# AN OVERVIEW OF RECENT RESULTS ON THE PARAMETRIC APPROACH TO ROBUST STABILITY

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#### **ABSTRACT**

This paper discusses recent results relating to the parametric approach to robust stability. The general problem of robust stability is defined and a review of Kharitonov type and edge theorems, zero exclusion results, interval matrix stability, maximal perturbation bounds, multilinear and nonlinear perturbations, time delay systems and numerical/graphical approaches is presented. Some open research directions are indicated, concluding remarks on the future of the approach are given and an extensive list of references is included.

## 1. INTRODUCTION

The fundamental justification for using feedback control is "to enforce good performance of control systems in the presence of uncertainty. Also, feedback is used to enable a process to work in the neighborhood of open-loop unstable operating conditions, that is, to stabilize unstable plants" [84]. Although most systems to be controlled are open-loop stable, the introduction of integrators in the loop (to suppress steady state errors) or more general feedback (to improve dynamic performance) makes the stabilization, in the presence of uncertain parameters and/or delays, very difficult. Contrary to the decade of the 1970s when most control efforts were concentrated on known mathematical models now "control engineers must live with uncertainty and understand that the impact of the level of modeling uncertainty on the design of controllers is crucial" [84]. Thus, questions of stability and stabilization of uncertain systems are among the most important issues in control engineering today. The aim of this paper is to review and discuss recent results on some of these issues, that is, what is now commonly called robust stability. This paper is an extension to [54] which was written for the practicing control engineer; here we give a more thorough and complete technical description of the recent results. The reader interested in tracing the history of robust stability in the parameter space is referred to recent surveys [11,115] and a number of books [2,23,64,107,114,131].

When dealing with a nominal system there are a number of well known methods of verifying stability. These include analytical tests such as the Routh-Hurwitz criterion or Lyapunov methods, as well as graphical tests such as the Nyquist criteria. Ensuring the practical stability of a control system, however, requires taking into consideration not only the nominal system but also all its reasonable (expected) perturbations. In the classical Bode-Nyquist design techniques the goal of practical stability is achieved by introducing uncertainty in gain and phase, leading to the notion of stability (gain and phase) margin. In many applications, however, the uncertain parameters are not the gain and/or phase but some other well defined physical quantities such as time constants, friction coefficients, loads, interconnection gains, chemical reaction rates, time delays, etc.

There are basically two well developed approaches available to problems of robust stability. One uses additive ( $G(s) + \Delta(s)$ )

or multiplicative ( $G(s)[1+\Delta(s)]$ ) perturbations of the nominal system transfer function (c.f. [48,49,65] and references therein) with  $H^\infty$ -norm or other bounds on  $\Delta(s)$ . The other takes into account the structure of the perturbations by assuming that some parameters (usually physical parameters) are known to lie within some bounds or tolerances (c.f. [2,64,114] for examples of uncertainty models in the parameter space). We will focus on the parametric approach since: i) parameter bounds can usually be obtained from physical considerations while it is difficult to find bounds on  $\Delta(s)$ , ii) models of perturbations in the frequency domain which proved successful (e.g. gain and/or phase perturbations) cannot represent parameter perturbation as a special case, iii) in many practical cases, after deriving an appropriate uncertainty model in the parameter space the remaining uncertainty in the frequency domain is negligible; it perturbs the Nyquist plot in a small neighborhood of the origin, iv) the classical gain and phase margin model becomes a special case of a more general uncertainty model in the space of transfer function coefficients, after introducing an uncertain complex gain coefficient.

## 2. PROBLEM DEFINITION AND PRELIMINARY REMARKS

Consider the continuous or discrete time closed-loop system with plant G and controller H. The characteristic equation is:

$$\det(1 + GH) = 0$$
 (1)

This can usually be represented in an equivalent form as:

$$p(r,\alpha) = 0 \tag{2}$$

where p(...) is a function (which we call the "characteristic function") in the vector of variables  $\mathbf{r} = [r_1,...,r_m]$  and the uncertain coefficient vector  $\mathbf{a} = [a_1,...,a_n]$  belongs to some known region of tolerances. The variables  $\mathbf{r}_k$  are functions of one complex variable associated with either the Laplace transform, or the z-transform, for example they may be of the form  $\mathbf{s}^k$ ,  $\mathbf{e}^{-\mathbf{s}T}$ , etc. The general robust stability problem can now be formulated as:

Given a family P of characteristic functions associated with an uncertain system, and a set D in the complex plane, provide computationally tractable techniques for determining the D-stability of P, that is, for checking whether the zeros of the functions in P remain within D.

