ELEC4410

Control System Design

Lecture 5: Reference & Disturbance Feedforward. Cascade Control

School of Electrical Engineering and Computer Science
The University of Newcastle

Outline

- Review Affine Parameterisation.

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- Reference Feedforward.

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- Disturbance Feedforward.

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Reference: Control System Design, Goodwin, Graebe & Salgado.

Review Affine Parameterisation - Open Loop Stable Model

- All sensitivity functions are affine in $Q(s)$.

$T_o(s) = Q(s)G_o(s)$	Complementary Sensitivity
$S_o(s) = 1 - Q(s)G_o(s)$	Sensitivity
$S_{io}(s) = (1 - Q(s)G_o(s))G_o(s)$	Input Disturbance Sensitivity
$S_{uo}(s) = Q(s)$	Control Sensitivity

Unlike the case of $C(s)$, which is nonlinear in the sensitivity functions, making it difficult to tune $C(s)$ to achieve a desired closed loop performance i.e.

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- The nominal loop is internally stable if and only if $Q(s)$ is a stable and proper transfer function and $C(s)$ is

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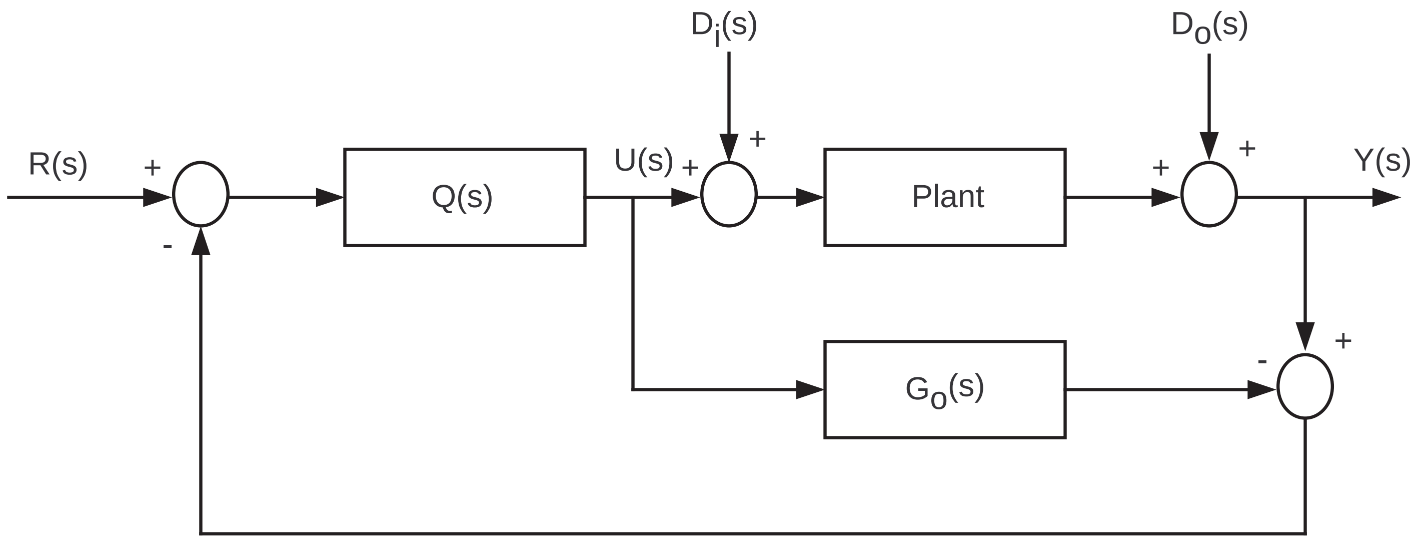
Review Affine Parameterisation - Open Loop Stable Model

Affine Parameterisation in terms of Q



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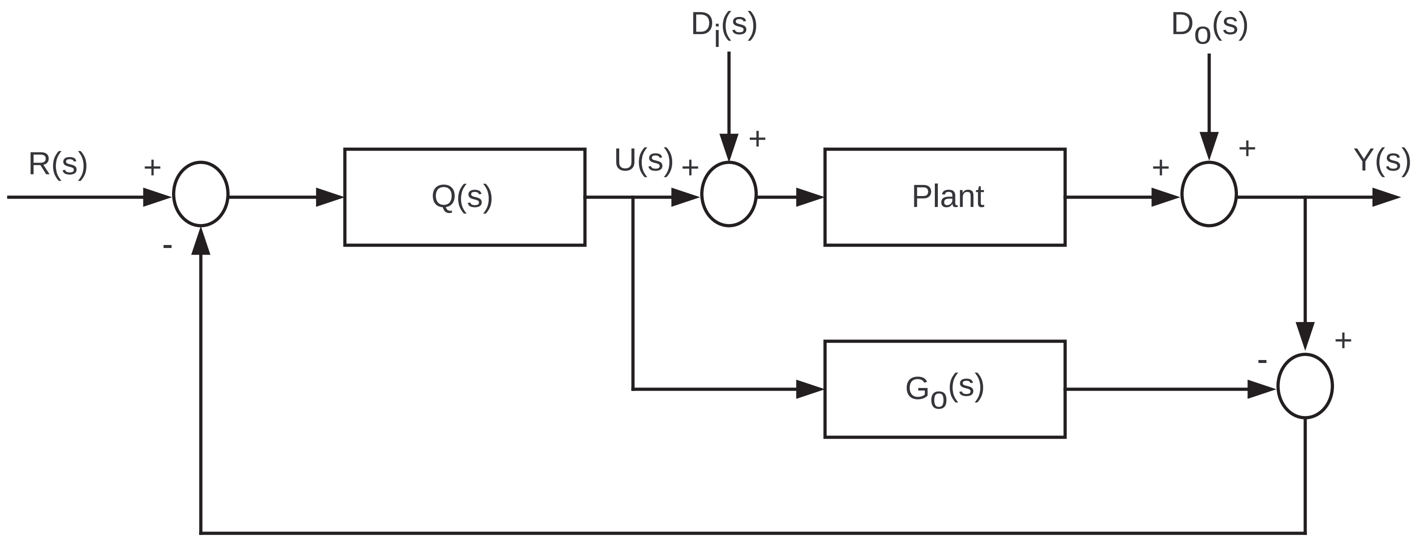

