ELEC4410

Control System Design

Lecture 3, Part 1: Introduction to the Principles of Feedback

School of Electrical Engineering and Computer Science
The University of Newcastle

Outline

- A statement of the fundamental nature of the control problem;

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Reference: Control System Design, Goodwin, Graebe & Salgado.

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- This behaviour altering effect of feedback is a key mechanism that control engineers exploit deliberately to achieve the objective of acting on a system to ensure that the desired performance specifications are achieved.

Definition of the control problem

Definition. The central problem in control is to find a technically feasible way to act on a given process so that the process behaves, as closely as possible, to some desired behaviour. Furthermore, this approximate behaviour should be achieved in the face of uncertainty of the process and in the presence of uncontrollable external disturbances acting on the process.

Prototype solution to the control problem via inversion

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  - say that we know what effect an action at the input of a system produces at the output,

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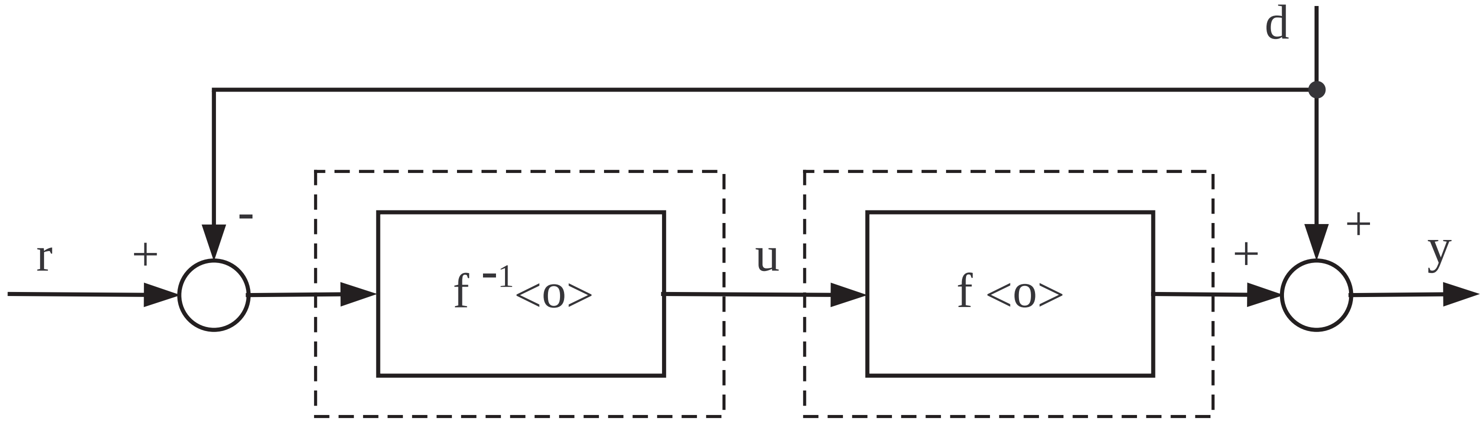
  - say that we have a desired behaviour for the system output, then one simply needs to invert the relationship between input and output to determine what input action is necessary to achieve the desired output behaviour.

Prototype solution to the control problem via inversion

- The above idea is captured in the following diagram:



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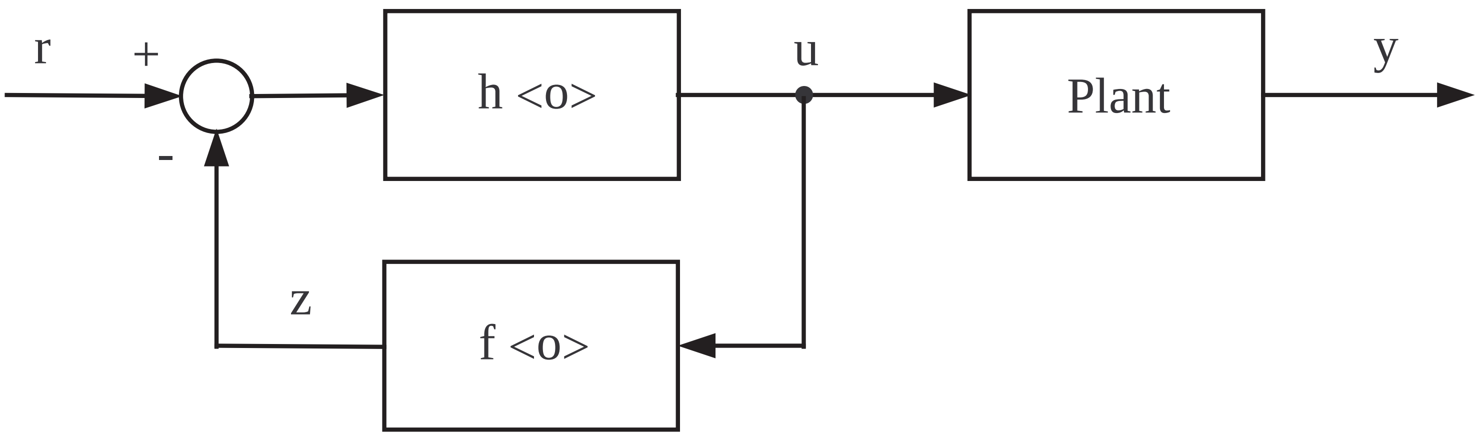
- Thus, all controllers implicitly generate an inverse of the process, in so far that this is feasible. However, the details of controllers will differ with respect to the mechanism used to generate the required approximate inverse.