Typical choices for D are the open left half plane (for continuous-time systems), the open unit disc (for discrete-time systems), or specified subsets thereof. The general problem is very difficult, and additional assumptions are required before concrete results can be given. To better understand some of the difficulties we consider the particular case where the characteristic function (polynomial) of a continuous time system is given by:

$$p(s,a) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0, a_n > 0$$
 (3)

We assume that the uncertain coefficients vary independently in the intervals  $a_i \in [\alpha_i, \beta_i]$ , i = 0,1,...,n. One way of checking the stability (in this case D is the open left half plane) would be to discretize the admissible parameter space and to check the stability of each p(s,a) at all discrete values of  $a_i$ . This is tractable

only when n is small and the discretization is coarse; for example, for n = 3, ten values of  $a_i$  i = 0,1,2,3 would require checking 10,000 polynomials. This approach is clearly not a good way to go, since it would be necessary to check thousands or millions of polynomials, even for small n. Furthermore, the discretization might be too coarse and this approach would not guarantee the correctness of the result. On the other hand, it is obvious, in view of the Hurwitz conditions that: for n = 2 it is sufficient to check whether  $\alpha_0 > 0$  and  $\alpha_1 > 0$ , for n = 3 it is possible to check the positivity of the lower bounds of the coefficients and the inequality  $\alpha_2\alpha_1 - \beta_3\beta_0 > 0$ , but for n > 3 the problem becomes cumbersome. Fortunately, recent results [78] obviate the necessity of pursuing this approach, at least for the p(s) in (3) and for independent coefficient variations.

## 3. RECENT RESULTS ON THE ROBUST STABILITY PROBLEM

In this section we briefly review recent results for special cases of the general robust stability problem.

## 3.1 Interval polynomials and Kharitonov-type results.

For the p(s,a) given in (3) a complete solution has been obtained by Kharitonov [78] under the assumptions that: (K1) P is a so called "interval polynomial", meaning that the coefficient variations are independent, that is

$$\alpha_i \in [\alpha_i, \beta_i], \quad 0 < \alpha_i \le \beta_i, \quad i = 0, ..., n$$
 (4)

(K2) the region D is the open left half plane. He demonstrated that P is D-stable if and only if exactly four polynomials corresponding to specially chosen extreme values of the coefficients are stable. The four polynomials are:

$$p_{\alpha\alpha}(s) = \alpha_0 + \alpha_1 s + \beta_2 s^2 + \beta_3 s^3 + \alpha_4 s^4 + \alpha_5 s^5 + \dots$$

$$p_{\alpha\beta}(s) = \alpha_0 + \beta_1 s + \beta_2 s^2 + \alpha_3 s^3 + \alpha_4 s^4 + \beta_5 s^5 + \dots$$

$$p_{\beta\beta}(s) = \beta_0 + \beta_1 s + \alpha_2 s^2 + \alpha_3 s^3 + \beta_4 s^4 + \beta_5 s^5 + \dots$$

$$p_{\beta\alpha}(s) = \beta_0 + \alpha_1 s + \alpha_2 s^2 + \beta_3 s^3 + \beta_4 s^4 + \alpha_5 s^5 + \dots$$
(5)

A simple way to memorize how these four polynomials are generated is to note the "melody"

$$\ldots, \alpha, \alpha, \beta, \beta, \alpha, \alpha, \beta, \beta, \ldots$$

which defines uniquely the above polynomials.

If the coefficients in (3) are complex (i.e. (K1) is modified such that each coefficient varies independently in a rectangle in the complex plane), Kharitonov [79] has shown that only eight polynomials must be checked. Kharitonov's results make the particular robust stability problem considered above straight-forward and a number of proofs of the theorem are available [6,29,30,33,34,37,92,142].

The type of systems which can be treated using Kharitonov's results is limited by assumptions (K1) and (K2) above. A considerable research effort has gone into attempts to relax these assumptions [9,13,19,25,27,31,36,38,43,50,51,53,55,61,82,85,91,93,97,98,122,128,145 and references therein], that is, to consider other D and/or dependent coefficient variations and/or systems with p(s,a) different than the one in (3). We note that if the system possesses only one unknown coefficient satisfying (4), then (5) reduces to checking two polynomials differing only in that one involves the minimum and the other the maximum value of the unknown coefficient. We refer to this as checking extreme values, and say that a given class of systems will in general admit a Kharitonov-like result if the stability for extreme values of the parameters guarantees robust stability. With this definition it has been shown that:

Neither discrete time [38,62] nor delay systems [55] admit Kharitonov-like results

For some special D regions and/or restricted types of polynomials, however, checking the extreme values may be sufficient [32,50,62,81,97].

