ELEC4410

Control System Design

Lecture 8: Design Limitations in Control

School of Electrical Engineering and Computer Science
The University of Newcastle

Outline

- Sensors & Actuators.

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- Disturbances.

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- Model deficiencies.

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- Structural issues.

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Reference: Control System Design, Goodwin, Graebe & Salgado.

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- Any error, or significant defect, in the measurement system will have a significant impact on performance.

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- Measurement noise is typically dominated by high frequencies.

Conclusion

> Measurement noise usually sets an upper limit on the bandwidth of the loop.

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Maximal Actuator Movement

- Recall that in a one d.o.f. loop, the controller output is given by:

$$U(s) = S_{uo}(s)(R(s) - D_o(s)) \quad \text{where} \quad S_{uo}(s) \triangleq \frac{T_o(s)}{G_o(s)}$$

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- If the loop bandwidth is much larger than that of the open loop model $G_o(s)$, then the transfer function $S_{uo}(s)$ will significantly enhance the high frequency components in $R(s)$ and $D_o(s)$.

Maximal Actuator Movement (Example)

- Consider a plant and associated closed loop given by:

$$G_o(s) = \frac{10}{(s+10)(s+1)} \quad \text{and} \quad T_o(s) = \frac{100}{s^2 + 12s + 100}$$

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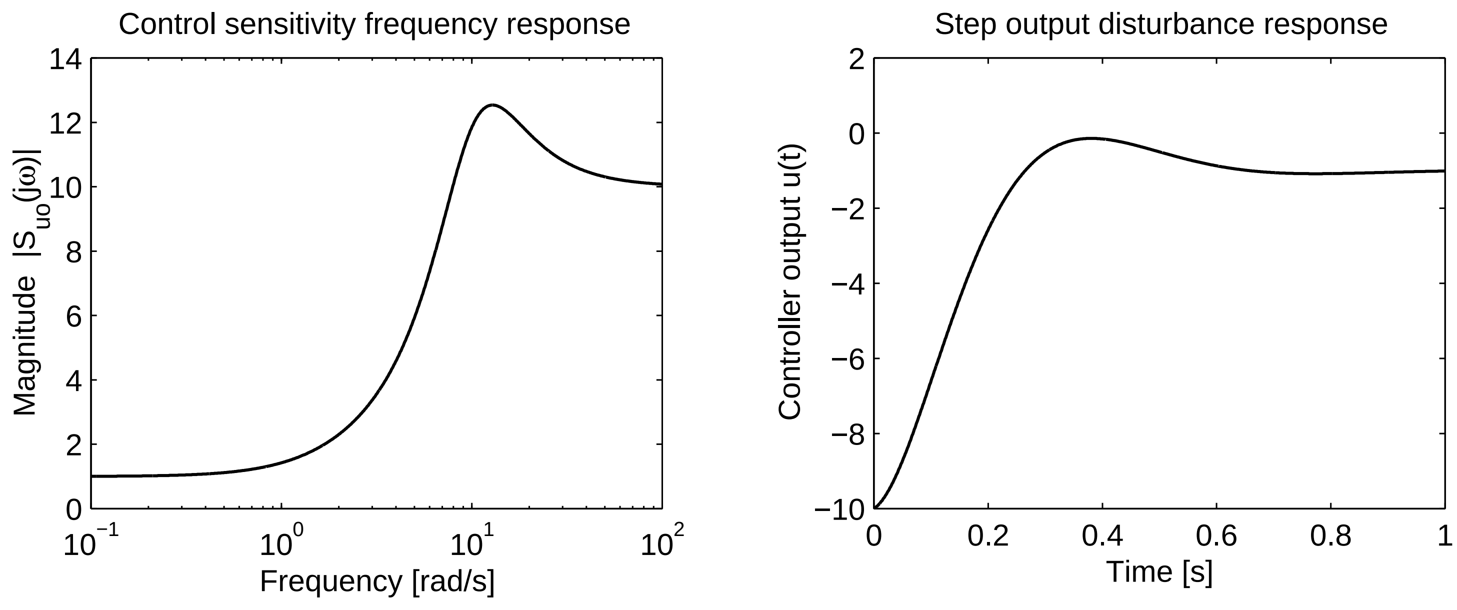
- This will yield a large initial control response in the presence of high frequency reference signals or disturbances.

