix as the $N-1$ time-delay subsystems. If the LMI holds with the corresponding parameter equal to the eigenvalues of Laplacian matrix, then we can obtain a gain matrix for the consensus protocol to guarantee the consensusability of the MASs.

Apparently, this result can be directly applied to first-order integrator systems, but for this case we employ a more direct frequency-domain analysis method to analyze the consensus condition. We prove that by choosing a appropriate gain, systems achieve consensus whether the time delay is big or small.

Notations: R denotes the real number field; $R^{m\times n}$ denotes the family of $m\times n$ dimensional real matrices; I_m denotes the $m\times m$ dimensional identity matrix; $\mathbf{1}_m$ denotes the m dimensional column vector with all components 1; $\mathbf{0}_m$ denotes the m dimensional column vector with all components 0; $\otimes$ denotes the Kronecker product; $X<0(>0)$ means that the matrix X is negative (positive) definite and $X^T=X$ with X^T denotes its transpose; $\lambda_i(X)$ denotes the ith eigenvalue of a matrix X; $x_{i,k\sim l}$ denotes a column vector composed of the kth component to the lth component of column vector x_i.

2 Algebraic Graph Theory

Let the directed graph $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{A}\}$ denotes the communication topology between multi-agents with the set of ver-

This work is supported by the Taishan Scholar Construction Engineering by Shandong Government, the National Natural Science Foundation of China under Grants 61120106011, 61203029.

tices $\mathcal{V} = \{1, 2, \ldots, N\}$ and the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The ith vertex represents the ith agent and the edge (i, j) denotes denotes that the agent j obtains information from the agent i. $\mathcal{E} \subset \{(i, j) : i, j \in \mathcal{V}\}$ is the edge set. The set of neighbors of the ith agent is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$. $\mathcal{A} = [a_{ij}] \in R^{N \times N}$ is called the weighted adjacency matrix of $\mathcal{G}$ with nonnegative elements and $a_{ij} > 0$ if and only if $i \in \mathcal{N}_i$. The in-degree of the ith vertex is denoted by $d_i = \sum_{j \in \mathcal{N}_i} a_{ij} = \sum_{j=1}^{N} a_{ij}$ and the in-degree matrix $\mathcal{D} = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$. The Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ is defined by $\mathcal{L} = \mathcal{D} - \mathcal{A}$. Note that $a_{ij} = a_{ji}, \forall i, j \in \mathcal{V}$ if and only if $\mathcal{G}$ is an undirected graph. Obviously, for an undirected graph, $\mathcal{L}$ is a symmetric, positive semi-definite matrix and all its eigenvalue are non-negative. Note that $\mathcal{L}\mathbf{1}_N = \mathbf{0}_N$. In this paper, x_i denotes the state of the ith node. For an undirected connected graph, the eigenvalues of $\mathcal{L}$ can be arranged as follows

$$0 = \lambda_1(\mathcal{L}) < \lambda_2(\mathcal{L}) \le \ldots \le \lambda_N(\mathcal{L}).$$

3 Consensus and Consensusability

In this paper, we will consider the consensus control for a network of continuous-time high-order linear time-invariant agents with the following dynamics

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, \ldots, N \tag{1}$$

where $x_i(t) \in R^n$ and $u_i(t) \in R^p$ are the state and control input of the ith agent. $A \in R^{n \times n}$, $B \in R^{n \times p}$ are constant matrices and the initial state is denoted by $x_i(0)$.

Definition 1 (consensus) The agents in the network achieve consensus if

$$\lim_{t \to \infty} \|x_j(t) - x_i(t)\| = 0, \quad \forall i, j \in \{1, \ldots, N\}$$

for any initial value $x_i(0)$.

Due to the presence of communication delay, the consensus protocol of the ith agent is given by

$$u_i(t) = K \sum_{j=1}^{N} a_{ij}(x_j(t - \tau) - x_i(t - \tau)) \tag{2}$$

where $K \in R^{p \times n}$ is a gain matrix. The time delay $\tau \in [0, \tau^*]$, $\tau^* \in R$ is known.