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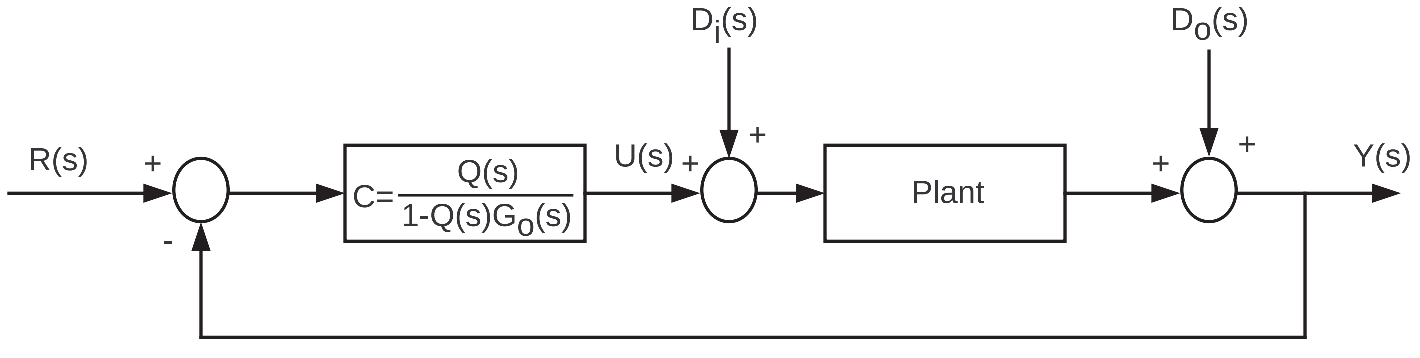

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- We know inversion is a key idea of control.

Review Affine Parameterisation - Open Loop Stable Model

- One way to design $Q(s)$ is

$$Q(s) = F_Q(s)[G_o(s)]^{-1}$$

However, recall, it is not always possible to invert $G_o(s)$ exactly.
Therefore use $G_o^i(s)$ which is a stable approximation to $[G_o(s)]^{-1}$

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- Note that the characteristic equation of $F_Q(s)$ will also be the characteristic equation of $T_o(s)$ (and of $S_o(s)$) if all the stable poles of $G_o(s)$ are included in the approximate inversion.

Review Affine Parameterisation - Open Loop Stable Model

- What about $S_{io}(s)$? We can see that the poles of $G_o(s)$ will appear in it. These poles will only be controllable from the disturbance!

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- What can we do about this? Slow poles in $G_o(s)$ will cause a transient associated with an input disturbance to decay at a rate dictated by these modes. The fix, essentially adding zeros to $S_o(s)$ at the location of the poles in $G_o(s)$ to be cancelled.

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- What about time delay, $e^{-s\tau}$? For small τ, use Padé approximation to model delay. Otherwise use Smith controller design, where $Q(s)$ is based on the rational part of the model only.

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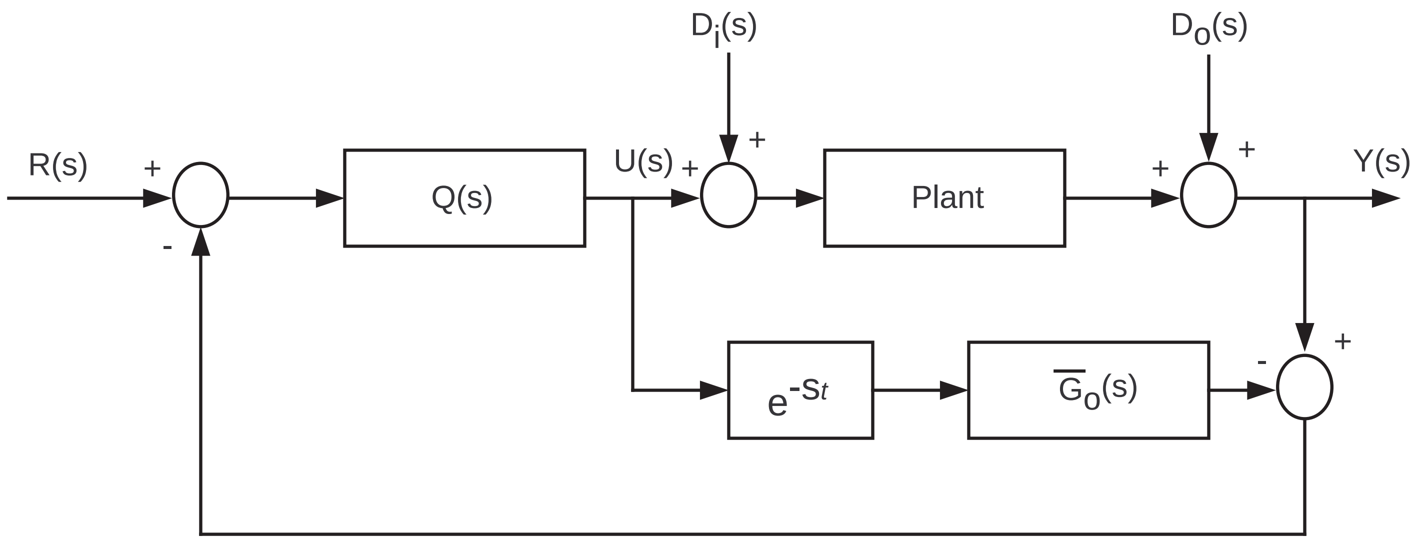
Smith Controller in Q Parameterisation Form