Maximal Actuator Movement (Example cont.)



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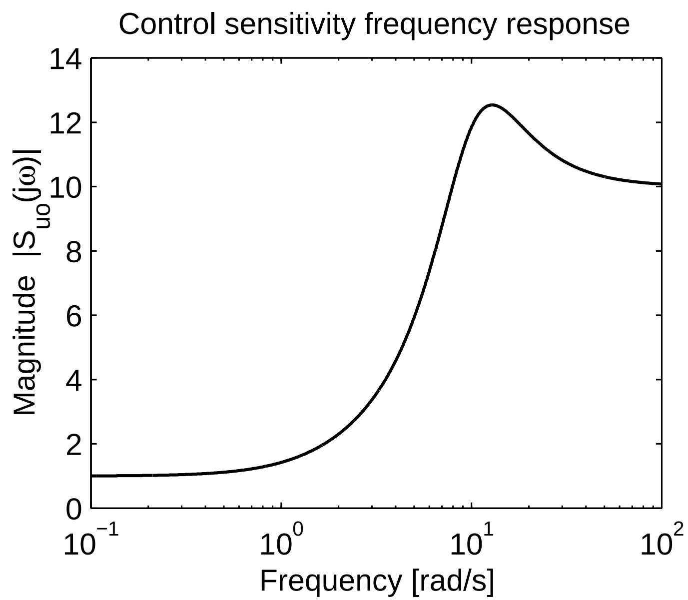

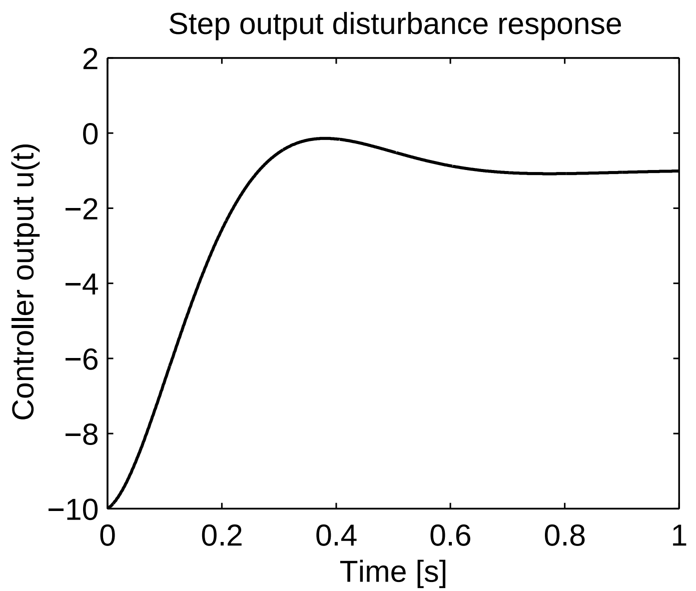

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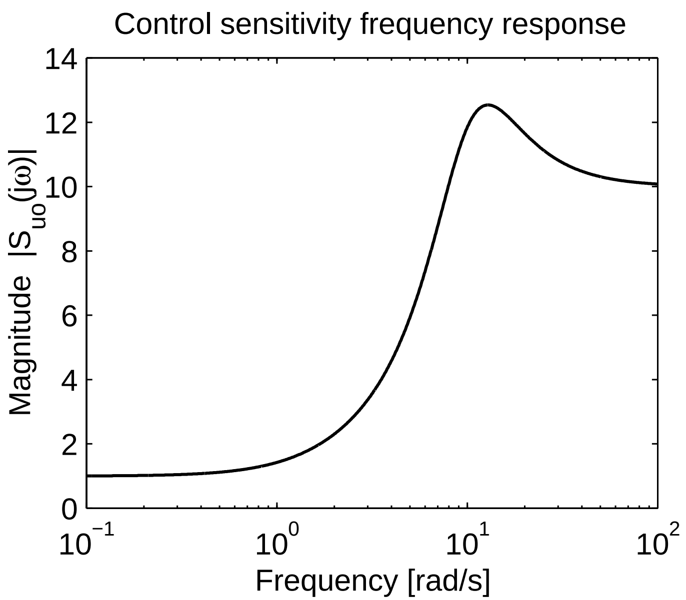