Let $u(t) = [u_1^T, \ldots, u_N^T]^T$, we consider the following admissible control set:

$$\begin{aligned} \mathcal{U} = \Big\{ & u(t) : [0, \infty) \to R^{pN} | \\ & u_i(t) = K \sum_{j=1}^{N} a_{ij}(x_j(t - \tau) - x_i(t - \tau)), \\ & \forall t \ge 0,\ K \in R^{p \times n},\ i = 1, \ldots, N \Big\}. \end{aligned}$$

Definition 2 (consensusable) For the system (1), if there is a $u(t) \in \mathcal{U}$ such that system (1) reaches consensus, then we say that the system (1) is consensusable with respect to (w.r.t.) $\mathcal{U}$.

It is obviously that if A is stable, then system (1) can reaches consensus by taking $u_i = 0$, so for the sake of making this problem interesting, without loss of generality, we assume that A is unstable (including the case where A has eigenvalues with zero real part).

In this paper, we focus on the consensusability of system (1) under the consensus protocol (2). Note from [6] that, when $\tau = 0$, the corresponding delay-free system can reach consensus under the following assumptions, which we will adopt for the delay case as well.

Assumption 1 The network topology $\mathcal{G}$ is an undirected connected graph.

Assumption 2 (A, B) is stabilizable.

We introduce a lemma which will be used in the proof the our main results.

Lemma For any constant matrix $M \in R^{m \times m}$, $M = M^T > 0$, scalar $\gamma > 0$, vector function $w : [0, \gamma] \to R^m$, such that the integrations in the following are well defined, then

$$\begin{aligned} & \gamma \int_{t-\gamma}^{t} w^T(s) M w(s) ds \\ & \ge \left(\int_{t-\gamma}^{t} w(s) ds \right)^T M \left(\int_{t-\gamma}^{t} w(s) ds \right). \end{aligned}$$

4 Consensusability of High-order Multi-agent Systems with Constant Communication Delay

Let $\delta_i(t) = x_1(t) - x_i(t)$, $i = 2, \ldots, N$, then we can obtain the dynamic of $\delta_i(t)$ as follows

$$\begin{aligned} \dot{\delta}_i(t) \ = \ & A\delta_i(t) + BK \Big[\sum_{j=2}^{N} (a_{ij} - a_{1j}) \delta_j(t - \tau) \\ & - d_i \delta_i(t - \tau) \Big], \quad i = 2, \ldots, N. \end{aligned}$$

Define $\delta(t) = \left[\delta_2^T(t), \ldots, \delta_N^T(t) \right]^T$, then we have

$$\begin{aligned} \dot{\delta}(t) \ = \ & (I_{N-1} \otimes A)\delta(t) - \left[(L_{22} + \mathbf{1}_{N-1}\alpha^T) \otimes BK \right] \\ & \delta(t - \tau) \end{aligned} \tag{3}$$

where

$$L_{22} = \begin{bmatrix} d_2 & -a_{23} & \cdots & -a_{2N} \\ -a_{32} & d_3 & \cdots & -a_{3N} \\ \vdots & \vdots & \vdots & \vdots \\ -a_{N2} & -a_{N3} & \cdots & d_N \end{bmatrix}, \ \alpha = \begin{bmatrix} a_{12} \\ a_{13} \\ \vdots \\ a_{1N} \end{bmatrix}.$$

It is obvious that systems (1)-(2) achieving consensus is also equivalent to $\lim_{t \to \infty} \|\delta(t)| = 0$. So next we will focus on the stability condition for system (3).

Note that $\mathcal{L} = \begin{bmatrix} d_1 & -\alpha^T \\ -\alpha & L_{22} \end{bmatrix}$ is a symmetric matrix, so L_{22} is also a symmetric matrix. Taking $S = \begin{bmatrix} 1 & \mathbf{0}_{N-1}^T \\ \mathbf{1}_{N-1} & I_{N-1} \end{bmatrix}$, then we have

$$S^{-1} \mathcal{L} S = \begin{bmatrix} 0 & -\alpha^T \\ \mathbf{0}_{N-1} & L_{22} + \mathbf{1}_{N-1}\alpha^T \end{bmatrix}. \tag{4}$$

From (4) we can see that the eigenvalues of $L_{22} + \mathbf{1}_{N-1}\alpha^T$ are the nonzero eigenvalues of $\mathcal{L}$, i.e., $\lambda_2(\mathcal{L}), \cdots, \lambda_N(\mathcal{L})$.