High gain feedback and inversion

- We next observe that there is a rather intriguing property of feedback, namely that it implicitly generates an approximate inverse of dynamic transformations, without the inversion having to be carried out explicitly.



The loop shown above implements an approximate inverse of $f\langle\circ\rangle$, i.e. $u = f^{-1}\langle r\rangle$, if

$$r - h^{-1}\langle u\rangle \approx r$$

High gain feedback and inversion

- Specifically,

$$u = h\langle r - z\rangle = h\langle r - f\langle u\rangle\rangle$$

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$$\begin{aligned} u &= f^{-1}\langle r - h^{-1}\langle u\rangle\rangle \\ &\cong f^{-1}\langle r\rangle \end{aligned}$$

- Provided $h^{-1}\langle u\rangle$ is small, i.e. if $h\langle\rangle$ is a *high gain* transformation.

High gain feedback and inversion

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- We conclude that an approximate inverse is generated provided we place the model of the system in a high gain feedback loop.

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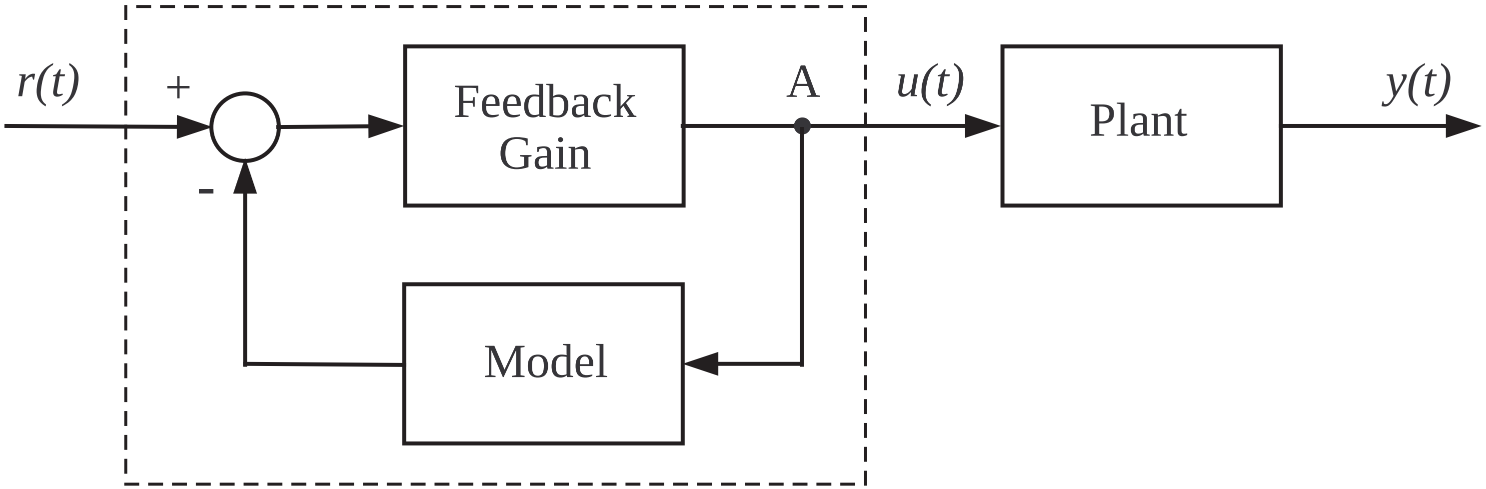
  - the model on which the design of the controller has been based is a very good representation of the plant,
  - the model and its inverse are stable, and
  - disturbances and initial conditions are negligible.
- We are thus motivated to find an alternative solution to the problem which retains the key features but which does not suffer from the above drawbacks.