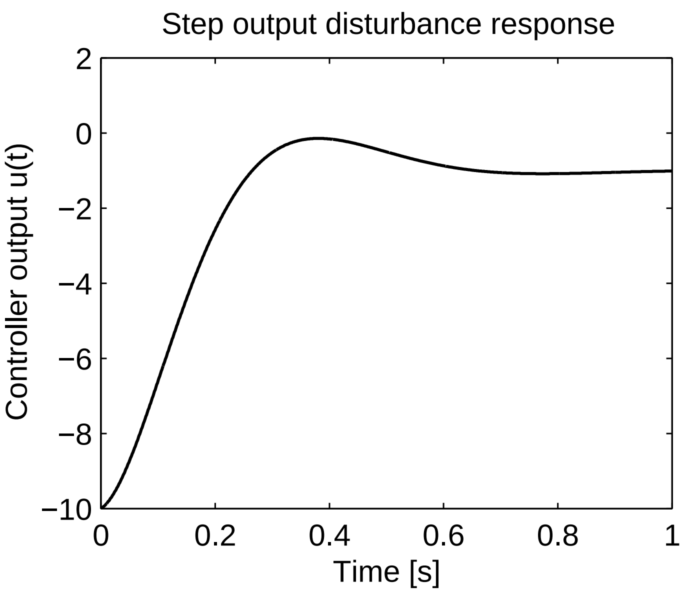

- The L.H. plot shows that the control sensitivity grows significantly at high frequencies.
- The input signal resulting from a unit step disturbance is shown on the right hand plot.
- Note that the initial value of the input is approximately ten times the size of the steady state input needed to cancel the input.

Maximal Actuator Movement

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- Hence, the rate of change of the input is given by

$$sU(s) = S_{uo}(s)[sR(s) - sD_o(s)] = \frac{T_o(s)}{G_o(s)}[sR(s) - sD_o(s)].$$

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- Thus, if the bandwidth of the closed loop is much larger than that of the plant dynamics, then the rate of change of the input signal will be large for fast changes in $r(t)$ and $d_o(t)$.

Conclusion

To avoid actuator saturation or slew rate problems, it will generally be necessary to place an upper limit on the closed loop bandwidth.

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- The manifestations of the problem are usually a self sustaining oscillation produced as the actuator goes through a cycle of sticking, moving, sticking and so on.
- The oscillation frequency is typically at or near the frequency where the loop phase shift is $\mathbf{180}^o$.

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- Then, it is clearly desirable to have small values for $|S_o(j\omega)|$ and $|S_{io}(j\omega)|$ in B_{wi} and B_{wo} respectively.
- Since $G(s)$ is fixed, this can only be achieved provided that $S_o(j\omega) \approx 0$, and hence $T_o(j\omega) \approx 1$ in the frequency band encompassing the union of B_{wi} and B_{wo}.

Conclusion

To achieve acceptable performance in the presence of disturbances, it will generally be necessary to place a lower bound on the closed loop bandwidth.

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- Modeling is normally good at low frequencies and deteriorates as the frequency increases, since then, dynamic features neglected in the nominal model become significant.
- This implies that $|G_\Delta(j\omega)|$ will become increasingly significant with rising frequency.

Conclusion

To achieve acceptable performance in the presence of model errors, it will generally be desirable to place an upper limit on the closed loop bandwidth.