Claim Let v_i denote the right eigenvector corresponding to the eigenvalue $\lambda_i(\mathcal{L})$, $i = 2, \ldots, N$, then $v_{i,2\sim N}^T$ is the left eigenvector of $L_{22} + \mathbf{1}_{N-1}\alpha^T$ corresponding to the same eigenvalue $\lambda_i(\mathcal{L})$, $i = 2, \ldots, N$.

Proof of claim In fact, we have

$$\mathcal{L}v_i = \begin{bmatrix} d_1 v_{i,1} - \alpha^T v_{i,2\sim N} \\ -\alpha v_{i,1} + L_{22} v_{i,2\sim N} \end{bmatrix} = \begin{bmatrix} \lambda_i(\mathcal{L}) v_{i,1} \\ \lambda_i(\mathcal{L}) v_{i,2\sim N} \end{bmatrix},$$

so

$$-\alpha v_{i,1} = \lambda_i(\mathcal{L}) v_{i,2\sim N} - L_{22} v_{i,2\sim N}. \qquad (5)$$

It is obviously that $\mathbf{1}_N^T v_i = 0$, $i = 2, \ldots, N$, thus we have

$$-\alpha v_{i,1} = \alpha \mathbf{1}_{N-1}^T v_{i,2\sim N}. \qquad (6)$$

From (5) and (6), we have

$$v_{i,2\sim N}^T (L_{22} + \mathbf{1}_{N-1}\alpha^T) = \lambda_i(\mathcal{L}) v_{i,2\sim N}^T.$$

That is to say, the component of the right eigenvector of $\mathcal{L}$ is the left eigenvector of $L_{22} + \mathbf{1}_{N-1}\alpha^T$ corresponding to the same eigenvalue $\lambda_i(\mathcal{L})$, $i = 2, \ldots, N$.

Because $\mathcal{L}$ is a symmetric matrix, it can be diagonalized by using its eigenvector matrix, i.e., there exists a matrix $U = [u_1, u_2, \ldots, u_N]$ composed of the mutually orthogonal eigenvectors of $\mathcal{L}$ such that

$$U^{-1}\mathcal{L}U = \mathrm{diag}\{0, \lambda_2(\mathcal{L}), \cdots, \lambda_N(\mathcal{L})\}.$$

We take $V = [u_{2,2\sim N}, \ldots, u_{N,2\sim N}]^T$, then V is an invertible matrix, and we have

$$V(L_{22} + \mathbf{1}_{N-1}\alpha^T)V^{-1} = \mathrm{diag}\{\lambda_2(\mathcal{L}), \ldots, \lambda_N(\mathcal{L})\}.$$

Let $\tilde{\delta}(t) = (V \otimes I_n)\delta(t)$, and we can obtain its dynamics as follws:

$$\begin{aligned}\dot{\tilde{\delta}}(t) &= (I_{N-1} \otimes A)\tilde{\delta}(t) - \big[\mathrm{diag}\{\lambda_2(\mathcal{L}), \cdots, \lambda_N(\mathcal{L})\} \\ &\quad \otimes BK\big]\tilde{\delta}(t - \tau).\end{aligned}$$

Take $\tilde{\delta}(t) = \left[\tilde{\delta}_2^T(t), \cdots, \tilde{\delta}_N^T(t)\right]^T$, then we can obtain the following theorem:

Theorem 1 Systems (1)-(2) achieve consensus if and only if the following $N-1$ time-delay subsystems associated with the eigenvalues of Laplacian matrix are simultaneous asymptotically stable:

$$\dot{\tilde{\delta}}_i(t) = A\tilde{\delta}_i(t) - \lambda_i(\mathcal{L})BK\tilde{\delta}_i(t - \tau), \quad i = 2, \ldots, N, \quad (7)$$

where the definitions of $\tilde{\delta}_i(t)$ can be found in the analysis above.

Remark 1 That is to say, if we find a matrix K such that every time-delay subsystem of (7) is stable, then we can say that system (1) is consensusable w.r.t. $\mathcal{U}$.