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- For example: if the roots of the characteristic equation are selected to provide a certain amount of relative damping, the maximum overshoot of the step response may still be excessive, owing to the zeros in the closed loop transfer function.
- Reference Feedforward can help reduce this and other effects as we will see.

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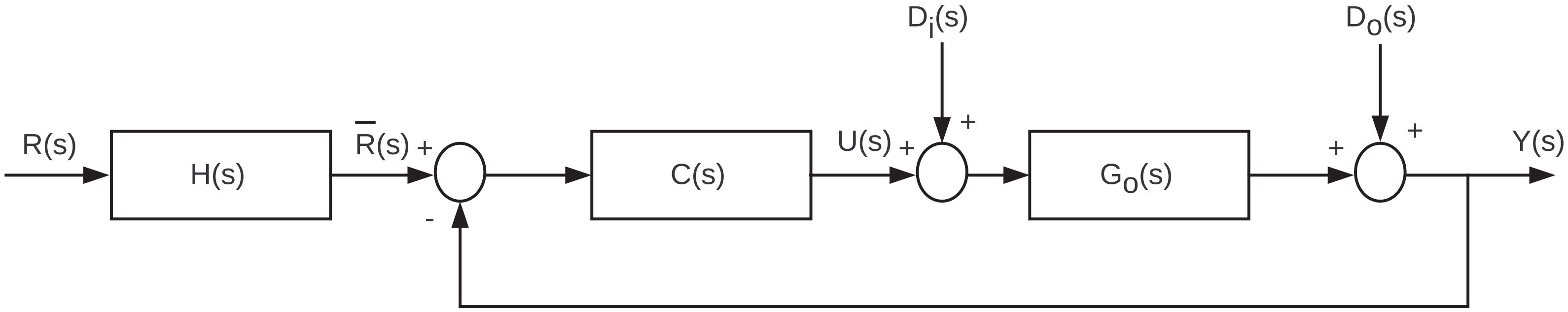

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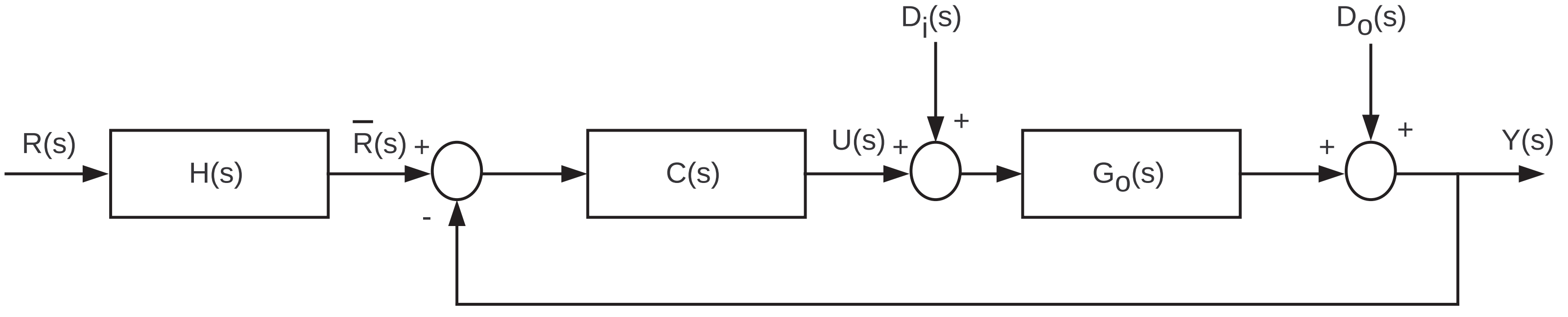

- The Feedforward control element is in the forward path of the feedback loop.