From open to closed loop architectures

Open loop control with built–in inverse



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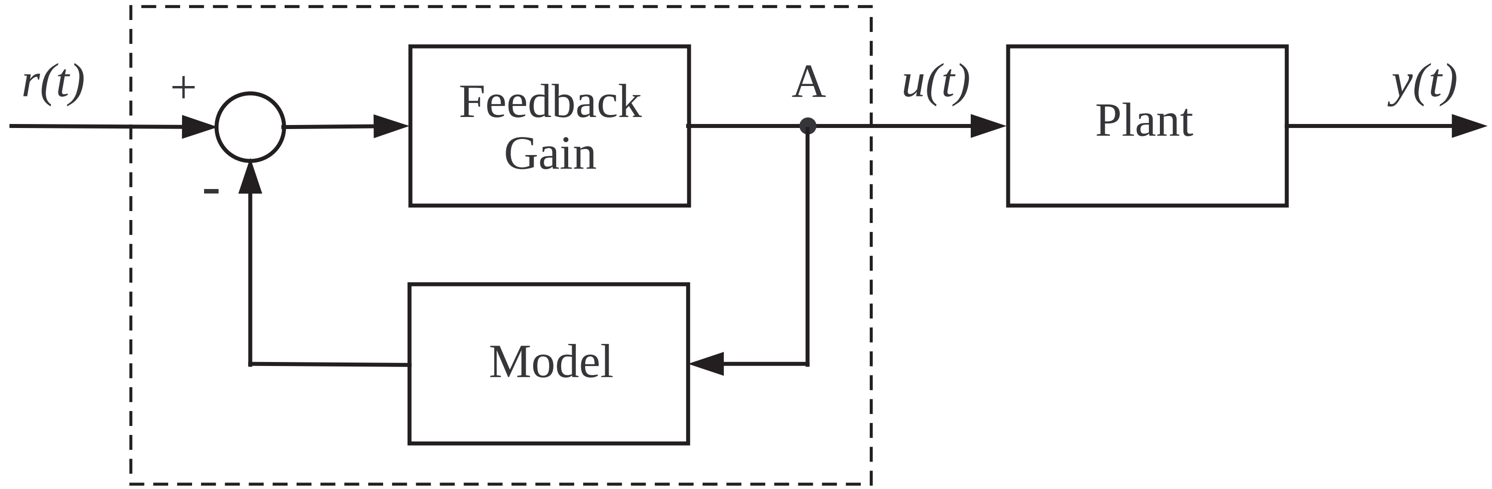
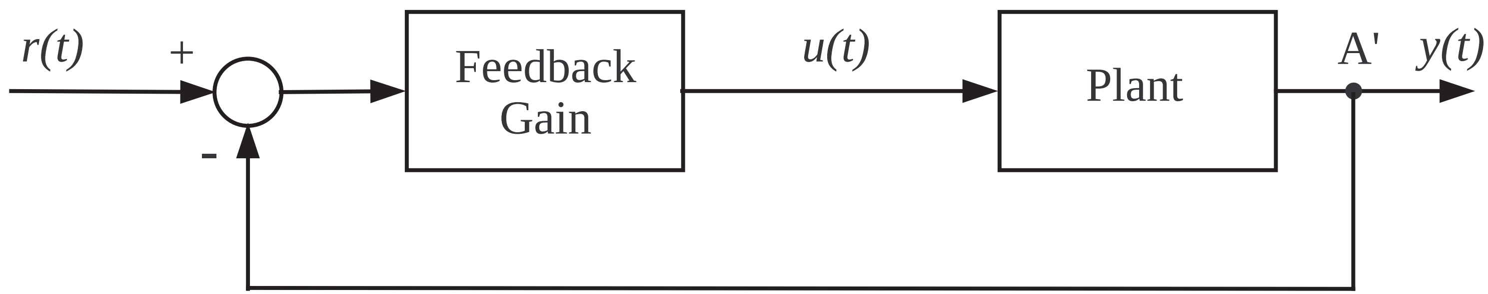


Closed loop control



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- The first thing to note is that, provided the model represents the plant exactly, and that all signals are bounded (i.e. the loop is stable), then both schemes are equivalent, regarding the relation between $r(t)$ and $y(t)$.

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- The key differences are due to disturbances and different initial conditions.
- In the open loop control scheme the controller incorporates feedback internally, i.e. a signal at point A is fed back.

From open to closed loop architectures

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- We will see later that this modified architecture has many advantages including:
  - insensitivity to modelling errors;
  - insensitivity to disturbances in the plant (*that are not reflected in the model*).

Trade-offs involved in choosing the feedback gain

- These preliminary insights would seem to imply that all that is needed to generate a controller is to put high gain feedback around the plant. This is true in so far that it goes. However, nothing in life is cost free and this also applies to the use of high gain feedback.

Trade-offs involved in choosing the feedback gain

- These preliminary insights would seem to imply that all that is needed to generate a controller is to put high gain feedback around the plant. This is true in so far that it goes. However, nothing in life is cost free and this also applies to the use of high gain feedback.
- For example, if a plant disturbance leads to a non-zero error, $e(t)$, then high gain feedback will result in a very large control action, $u(t)$. This may lie outside the available input range and thus invalidate the solution.

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- Instability is characterised by self sustaining (or growing) oscillations.
- As an illustration, you may have witnessed the high pitch whistling sound that is heard when a loudspeaker is placed too close to a microphone. This is a manifestation of instability resulting from excessive feedback gain.

Summary

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- When choosing the feedback gain one needs to make a conscious trade-off between competing issues.