Theorem 2 For a given constant $\bar{\tau}$, if there exist symmetric positive definite matrices W, X and Y of appropriate dimensions such that the following LMI holds:

$$\begin{bmatrix} XA^T + AX - aY^TB^T - aBY & -aBY & \bar{\tau}(XA^T - aY^TB^T)W \\ -aY^TB^T & -XWX & -a\bar{\tau}Y^TB^TW \\ \bar{\tau}W(AX - aBY) & -a\bar{\tau}WBY & -W \end{bmatrix} < 0, \quad (8)$$

then the following system

$$\dot{\eta} = A\eta(t) - aBu(t - \tau), \quad a \in R \qquad (9)$$

with the state feedback control law given by

$$u(t) = F\eta(t) \qquad (10)$$

is asymptotically stable for any $\tau \in [0, \bar{\tau}]$ and the gain matrix can be taken as $F = YX^{-1}$.

Proof: Systems (9)-(10) can be rewritten as

$$\dot{\eta} = A\eta(t) - aBF\eta(t - \tau). \qquad (11)$$

We introduce in a Lyapunov functional for (11) as follows:

$$V(\eta(t)) = \eta^T(t)S\eta(t) + \bar{\tau}\int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{\eta}^T(s)W\dot{\eta}(s)ds,$$

where S and W are both symmetric positive definite matrices. Thus we can obtain the derivative of $V(\eta(t))$ about t as follows

$$\begin{aligned}\dot{V}(\eta(t)) &= \dot{\eta}^T(t)S\eta(t) + \eta^T(t)S\dot{\eta}(t) + \bar{\tau}\tau\dot{\eta}^T(t)W\dot{\eta}(t) \\ &\quad -\bar{\tau}\int_{t-\tau}^{t}\dot{\eta}^T(s)W\dot{\eta}(s).\end{aligned}$$

Then from lemma we have

$$\begin{aligned}\dot{V}(\eta(t)) &\leq \dot{\eta}^T(t)S\eta(t) + \eta^T(t)S\dot{\eta}(t) + \bar{\tau}\tau\dot{\eta}^T(t)W\dot{\eta}(t) \\ &\quad -\frac{\bar{\tau}}{\tau}\left(\int_{t-\tau}^{t}\dot{\eta}^T(s)ds\right)W\left(\int_{t-\tau}^{t}\dot{\eta}^T(s)ds\right).\end{aligned}$$

Since $W > 0$, then we have

$$\begin{aligned}\dot{V}(\eta(t)) &\leq \dot{\eta}^T(t)S\eta(t) + \eta^T(t)S\dot{\eta}(t) + \bar{\tau}\tau\dot{\eta}^T(t)W\dot{\eta}(t) \\ &\quad -\left(\int_{t-\tau}^{t}\dot{\eta}^T(s)ds\right)W\left(\int_{t-\tau}^{t}\dot{\eta}^T(s)ds\right) \\ &= \xi^T(t)Z\xi(t),\end{aligned}$$

where $\xi(t) = [\eta^T(t), \eta^T(t-\tau)]^T$ and

$$Z = \begin{bmatrix} A^TS + SA + \bar{\tau}^2A^TWA - W & -aSBF - a\bar{\tau}^2A^TWBF + W \\ -aF^TB^TS - a\bar{\tau}^2F^TB^TWA + W & a^2\bar{\tau}^2F^TB^TWBF - W \end{bmatrix}.$$

If the LMI (8) holds, take $S = X^{-1}$ and $F = YS$, then we have

$$\begin{bmatrix} S^{-1}A^T + AS^{-1} - aS^{-1}F^TB^T - aBFS^{-1} & -aBFS^{-1} & \bar{\tau}S^{-1}(A^T - aF^TB^T)W \\ -aS^{-1}F^TB^T & -S^{-1}WS^{-1} & -a\bar{\tau}S^{-1}F^TB^TW \\ \bar{\tau}W(A - aBF)S^{-1} & -a\bar{\tau}WBFS^{-1} & -W \end{bmatrix} < 0.$$

By linearization technique, we obtain the following equivalent LMI

$$\begin{bmatrix} A^TS + SA - W & -aSBF + W & A^T \\ -aF^TB^TS + W & -W & -aF^TB^T \\ A & -aBF & -\frac{1}{\bar{\tau}^2}W^{-1} \end{bmatrix} < 0.$$

By using Schur's complement, we see that $Z < 0$, so $\dot{V}(\eta(t)) < 0$, thus from Lyapunov stability theorem we know that (9)-(10) are stable with $F = YX^{-1}$.