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Structural Limitations - Delays

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- The output sensitivity can, at best, be given by:

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  where τ is the delay.
- To achieve this ideal result requires use of a Smith Predictor plus an ideal controller.

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- This has gain 1 at all frequencies.
- Hence high frequency model errors will lead to instability unless the bandwidth is limited.
- Errors in the delay are particularly troublesome.

Conclusion

1. Delays limit disturbance rejection by requiring that a delay occur before the disturbance can be cancelled. This is reflected in the ideal sensitivity $S_o^*(s)$
2. Delays further limit the achievable bandwidth due to the impact of model errors.

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- We have seen above that delays (where the response does not *move* for a given period) represent a very important source of structural limitations in control design.

- We might then conjecture that non-minimum phase behaviour (where the response initially *goes in the wrong direction*) may present even harder challenges to control system design?

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- We begin by examining the so-called interpolation constraints which show how open loop poles and zeros are reflected in the poles and zeros of the various closed loop sensitivity functions.

Structural Limitations - Open Loop Poles and Zeros

- Recall that the relevant nominal sensitivity functions for a nominal plant $G_o(s) = \frac{B_o(s)}{A_o(s)}$ and a given unity feedback controller $C(s) = \frac{P(s)}{L(s)}$ are:

$$T_o(s) = \frac{G_o(s)C(s)}{1 + G_o(s)C(s)} = \frac{B_o(s)P(s)}{A_o(s)L(s) + B_o(s)P(s)}$$

$$S_o(s) = \frac{1}{1 + G_o(s)C(s)} = \frac{A_o(s)L(s)}{A_o(s)L(s) + B_o(s)P(s)}$$

$$S_{io}(s) = \frac{G_o(s)}{1 + G_o(s)C(s)} = \frac{B_o(s)L(s)}{A_o(s)L(s) + B_o(s)P(s)}$$

$$S_{uo}(s) = \frac{C(s)}{1 + G_o(s)C(s)} = \frac{A_o(s)P(s)}{A_o(s)L(s) + B_o(s)P(s)}$$

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Structural Limitations - Open Loop Poles and Zeros (Effect of Open Loop Integrators)

Lemma. We assume that the plant is controlled in a one d.o.f. configuration and that the open loop plant and controller satisfy:

$$A_o(s)L(s) = s^i(A_o(s)L(s))' \qquad i \geq 1$$

$$\lim_{s\to 0}(A_o(s)L(s))' = c_0 \neq 0$$

$$\lim_{s\to 0}(B_o(s)P(s)) = c_1 \neq 0$$

i.e. the plant-controller combination has i poles at the origin. Then, for a step output disturbance or step set point, the control error, $e(t)$, satisfies

$$\lim_{t\to\infty} e(t) = 0 \qquad \forall i \geq 1$$

$$\int_0^\infty e(t)dt = 0 \qquad \forall i \geq 2$$

Structural Limitations - Open Loop Poles and Zeros (Effect of Open Loop Integrators)

Lemma. (cont.) Also, for a negative unit ramp output disturbance or a positive unit ramp reference, the control error, $e(t)$, satisfies