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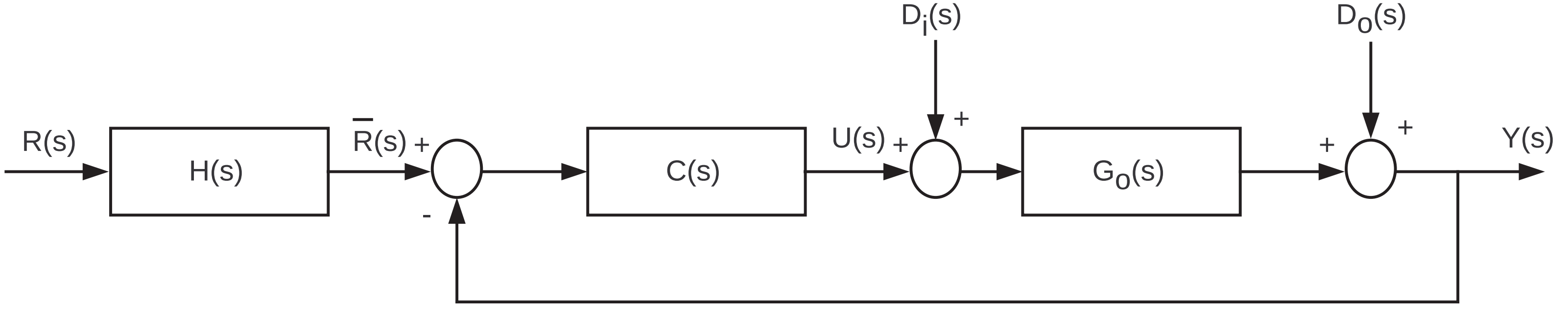

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- Another way of describing the flexibility of the two-degree-of-freedom controller is that the controller $C(s)$ is usually designed to provide a certain degree of system stability and performance, but since the zeros of $C(s)$ always become the zeros of the closed loop transfer function, unless some zeros are cancelled by the poles of the process, these zeros may cause a large overshoot in the system output even when the relative damping as determined by the characteristic equation is satisfactory.

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- In this case and for other reasons, the transfer function $H(s)$ may be used for the control or cancellation of the undesirable poles or zeros of the closed loop transfer function, while keeping the characteristic equation intact.
- Of course we could introduce zeros in $H(s)$ to cancel some of the undesirable poles of the closed loop transfer function that result from the controller $C(s)$.

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- Note that, by this strategy, one can avoid using high gain feedback to bring $T_o(\omega_i)$ to $\mathbf{1}$.
- Note, however, that use of reference feedforward in this way does not provide perfect tracking if there is a change in the model.

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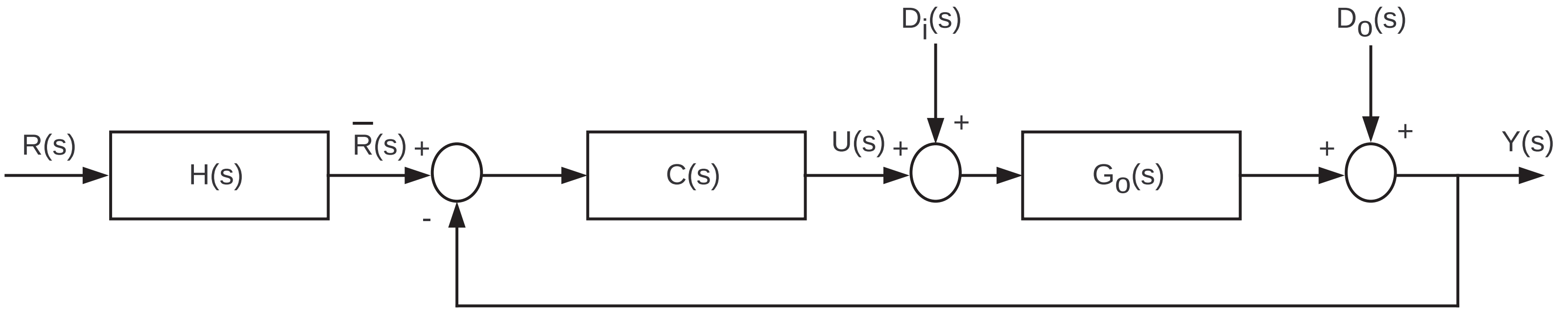

- Recall,

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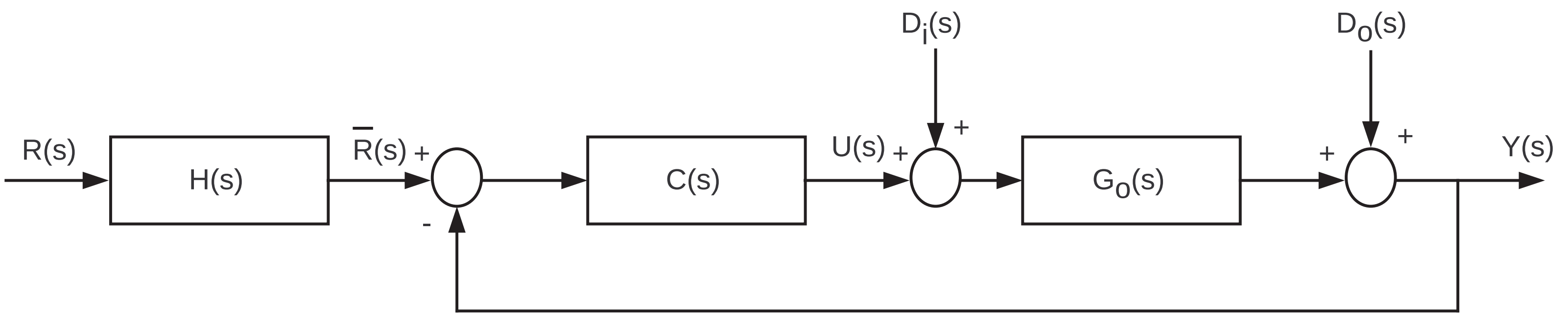