We now consider in detail assumption (K1) which is often not satisfied in typical situations. To see this, consider the transfer function:

$$GH(s) = \frac{k(s^2 + s + 5)}{s^3 + 1.4s^2 + 1.4s + 1}$$
 (6)

with uncertain gain parameter  $k \in [0.5,3]$ . The characteristic equation is given by (3) with n=3,  $a_3=1$ ,  $a_1=a_2=1.4+k$  and  $a_0=1+5k$ . Clearly the coefficient variations are linearly dependent, and do not satisfy assumption (K1). On the other hand, the stability is preserved if and only if  $a_1a_2>a_0$  (Note that the positivity of  $a_i$ , i=0,...,3 is assured). Checking this inequality for k=0.5,1 and 3 we conclude that the system is, respectively, stable, unstable and again stable. Hence stability for the extreme values of the uncertain parameter does not guarantee stability for the entire region of uncertainty and a Kharitonov like result can not, in general, be obtained for systems where the coefficient variations are linearly dependent. This seems to indicate that progress in the theory of robust stability to include linearly dependent coefficients is hard to achieve. Fortunately, however, some recent results bring good news.

## 3.2 Polytopes of polynomials and edge results.

Consider the polynomial given in (3) with n=3,  $a_3=1$ ,  $a_2=a_1$  and assume that  $a_1$  and  $a_0$  are bounded by a triangular region. It can be seen, using the Routh - Hurwitz test, that in order to check the stability of the whole triangular region it is sufficient to check the stability of its edges, which reduces the dimensionality of the problem (note that  $\alpha_0 = \alpha_1^2$  defines the boundary of the stability region).

The above example generalizes to the multidimensional case: the so called "Edge Theorem" of Bartlett, Hollot and Lin [19] states that if we are given a polytope in the space of coefficients  $a_0,...,a_n$  then the stability of the whole polytope is equivalent to the stability of its edges. This yields a great reduction in computational complexity; not as dramatic as Kharitonov's result, but still substantial since checking the edges can be executed by checking the root locations of certain easily derivable matrices. More specifically, let

$$P = \{p(s, \underline{\lambda}): p(s, \underline{\lambda}) = \lambda_1 p_1(s) + \dots + \lambda_m p_m(s),$$

$$\lambda_i \ge 0, \sum_{i=1}^m \lambda_i = 1\}$$
(7)

be a general polytope of polynomials. Note that our previous example is also a polytope

$$\{s^3 + (1.4 + k)s^2 + (1.4 + k)s + 1 + 5k \colon k \in [0, 3]\} =$$

$$\{\lambda(s^3 + 1.4s^2 + 1.4s + 1) + (1 - \lambda)(s^3 + 4.4s^2 + 4.4s + 16) \colon \lambda \in [0, 1]\}$$

To check the stability of an edge

$$\{\lambda_1 p_1(s) + \lambda_2 p_2(s) : \lambda_1 + \lambda_2 = 1, \lambda_i \ge 0\}$$

it is necessary and sufficient that  $p_1$  be stable and the matrix  $H_1^{-1}H_2$  (or  $H_2H_1^{-1}$ ) does not have eigenvalues in  $(-\infty,0]$  where  $H_1$  and  $H_2$  are Hurwitz matrices corresponding to  $p_1$  and  $p_2$  respectively [25]. The counterpart of this condition for discrete-time systems can be found in [3,18]. In addition, an attractive graphical test has been found for checking edges [55,56]: it is sufficient to check stability of  $p_1$  (or  $p_2$ ) and then verify whether

$$-\pi < \arg\left(\frac{p_2(j\omega)}{p_1(j\omega)}\right) < \pi \tag{8}$$

for all  $\omega$ . For example, if  $p_1(s) = (s + 1)^2$  and  $p_2(s) = (s - 1)^2$  then the continuous function

$$arg\left(\frac{p_2(j\omega)}{p_1(j\omega)}\right) = -4arctg(\omega)$$

reaches  $-\pi$  at  $\omega=1$  thus indicating instability of some convex combination of  $p_1(s)$  and  $p_2(s)$ . In general, the practical computation can be executed over a finite interval, since for large  $\omega$  the ratio  $p_2(j\omega)/p_1(j\omega)$  approaches a constant value.

We can now ask whether the edge theorem holds for a wider class of systems. The answer is that:

The Edge Theorem holds for the case with a reduced system order (vanishing highest order coefficients) [63], for discrete time systems [3,18,19,44,143] and can be extended to time delay systems [55,56] as well as almost arbitrary D regions (some mild assumptions on D are required [53,55]).