$$\lim_{t\to\infty} e(t) = \frac{c_0}{c_1} \quad \text{for } i = 1$$

$$\lim_{t\to\infty} e(t) = 0 \quad \forall i \geq 2$$

$$\int_0^\infty e(t)dt = 0 \quad \forall\ i \geq 3$$

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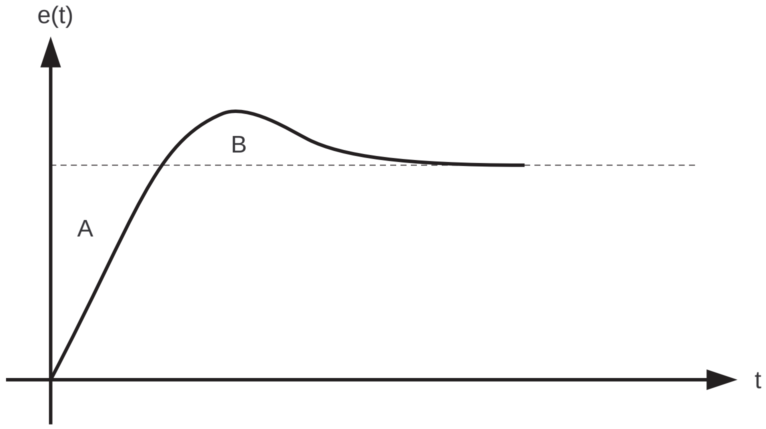
Example: Equal Area Result

Say $G_o(s)C(s)$ contains a double integrator $\Rightarrow$ $S_o(s)$ has a double zero at $s = 0$.

$$\begin{aligned}
\int_0^\infty e(t) &= \lim_{s\to 0} \int_0^\infty e(t)e^{-st}dt \\
&= \lim_{s\to 0} E(s) \\
&= S_o(s)\frac{1}{s} \qquad \text{(for unit step)} \\
&= 0
\end{aligned}$$



A=B!

The above holds for a one d.o.f. feedback control system. Overshoot can actually be avoided if the architecture is changed to a two-degree-of-freedom control system.

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- This, in turn, implies that the error must change sign, i.e. overshoot must occur.
- Thus it is impossible to have zero steady state error to ramp type disturbances together with no overshoot to a step reference.

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In the case of input disturbances, the numerator of $S_{io}(s)$ is $B_o(s)L(s)$ rather than $A_o(s)L(s)$ as was the case for $S_o(s)$. This implies that *integration in the plant does not impact on the steady state compensation of input disturbances*. Thus we need to modify the previous Lemma:

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Lemma. Assume that the controller satisfies:

$$L(s) = s^i(L(s))' \qquad i \geq 1$$

$$\lim_{s\to 0}(L(s))' = l_i \neq 0$$

$$\lim_{s\to 0}(P(s)) = p_0 \neq 0$$

the controller alone has i poles at the origin. Then, for a step input disturbance, the control error, $e(t)$, satisfies

$$\lim_{t\to\infty} e(t) = 0 \qquad \forall i \geq 1$$

$$\int_0^\infty e(t)dt = 0 \qquad \forall i \geq 2$$

Structural Limitations - Open Loop Poles and Zeros (Effect of Open Loop Integrators)

Lemma. (cont.) Also, for a negative unit ramp input disturbance, the control error, $e(t)$, satisfies

$$\lim_{t\to\infty} e(t) = \frac{l_i}{p_0} \quad \text{for} \quad i = 1$$

$$\lim_{t\to\infty} e(t) = 0 \quad \forall i \geq 2$$

$$\int_0^\infty e(t)dt = 0 \quad \forall i \geq 3$$

Structural Limitations - Open Loop Poles and Zeros (More General Effects)

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- The results above depend upon the zeros of the various sensitivity functions at the origin.
- However, it turns out that zeros in the right half plane have an even more dramatic effect on achievable transient performances of feedback loops.
- Hence, we shall develop a series of integral constraints that apply to the transient response of feedback systems having various combinations of open loop poles and zeros.

Structural Limitations - Open Loop Poles and Zeros (More General Effects)

Lemma. Consider a feedback control loop having stable closed loop poles located to the left of $-\alpha$ for some $\alpha > 0$. Also assume that the controller has at least one pole at the origin. Then, for an uncancelled plant zero z_0 or an uncancelled plant pole η_0 to the right of the closed loop poles, i.e. satisfying $\Re\{z_0\} > -\alpha$ or $\Re\{\eta_0\} > -\alpha$ respectively, we have

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(i) For a positive unit reference step or a negative unit step output disturbance, we have

$$\int_0^\infty e(t)e^{-z_0 t}dt = \frac{1}{z_0}$$

$$\int_0^\infty e(t)e^{-\eta_0 t}dt = 0$$

Structural Limitations - Open Loop Poles and Zeros (More General Effects)

Lemma. (cont.)