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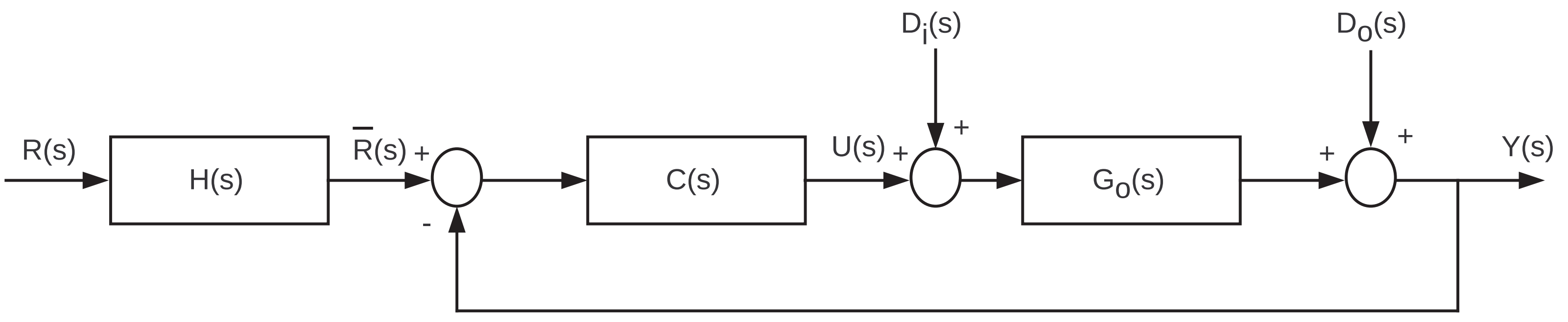

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- Can now shape the sensitivity from R(s) to Y(s) independent of the other sensitivities.

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- Good for set point tracking loops.

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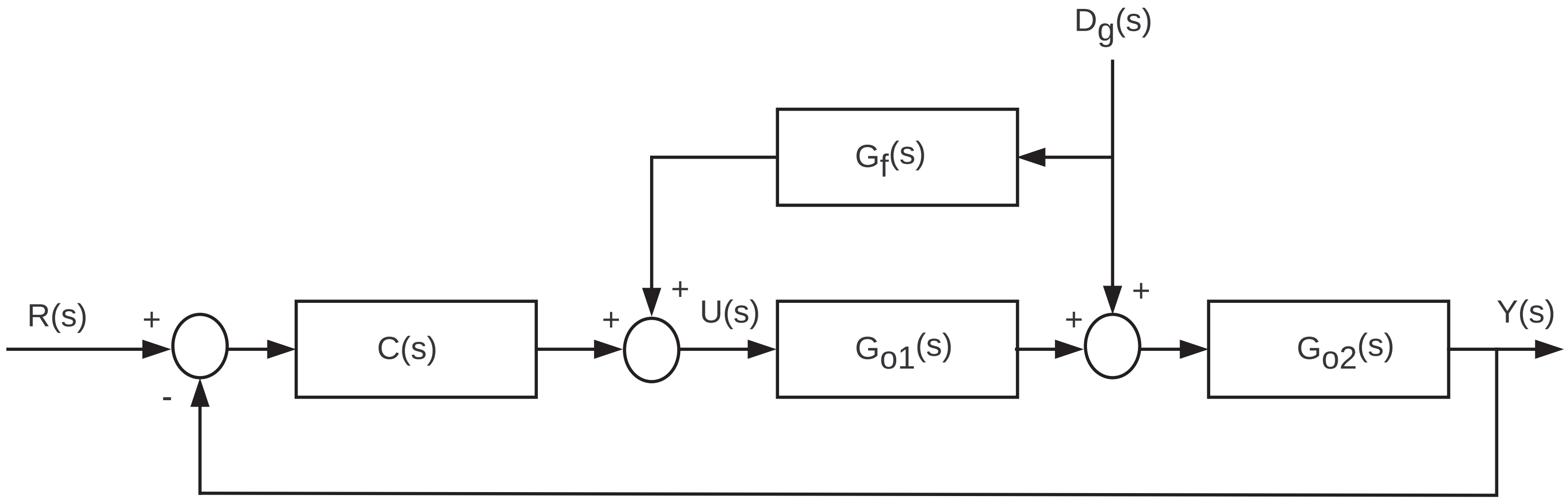
- We want to use the control signal, $U(s)$, to cancel the disturbance, $D_g(s)$, at the point where it enters the process.



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- $G_f(s)$ would be expected to be high pass as $G_{o1}(s)$ will typically possess a low pass characteristic. Therefore will have to include “fast” poles in $G_f(s)$ to make proper.
- Gives more flexibility in the design as trade-offs can be relaxed.