Remark 2 If the LMI (8) holds, then we have $XA^T + AX - aY^TB^T - aBY < 0$, so the matrix $A - aBF$ is a stable matrix. That is to say, when $\tau = 0$, the corresponding delay-free system is asymptotically stable.

From Theorems 1 and 2, we obtain the following result:

Corollary 1 Systems (1)-(2) reach consensus if the LMI (8) holds for all $a = \lambda_i(\mathcal{L})$ with $W = W_i$, $i = 2, \ldots, N$. In this case, the common gain matrix can be taken as $K = YX^{-1}$.

5 Consensusability of First-order Integrator System with Constant Communication Delay

Next we will consider the consensusability of a class of first-order integrator system:

$$\dot{x}_i(t) = u_i(t), \quad i = 1, \cdots, N \tag{12}$$

with the consensus protocol of the ith agent is given by

$$u_i(t) = k\sum_{j=1}^{N} a_{ij}(x_j(t-\tau) - x_i(t-\tau)) \tag{13}$$

where $k \in R$, time delay $\tau \geq 0$.

From Theorem 1, we get the following result for systems (12)-(13).

Corollary 2 Systems (12)-(13) achieve consensus if and only if the following $N-1$ time-delay subsystems are simultaneously asymptotically stable:

$$\dot{\tilde{\delta}}_i(t) = -\lambda_i(\mathcal{L})k\tilde{\delta}_i(t-\tau), \quad i = 2, \ldots, N. \tag{14}$$

where the definitions of $\tilde{\delta}_i(t)$ are same as Theorem 1.

Due to the simplicity of system (14), we can analyze its stability by using the frequency domain method.

Theorem 3 Systems (12)-(13) can always reach consensus if $k > 0$ and $\tau < \tau_{\max}$, where $\tau_{\max} = \frac{\pi}{2k\lambda_{\max}(\mathcal{L})}$.

Proof: For every time-delay subsystem of (14), its characteristic equation is

$$s + \lambda_i(\mathcal{L})ke^{-\tau s} = 0, \quad i = 2, \cdots, N. \tag{15}$$

We should note that k should stabilize the delay-free system of (14), that is to say, when $\tau = 0$, the following $N-1$ delay-free subsystems are simultaneously stable,

$$\dot{\tilde{\delta}}_i(t) = -\lambda_i(\mathcal{L})k\tilde{\delta}_i(t), \quad i = 2, \ldots, N,$$

i.e., $-\lambda_i(\mathcal{L})k < 0$, $i = 2, \ldots, N$, thus we only need $k > 0$.

By the continuous dependence of the roots for parameter τ, we only need to consider that there is no imaginary axis root and zero root of (15) for $\tau < \tau_{\max}$, where $\tau_{\max}$ is a parameter to be determined.

Note that the complex roots of (15) are distributed symmetrically on the complex plane with respect to the real axis. Thus we only need to consider the case of $s = \iota w_i$ ($\iota^2 = -1$), $w_i \geq 0$, $i = 2, \ldots, N$ for (15). That is to say, we only need to guarantee that the following equations

$$\iota w_i + \lambda_i(\mathcal{L})ke^{-\iota w_i \tau} = 0, \quad i = 2, \cdots, N$$

do not have solutions $w_i \geq 0$, $i = 2, \cdots, N$.

If there is a $i^* \in \{2, \cdots, N\}$ such that

$$\iota w_{i^*} + \lambda_{i^*}(\mathcal{L})ke^{-\iota w_{i^*}\tau} = 0, \tag{16}$$

where $w_{i^*} \geq 0$, then by separating the real part and imaginary part of (16), we get that