(ii) For a positive unit step reference and for z_0 in the right half plane, we have

$$\int_0^\infty y(t)e^{-z_0 t}dt = 0$$

Structural Limitations - Open Loop Poles and Zeros (More General Effects)

Lemma. (cont.)

(ii) For a positive unit step reference and for z_0 in the right half plane, we have

$$\int_0^\infty y(t)e^{-z_0 t}dt = 0$$

(iii) For a negative unit step input disturbance, we have

$$\int_0^\infty e(t)e^{-z_0 t}dt = 0$$

$$\int_0^\infty e(t)e^{-\eta_0 t}dt = \frac{L(\eta_0)}{\eta_0 P(\eta_0)}$$

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- The above integral constraints show that (irrespective of how the closed loop control system is designed) the closed loop performance is constrained in various ways.

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- A real stable (LHP) zero to the right of all closed loop poles produces overshoot in the step response.
- A real unstable (RHP) zero always produces undershoot in the step response. The amount of undershoot grows as the zero approaches the origin.
- Any real open loop pole to the right of all closed loop poles will produce overshoot in a one d.o.f. control architecture.

Conclusion

To avoid poor closed loop transient performance:

1. The bandwidth should in practice be set less than the smallest non minimum phase zero.
2. It is advisable to set the closed loop bandwidth greater than the real part of any unstable pole.

Structural Limitations - Open Loop Poles and Zeros (More General Effects)

Example: Effect of different locations of poles and zeros in the loop performance.

Consider a nominal plant model given by

$$G_o(s) = \frac{s - z_p}{s(s - p_p)}.$$

The closed loop poles were assigned to $\{-1, -1, -1\}$. Then, the general controller structure is given by

$$C(s) = K_c \frac{s - z_c}{s - p_c}.$$

Five different cases are considered.

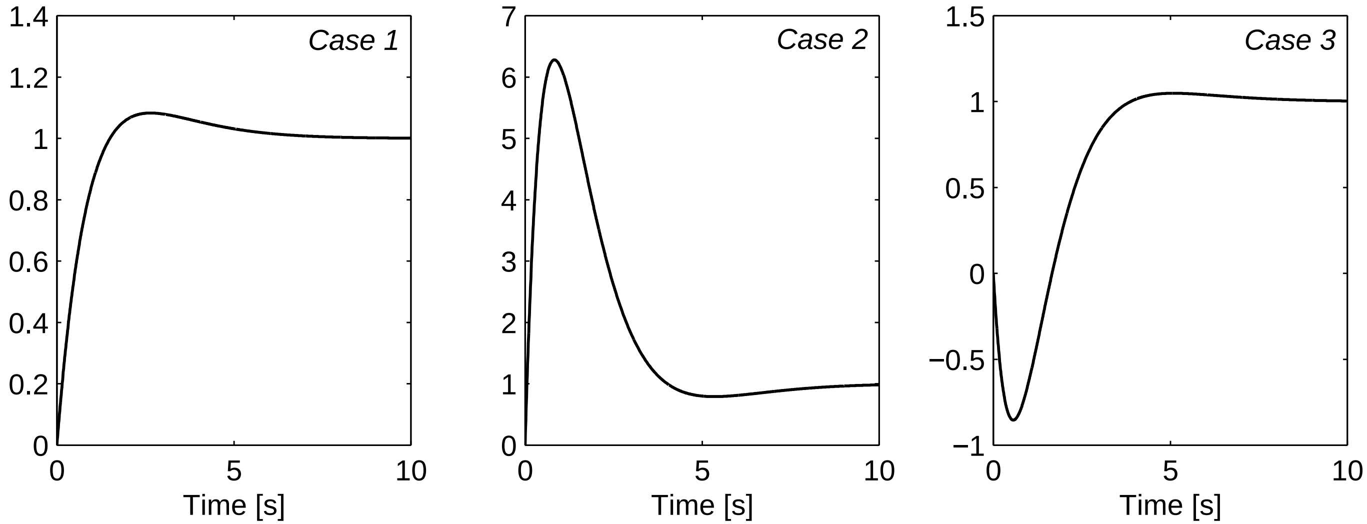
Structural Limitations - Open Loop Poles and Zeros (More General Effects)