Also the graphical tests for checking the edges extend to these cases [55] where they are of even greater interest since, e.g., no general analytic tests exist for checking stability of time delay systems.

### 3.3 Interval matrices and other state space uncertainty models

State space models with perturbed system matrices are a natural counterpart to uncertainty models based on characteristic polynomials. The corresponding stability problems are much more complicated than the one of a polytope of polynomials because the coefficients of the characteristic function of the system are multilinear functions in the elements of the system matrix. Among the many possible matrix perturbations the simplest are "interval matrices" where it is assumed that all (or some) matrix elements vary independently in prescribed intervals. For this model it would seem reasonable to expect the existence of Kharitonov type results; this expectation led to claims [26,69] of necessary and sufficient conditions for stability of interval matrices in the continuous time [26] and discrete time [69] cases. Unfortunately, both claims are incorrect as pointed out by respectively [12] and [57,77,80,105,110,120,144]. Furthermore, the edge theorem does not extend to a polytope of matrices (or even a hyperectangle) [14].

On the positive side, to check the D stability of a polytope of matrices, results [39,40] indicate that when the dimension of the system matrix is n, it is necessary and sufficient to only check either (2n - 4) or (2n -2) faces depending on the structure of D. Furthermore, Kharitonov type necessary and sufficient conditions have been found for very special classes of matrices [137]. There are also a number of conservative results offering sufficient conditions [1,20, 21, 22, 41, 58, 66-68, 70-76, 83, 86, 87, 89, 90,95,99,109, 111,117,119,138-141,145].

An open research question is whether it is possible to find simple necessary and sufficient conditions for general interval matrices or polytopes of matrices. By simple, we mean conditions which would require either finitely many arithmetic operations or solving finitely many polynomial equations or both.

An interesting direction has been proposed in the case of nonlinear dependence on uncertain parameters [108,130] where a single parameter perturbation has been reduced to checking the root locations of a special matrix. These papers also touch on the two parameter perturbation case.

## 3.4 Maximal perturbation bounds, stability radii and similar approaches

An interesting open area in which results are just beginning to appear [7,8,10,24,28,35,46,51,52,59,60,88,100,102-104,118,121,123,125,126,130,132,141] is that of finding "maximal perturbation bounds." By this we mean finding the maximal value of a scaling factor for the set of uncertain parameters such that robust stability is preserved. More precisely, given a set of uncertain physical parameters  $\mathcal{A}$ , a nominal value  $a_0 \in \mathcal{A}$  and an expansion-contraction transformation described by

$$f(a) = a_0 + k(a - a_0)$$

we seek a maximum value for a real positive k such that the system remains robustly stable  $\forall a \in f(A)$ . The interest here is that in contrast to cases where the perturbation bounds are assumed known, this approach attempts to find the largest set of perturbations for which the system remains D-stable. This may be of particular interest for the robust controller synthesis problem where design parameters need to be chosen. For example, in [52] a closed form description is given for the maximal perturbation bound of Hurwitz stable interval polynomials. For the polytope of polynomials defined in (7) solutions are given in [51,130].

The nature of existing results is that for simple perturbation models (e.g. one dimensional segments in the vector space of uncertain parameters) analytical tests are available which are easily computable. For more general perturbation models, however, the use of optimization algorithms is required to carry out the tests. An open question is whether the optimization problems can be simplified.

#### 3.5 Results based on the "zero exclusion criterion."

The main idea behind the so-called "zero exclusion criterion (or principle)" is the fundamental property that the roots of polynomials are continuous as functions of coefficients. To explain this, consider the robust stability of a continuous-time system with coefficients of the characteristic polynomial depending continuously on a number of independent parameters located within specified intervals. Then the robust stability is guaranteed if (1) a nominal system is stable and, (2) for any admissible values of the parameters, the characteristic polynomial does not have zeros on the imaginary axis. Statement 2 means that no polynomial can assume a zero value at the boundary of the left half-plane (zero exclusion). This principle can be developed further to obtain numerical tests for robust stability. A number of results [9,13,30,42,43,51,133] are based on this criterion. In [9], the zero exclusion criterion is used to generalize the concept of four Kharitonov functions for a polytope of polynomials. This generalization would seem to be of real interest for reducing the computation time when treating cases involving many perturbation parameters. The results of [9] are extended in [13] to include unmodelled dynamics. The issue of robust stability of polytopes of functions which are not necessarily polynomials is addressed in [42] and a more general form of the zero exclusion criterion is developed. Fu [51] proposes an approach based on the zero exclusion criterion for the D-stability of polytopes of polynomials, which provides a closed-form description for the "maximal" size of a polytope of D-stable polynomials. Numerical algorithms are also given in [51] for calculating the maximal size.