Cascade Control

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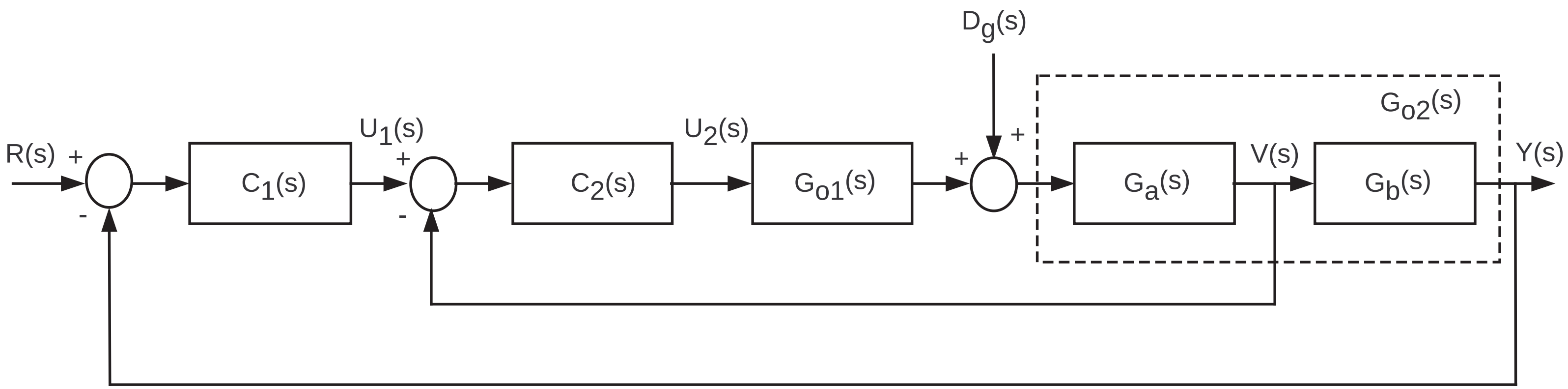

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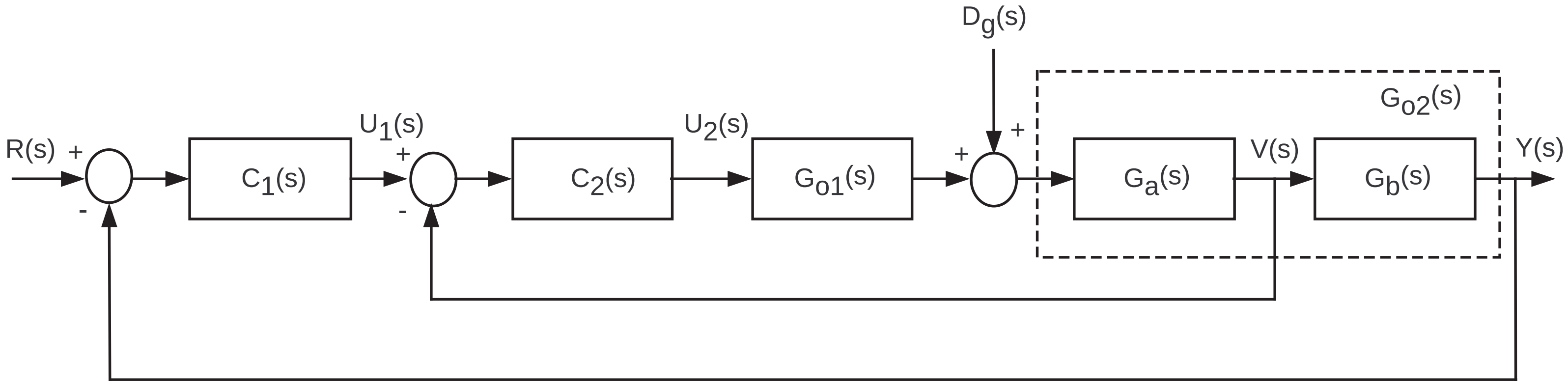

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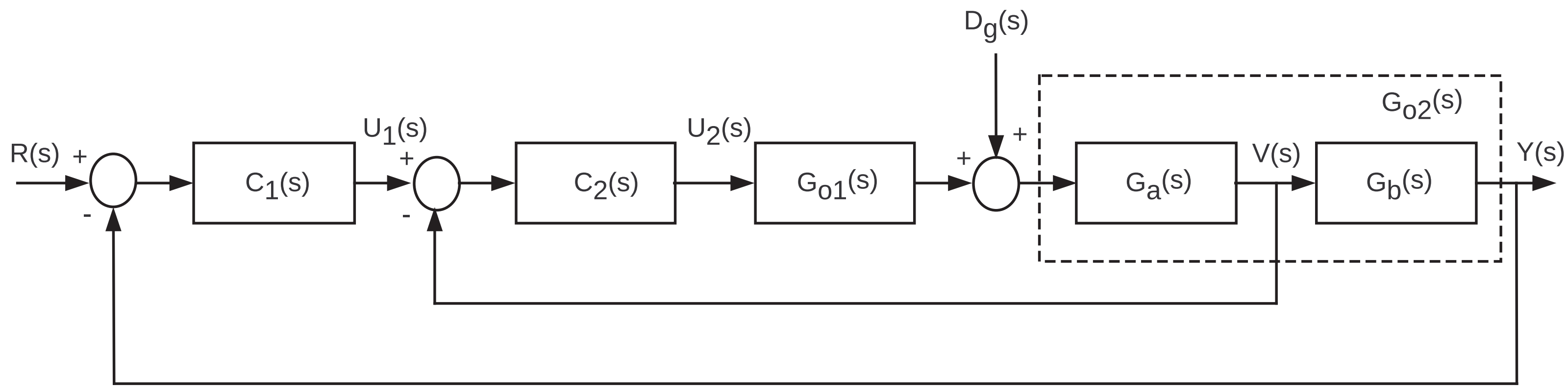