$$\lambda_{i^*}(\mathcal{L})k\cos(w_{i^*}\tau) = 0, \tag{17}$$

$$w_{i^*} - \lambda_{i^*}(\mathcal{L})k\sin(w_{i^*}\tau) = 0. \tag{18}$$

From (17) we have $\cos(w_{i^*}\tau) = 0$, so $w_{i^*} \neq 0$ and there must be $w_{i^*}\tau = l\pi + \frac{\pi}{2}$, $l = 0, 1, \ldots$. From (18) we have $\frac{w_{i^*}}{\sin(w_{i^*}\tau)} = \lambda_{i^*}(\mathcal{L})k > 0$, so $\sin(w_{i^*}\tau) > 0$, thus $w_{i^*}\tau = 2l\pi + \frac{\pi}{2}$, $l = 0, 1, \ldots$. That is to say, $k\lambda_{i^*}(\mathcal{L}) = w_{i^*} = \frac{2l\pi + \frac{\pi}{2}}{\tau}$, $l = 0, 1, \ldots$. So we have $\tau = \frac{2l\pi + \frac{\pi}{2}}{k\lambda_{i^*}(\mathcal{L})}$, thus we can obtain a bound of time delay, i.e., $\tau_{\max} = \frac{\pi}{2k\lambda_{\max}(\mathcal{L})}$.

Remark 3 From Theorem 3 we know that by choosing a relatively small (or big) gain k, we can guarantee that systems (12)-(13) achieve consensus when the time delay is big (or small).

6 Simulation

We assume that there are three agents in a network. The adjacency matrix $\mathcal{A} = \begin{bmatrix} 0 & 0.5 & 1 \\ 0.5 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then for first-order integrator system $\dot{x}_i(t) = u_i(t)$, $i = 1, 2, 3$, with the consensus protocol of the ith agent is given by $u_i(t) = k\sum_{j=1}^{3} a_{ij}(x_j(t-\tau) - x_i(t-\tau))$. Through calculation we know that $\lambda_{\max}(\mathcal{L}) = \frac{3+\sqrt{3}}{2}$, then from Theorem 3 we know that this network can reaches consensus if only $\tau \leq \tau_{\max} = \frac{\pi}{(3+\sqrt{3})k}$. When $k = 2$ and $k = 0.2$, the corresponding $\tau_{\max} = \frac{\pi}{2(3+\sqrt{3})}$ and $\tau_{\max} = \frac{5\pi}{3+\sqrt{3}}$, respectively. Fig. 1 and Fig. 2 can display our results.

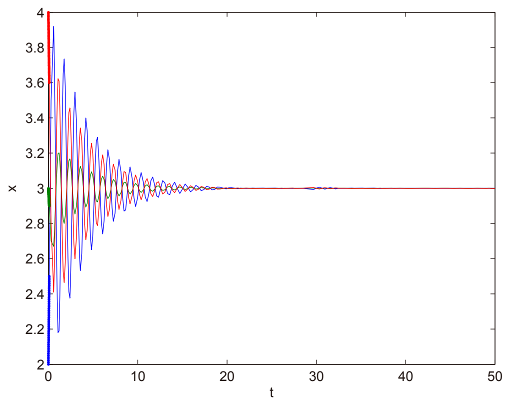

Fig. 1: $k = 2$ and $\tau = 0.3$

7 Conclusion

This paper studied the consensusability of high-order linear MASs with uniform constant communication delay in an undirected network. The consensusability problem of N agents can be turned into simultaneous stability of $N-1$ time-delay subsystems associated with the eigenvalues of Laplacian matrix. We presented a sufficient condition of MASs'

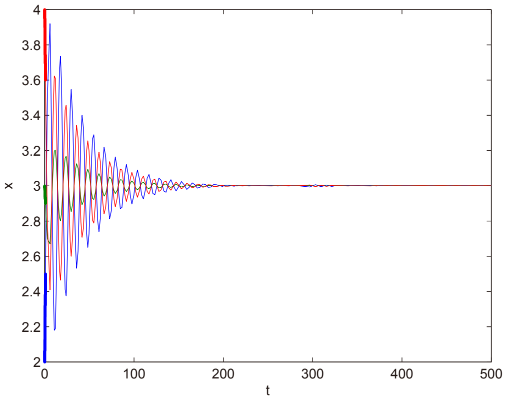

Fig. 2: $k = 0.2$ and $\tau = 3$

reaching consensus in the form of an LMI. We also considered the consensus condition of a class of first-order integrator system using frequency domain analysis and showed that by choosing a relatively small (or big) gain k, we can guarantee that systems achieve consensus when the time delay is big (or small).

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