### 3.6 Multilinear and nonlinear perturbations

An important class of largely open problems is the multilinear case. This case assumes that the characteristic function is given by (3) where the ai are multilinearly dependent on a set of other parameters (as noted in Section 3.3, the interval matrix model leads to multilinear perturbations). Unfortunately, no efficient results currently exist for treating the general case of multilinear or nonlinear parameter perturbations. Since the multilinear case is often encountered in applications, it represents a major barrier which needs to be overcome through future research on the parametric approach to robust stability.

Only some preliminary results are available. For instance, in [45] it has been shown that under some "shaping conditions" a value set of a polynomial dependent bilinearly on two parameters  $\alpha_1$  and  $\alpha_2$  varying in some intervals is a polygon for each  $s=j\omega$ . This resembles the situation with a linear (polytope) structure with respect to  $\alpha_1$  and  $\alpha_2$ . The reduction of robust stability to root location of a simple matrix [108,130] already mentioned in Sec. 3.3 is another example of an interesting preliminary research direction. Of course, when one wants to try conservative sufficient conditions many of the results for matrix perturbations cited in Sec. 3.3 apply. In addition, there are papers offering sufficient

conditions for perturbations in the coefficients of the characterconditions for perturbations in the coefficients of the characteristic polynomial [101,116,127,136] or exploiting a special structure of the uncertain parameters (perturbed gains, etc., [96,106,113]). In principle, it is possible (by using the zero exclusion principle) to reduce the general problem with polynomial dependence on uncertain parameters to testing positivity of multivariable polynomials. For the latter, the existing tests based on resultants are very complicated and do not simplify easily for particular parameter structures. Perhaps only special numerical approaches may be capable of handling the general nonlinear problems.

### 3.7 Time delay systems

As already indicated, Kharitonov's theorem does not extend to time delay systems [55] but the edge theorem does [55,56]. For some special perturbations Kharitonov-like results can be obtained (see e.g., Theorem 2 in [93], however Theorems 3 and 5 in [93] are invalid as evidenced by the counterexample in [55]). There is little other literature on this subject ([135] offers some sufficient conditions, [15] proposes a numerical approach in the frequency domain, [94] and [146] show destabilizing effects of time delays and other parameters in pole placement design, [147] gives analytic tools to deal with systems of order less or equal to four). This is somewhat understandable since the work for nondelayed systems is far from being completed. On the other hand, there are a number of relatively old results concerning stability independent of time-delays (see [149] and references therein) where algebraic methods were proposed. However, one warning applies: If time delays grow to infinity, system eigenvalues either enter the right half plane or approach the imaginary axis [148].

### 3.8 Numerical and graphical approaches

There are two main directions in this area: one trying to execute reduction of dimensionality offered by the theory), and another working in the complex domain of values of characteristic polynomials or transfer functions. The second direction can be attractively visualized using computer graphics. To compare the two, let us recall a problem of checking the stability of edges (see Sec. 3.2 and compare the graphical test with calculating eigenvalues). The second direction seems to be more effective based on the fact that graphical tests for checking edges extend to time-delay systems [55] whereas many other methods do not (similarly, the frequency domain method in [9] extends to a delayed case [15]). Two recent papers following the first direction are [127,134], whereas frequency domain or value sets are used in [5,15,129]. In [4] a case study is presented involving root locus, value sets, Hurwitz tests and stability boundary testing in the parameter space. Summarizing, the research in numerical approaches is only preliminary and more work is necessary before a reliable software becomes feasible.

## 4. REMARKS AND CONCLUSIONS

The necessity of developing tools to deal with uncertainty in control systems is now widely recognized, and in the last five years the parametric approach to robust stability has been receiving increasing interest. If knowledge of the system justifies assuming independent or linearly dependent coefficient perturbations in the characteristic function, the theory underlying the parametric approach to robust stability is reasonably well developed. Furthermore, conditions under which Kharitonov type results or edge results apply are now fairly clear. In addition, the zero exclusion criterion offers a method of determining the maximal size of the polytope of polynomials for which stability can be achieved. Thus, for a wide class of systems we believe the theory is sufficiently well developed that work can begin on developing efficient software to aid control engineers in incorporating the parametric approach into their analysis and design toolboxes.

The major stumbling block to the extension of the parametric approach to a wider class of systems is the lack of a theory for treating multilinear and/or nonlinear cases. Currently, the only

practical results concern sufficient conditions for limited cases. This is the main area which begs for results and we feel that numerical approaches represent a promising direction.

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