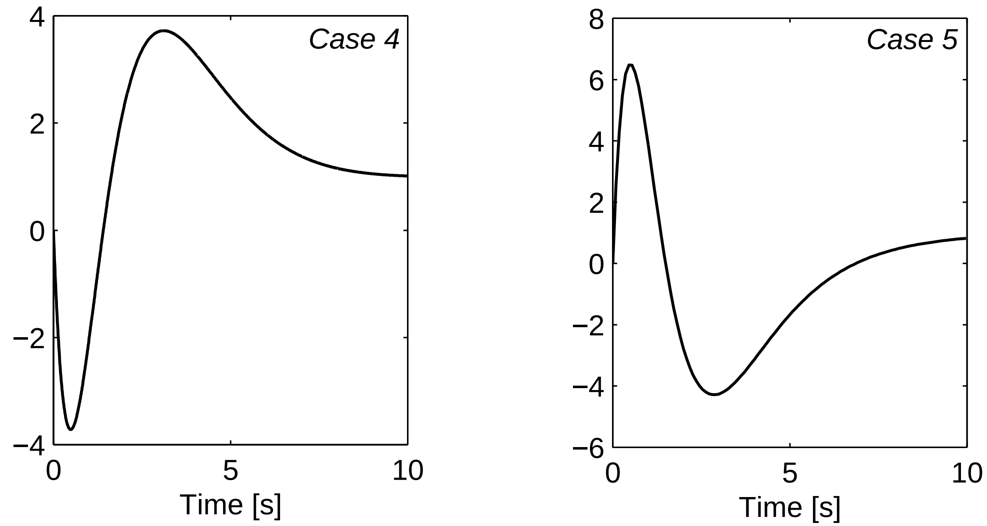
Example: (cont.)

	Case 1	Case 2	Case 3	Case 4	Case 5
	$p_p = -0.2$ $z_p = -0.5$	$p_p = -0.5$ $z_p = -0.1$	$p_p = -0.5$ $z_p = 0.5$	$p_p = 0.2$ $z_p = 0.5$	$p_p = 0.5$ $z_p = 0.2$
K_c	1.47	20.63	-3.75	-18.8	32.5
p_c	-1.33	18.13	-6.25	-22.0	29.0
z_c	-1.36	-0.48	-0.53	-0.11	0.15

Structural Limitations - Open Loop Poles and Zeros (More General Effects)

Example: (cont.) The different designs were tested with a unit step reference and, in every case, the plant output was observed.





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Case 5 (Small unstable zero, large unstable pole) Here the undershoot is produced by the RHP zero and the overshoot by RHP pole. In this case the overshoot is significantly larger than in Case 4, due to the fact that the unstable pole is further into the RHP.

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- This suggests that the one to overcome these limitations is to consider changing the basic architecture of the problem.
- We only examined the effects with respect to a one d.o.f. architecture.

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- With a two d.o.f. controller it is possible to cancel these zeros outside the loop.
- Note that they remain a difficulty inside the loop and thus contribute to design trade-offs regarding robustness, disturbance rejection, etc.

Structural Limitations - Example

Effect of Two Degree of Freedom Architecture on Closed Loop Response with PI Control.

- Consider the feedback control of plant with a PI controller:

$$G_o(s) = \frac{1}{s}; \qquad C(s) = \frac{2s + 1}{s}$$

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- Closed loop poles are $(-1; -1)$ and the controller has a zero at (-0.5).
- If we prefilter the reference by $H(s) = \frac{1}{2s+1}$, then no overshoot occurs in response to a step change in the reference signal.