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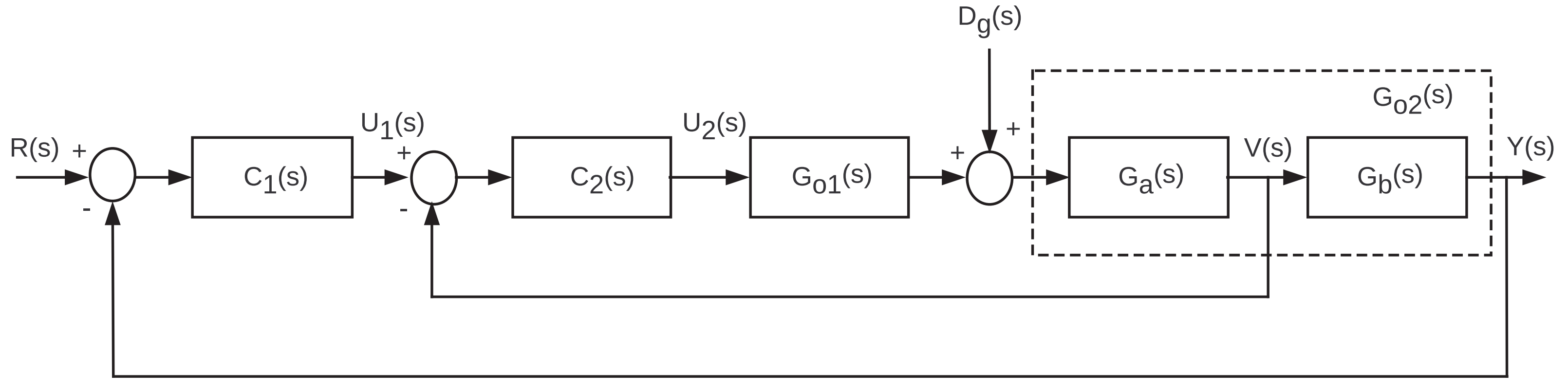

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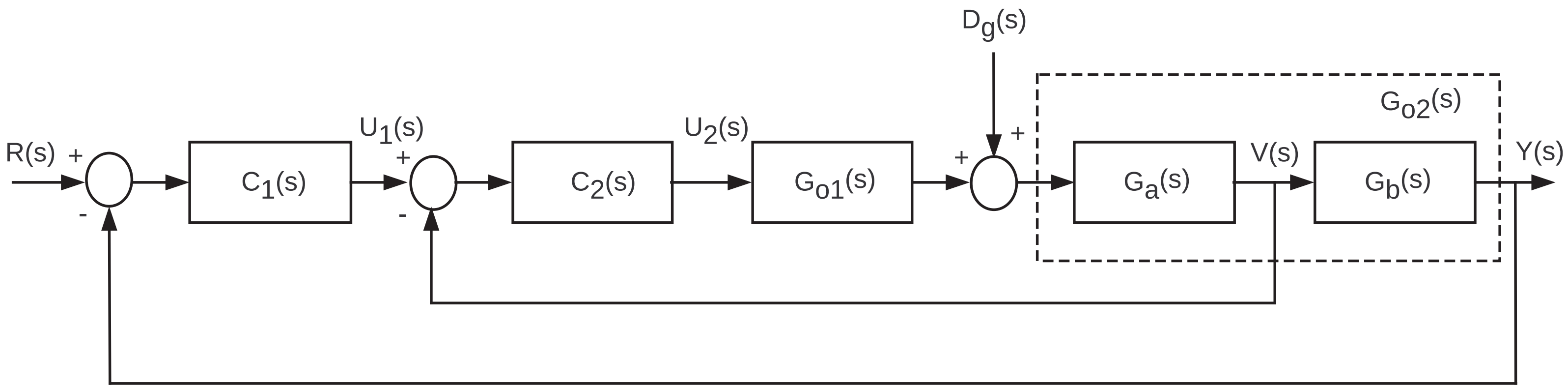

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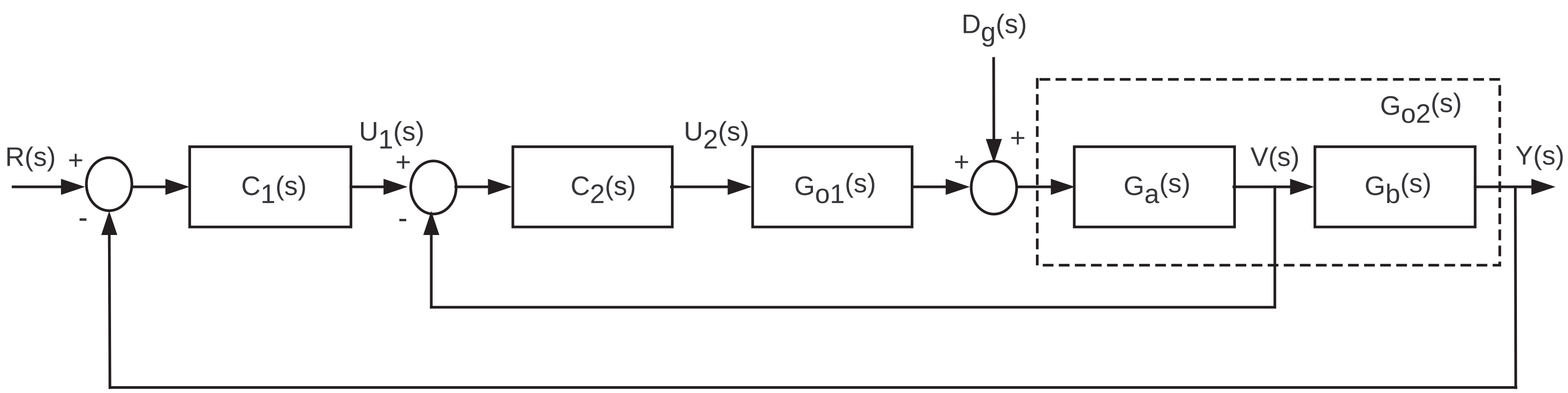

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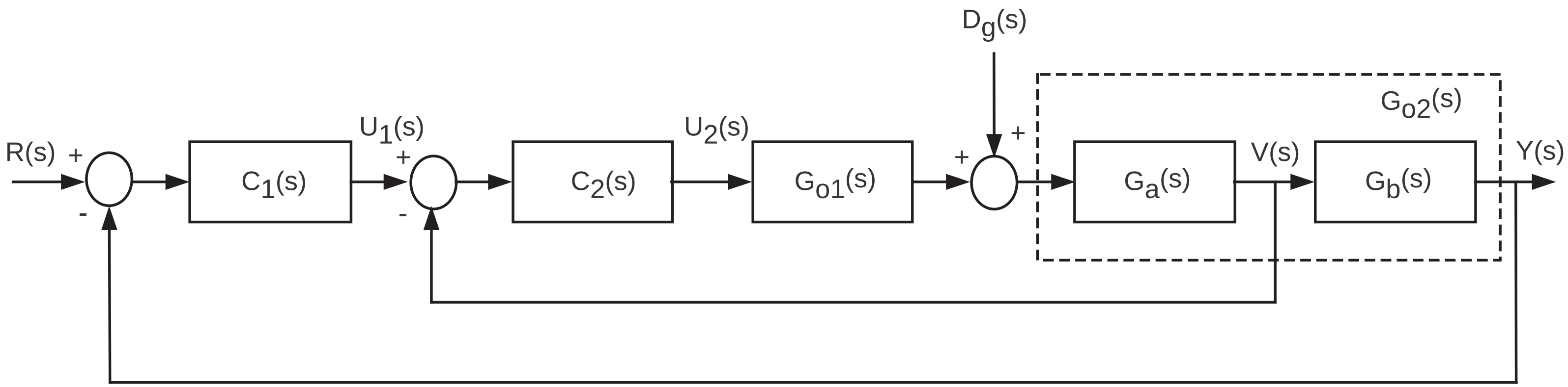
  - $G_a(s)$ contains nonlinearities that limit the loop performance
  - $G_b(s)$ is N.M.P. and / or contains time delays that limit B.W.



Cascade Control

- The output of the system is given by:

$$Y(s) = C_2(s)G_o(s)S_{o2}(s)U_1(s) + G_{o2}(s)S_{o2}(s)D_g(s)$$

$$G_o(s) = G_{o1}(s)G_{o2}(s)$$

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- This can be re-written as:

$$Y(s) = G_b(s)T_{o2}(s)U_1(s) + G_{o2}(s)S_{o2}(s)D_g(s) \tag{3}$$

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- This can be re-written as:

$$Y(s) = G_b(s)T_{o2}(s)U_1(s) + G_{o2}(s)S_{o2}(s)D_g(s) \qquad (5)$$

- If we don't have an inner loop, the output is given by

$$Y(s) = G_o(s)U(s) + G_{o2}(s)D_g(s) \qquad (6)$$

Cascade Control

- The output of the system is given by:

$$Y(s) = C_2(s)G_o(s)S_{o2}(s)U_1(s) + G_{o2}(s)S_{o2}(s)D_g(s)$$

$$G_o(s) = G_{o1}(s)G_{o2}(s)$$

- This can be re-written as:

$$Y(s) = G_b(s)T_{o2}(s)U_1(s) + G_{o2}(s)S_{o2}(s)D_g(s) \tag{7}$$

- If we don't have an inner loop, the output is given by

$$Y(s) = G_o(s)U(s) + G_{o2}(s)D_g(s) \tag{8}$$

- It can be seen in (1) that the disturbance will be somewhat attenuated when compared to (2).

Cascade Control

- The secondary controller is usually designed first.

Cascade Control

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- The primary controller is then designed based on an equivalent plant.

$$G_{eq} \triangleq G_b(s)T_{o2}(s)$$

Cascade Control

- The secondary controller is usually designed first.
- The primary controller is then designed based on an equivalent plant.

$$G_{eq} \triangleq G_b(s)T_{o2}(s)$$

- Generally, the secondary controller is designed such that

$$\text{B.W of } T_{o2}(s) > \text{B.W of } T_{o1}(s)$$